Parametric Cycle Analysis of Real Turbofan

Introduction

- Parametric cycle analysis of a real turbofan
- The behaviour of a turbofan operating at optimum bypass ratio
Cycle Analysis of Real Turbofan

- Assumptions:
  - Upstream of main burner: perfect gas with constant properties $\gamma_c$, $R_c$, $c_{pc}$, etc.
  - Downstream of main burner: perfect gas with constant properties $\gamma_t$, $R_t$, $c_{pt}$, etc.
  - All components are adiabatic (turbine cooling is neglected)
  - Constant polytropic efficiencies $\eta_f$, $\eta_c$, $\eta_t$ for fan, compressor, and turbine

Real Turbofan

Input: $M_0, T_0, \gamma_c, c_{pc}, \gamma_t, c_{pt}, h_{PB}, \pi_{d\ max}, \pi_B, \pi_R, \pi_{f\ max}, \pi_{f\ max}, \pi_{f\ max}, \eta_f, \eta_c, \eta_t$

Output: $\eta_B, \eta_m, P_b/P_g, P_b/P_{19}, T_{14}, \pi_c, \pi_f, \alpha$
Real Turbofan Equations

\[ R_c = \frac{Y_c - 1}{Y_c} c_{pc} \]
\[ R_t = \frac{Y_t - 1}{Y_t} c_{pt} \]
\[ a_0 = \sqrt{Y_c R_c g_c T_0} \]
\[ V_0 = a_0 M_0 \]
\[ \tau_r = 1 + \frac{Y_c - 1}{2} M_0^2 \]
\[ \pi_r = \tau_r \sqrt{Y_c} \left( Y_c - 1 \right) \]
\[ \eta_r = 1 \quad \text{for} \quad M_0 \leq 1 \]
\[ \eta_r = 1 - 0.075 (M_0 - 1)^{1.35} \quad \text{for} \quad M_0 > 1 \]
\[ \pi_d = \pi_{d \text{ max}} \eta_r \]
Real Turbofan Equations

\[ \tau_\lambda = \frac{c_{pe}T_{ts}}{c_{pc}T_0} \]
\[ \tau_c = (\pi_c)^{(\gamma_c-1)/(\gamma_c\epsilon)} \]
\[ \eta_c = \frac{\pi_c^{\gamma_c-1}/\gamma_c - 1}{\tau_c - 1} \]
\[ \tau_f = (\pi_f)^{(\gamma_f-1)/(\gamma_f\epsilon_f)} \]
\[ \eta_f = \frac{\pi_f^{\gamma_f-1}/\gamma_f - 1}{\tau_f - 1} \]
\[ f = \frac{\tau_\lambda - \tau_c}{\eta_b R_{n}/(c_{pc}T_0) - \tau_\lambda} \]
\[ \tau_t = 1 - \frac{1}{\eta_m(1 + f)} \frac{\tau_c}{\tau_\lambda} \left[ \tau_c - 1 + \alpha (\tau_f - 1) \right] \]
\[ \pi_t = \tau_f \gamma_t/(\gamma_t-1) \gamma_t \]
\[ \eta_t = \frac{1 - \tau_t}{1 - \tau_f \gamma_t^{-1}} \]

Source: "Elements of Propulsion: Gas Turbines and Rockets" by Jack D. Mattingly
Real Turbofan Equations

\[ F = \frac{1}{\dot{m}_0} \frac{a_o}{1 + \alpha g_c} \left[ (1 + f) \frac{V_{t9}}{a_o} - M_0 + (1 + f) \frac{R_t T_9 / T_0}{R_c V_{t9} / a_o} \frac{1 - P_0 / P_9}{\gamma_c} \right] \]

\[ + \frac{\alpha}{1 + \alpha g_c} \frac{V_{t9}}{a_o} - M_0 + \frac{T_{t9} / T_0}{V_{t9} / a_o} \frac{1 - P_0 / P_9}{\gamma_c} \]

\[ (1 + f) \frac{V_{t9}}{a_o} - M_0 + (1 + f) \frac{R_t T_9 / T_0}{R_c V_{t9} / a_o} \frac{1 - P_0 / P_9}{\gamma_c} \]

\[ FR = \frac{V_{t9} - M_0 + V_{t9} / a_o}{V_{t9} / a_o} \frac{1 - P_0 / P_9}{\gamma_c} \]

\[ \eta_p = \frac{2 M_0 [(1 + f) V_{t9} / a_o + \alpha (V_{t9} / a_o) - (1 + \alpha) M_0]}{\frac{1}{\gamma_c} (1 + f) \frac{V_{t9} / a_o} + \alpha (V_{t9} / a_o)^2 - (1 + \alpha) M_0^2} \]

\[ \eta_f = \frac{a_o^2 \left[(1 + f) (V_{t9} / a_o)^2 + \alpha (V_{t9} / a_0)^2 - (1 + \alpha) M_0^2 \right]}{2 g_c f h_{PR}} \]

\[ S = \frac{1}{(1 + \alpha)} \frac{f}{F / \dot{m}_0} \]

\[ \eta_0 = \eta_f \eta_p \]

Cycle Analysis of Real Turbofan

\[ T_0 = 216.7 \text{ K} \quad \pi_{d\text{max}} = 0.98 \quad e_c = 0.90 \quad \text{Baseline} \]

\[ \gamma_c = 1.4 \quad \pi_\text{d} = 0.98 \quad e_f : 0.91 \quad M_0 = 0.9 \]

\[ c_{\text{in}} = 1.004 \text{ kJ/(kg. K)} \quad n_c = 0.998 \quad \pi_f = 2 \]

\[ Y_c = 1.35 \quad \eta_c = 0.99 \quad h_{\text{PR}} = 42,800 \text{ kJ/kg} \]

\[ \eta_{\text{sh}} = 1.096 \text{ kJ/(kg. K)} \quad \eta_{\text{in}} = 0.98 \quad T_{\text{H}} = 1670 \text{ K} \]

Turbofan with Losses: Specific Thrust vs Comp Pressure Ratio

Turbofan with Losses: Fuel-air Ratio vs Comp Pressure Ratio

Source: “Elements of Propulsion: Gas Turbines and Rockets” by Jack D. Mattingly
As $\alpha$ increases, difference in $S$ between real engine cycle and ideal engine cycle increases:

- Due mainly to the much higher $f$ for the "real" engine.
Optimum BPR, a* Turbofan

• Optimum bypass ratio, in a turbofan with losses, that gives the minimum thrust specific fuel consumption

\[ a^* = \eta_m \frac{(1+f)\tau_x(1-\tau_x^*) - \tau_t(\tau_c - 1)}{\tau_t(\tau_f - 1)} \]

\[ \Pi = \frac{(\pi_c \pi_d \pi_e \pi_b \pi_r)(\tau_r - 1)/\tau_x}{1 + \left[ 1 - \frac{\tau_r(\tau_f - 1)}{2\eta_m V_i/V_0 - 1} \right]^{2/\tau_f}} \]

Optimum BPR, a* Turbofan

• An optimum-bypass-ratio turbofan engine has the following characteristics:
  – \( \pi_c \) has very little effect on the specific thrust
  – Increasing the \( \pi_f \) increases the specific thrust
  – \( a^* \) increases with \( \pi_c \) and decreases with \( \pi_f \)
  – Specific fuel consumption decreases with increasing \( \pi_c \)
  – Specific fuel consumption increases with increasing \( \pi_f \)
  – Specific thrust decreases with \( M_0 \) up to a critical value before increasing again.
  – Specific fuel consumption increases with increasing \( M_0 \)
  – Optimum bypass ratio decreases with increasing \( M_0 \)
• $\pi_c$ has very little effect on the specific thrust

• Increasing the $\pi_f$ increases the specific thrust

• $\alpha^*$ increases with $\pi_c$ and decreases with $\pi_f$

Optimum BPR, $a^*$ Turbofan

- Specific fuel consumption decreases with increasing $\pi_c$
- Specific fuel consumption increases with increasing $\pi_f$

Source: Example 7.8
"Elements of Propulsion: Gas Turbines and Rockets" by Jack D. Mattingly
Specific thrust decreases with $M_0$ to a critical value before increasing again.

Optimum bypass ratio decreases with increasing $M_0$.

Specific fuel consumption increases with increasing $M_0$. 
Summary

- Parametric cycle analysis of a real turbofan
  - Consideration for losses
  - More real-world assumptions
  - Comparison with ideal turbofan
- Evaluating a turbofan operating at optimum bypass ratio

Reflection Question

- What is defined as a turbofan engine with optimum bypass ratio, and what are the characteristics of such an engine?