Parametric Cycle Analysis of Real Turbojet

Introduction

- Parametric cycle analysis of a real turbojet
- Parametric cycle analysis of a real turbojet with afterburner
Introduction

• Parametric cycle analysis of ideal gas turbine engine:
  – Engine components are idealized and
  – Working fluid is assumed to behave as a perfect gas with constant $c_p$
  – Analysis of engine performance trends

• Component Performance:
  – Variation of specific heat with temperature and fuel/air ratio
  – More realistic assumptions, e.g. component losses
  – Developed figures of merit of various engine components

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Real Engines

• Parametric cycle analysis of real engines:
  – Equations for different engine cycles
  – Determine their performance
  – Determine the effects of real components with losses

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• Assume one-dimensional flow
• Similar cycle analysis as ideal engine except
  – **Variation of specific heats**: $c_{pc}$ and $c_{pt}$ for upstream and downstream of the combustor respectively
  – Inclusion of **component losses**, 
  – **Mass flow** of fuel through components
  – Pressure at exhaust nozzle is not necessarily atmospheric

**Real Turbojet**

**INPUTS:**

\[ M_0, T_0, \gamma_c, \gamma_{pc}, \gamma_t, \gamma_{pt}, h_{IG}, h_{T_max}, \eta_b, \eta_n, \eta_c, e_c, e_t, \eta_m, P_0/P_a, T_{ts}, \eta_c \]

**OUTPUTS:**

\[ \frac{F}{\dot{m}_0}, f, S, \eta_T, \eta_P, \eta_0, \eta_c, \eta_t \]
Real Turbojet

\[
R_c = \frac{Y_c - 1}{Y_c} \quad c_{pc}
\]

\[
R_t = \frac{y_t - 1}{y_t} \quad c_{pt}
\]

\[
a_0 = \sqrt{y_c R_c g_c T_0}
\]

\[
V_0 = a_0 M_0
\]

\[
\tau_r = 1 + \frac{Y_c - 1}{2} M_0^2
\]

\[
\pi_r = \frac{\tau_r y_c}{\gamma (Y_c - 1)}
\]

\[
\eta_r = \frac{1}{M_0} \quad \text{for } M_0 \leq 1
\]

\[
\eta_r = 1 - 0.075 (M_0 - 1)^{1.35} \quad \text{for } M_0 > 1
\]

\[
\eta_d = \eta_d \max \eta_r
\]
Real Turbojet

\[
\frac{T_b}{T_0} = \frac{T_I T_e}{(P_{\text{out}}/P_0)^{\gamma - 3}/\gamma} \frac{c_{pc}}{c_{pt}}
\]

\[
\frac{V_0}{a_0} = \frac{M_0}{\sqrt{\gamma \pi c T_0}}
\]

\[
F = \frac{a_0}{\rho_0 c} \left[ (1 + f) \frac{V_0}{a_0} - M_0 + (1 + f) \frac{R_2 T_0}{\gamma \pi V_0} \left( \frac{1 - P_0/P_0}{\gamma} \right) \right]
\]

\[
S = \frac{f}{F/\rho_0}
\]

\[
\eta_F = \frac{a_0^2 \left[ (1 + f) \frac{V_0}{a_0} \right]^2 - M_0^2}{2 \rho_0 c f \rho R}
\]

\[
\eta_p = \frac{2 \rho_0 c \eta_p (F/\rho_0)}{a_0^2 \left[ (1 + f) \frac{V_0}{a_0} \right]^2 - M_0^2}
\]

\[
\eta_o = \eta_F \eta_P
\]

Effect of \(e_c\) on Turbojet Cycle

Mission consideration:
Short range interceptor - Low \(\pi_c\) preferred for high \(F/\rho_0\), as compressor size is small and weight is reduced

As \(\pi_c\) increases, the effect of engine losses increases.

Optimum \(\pi_c\) is affected by \(e_c\)

Long-range transport – prefer high \(\pi_c\) to give lower \(S\) for operating efficiency

Source: Example 7.3

"Elements of Propulsion: Gas Turbines and Rockets" by Jack D. Mattingly
**Effect of Nozzle Off-Design Conditions**

R-1, Example 7.3

\[ \pi_c = 16; \, \varepsilon_c = 0.92 \]

Effect of slight exit pressure mismatch is small.

**Fig. 7.6** Effect of nozzle off-design conditions on thrust-specific fuel consumption.

Source: Example 7.3

"Elements of Propulsion: Gas Turbines and Rockets" by Jack D. Mattingly

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**Effect of Speed on Real Turbojet**

- \( \text{\( M_0 = 1 \rightarrow 4 \), \( T_s = 216.7 \, \text{K} \)} \), \( \gamma = 1.4, \, c_p = 1.004 \, \text{kJ/(kg \cdot K)} \), \( \gamma = 1.35 \)
- \( \text{\( c_p = 1.096 \, \text{kJ/(kg \cdot K)} \), \( \text{\( h_{fg} = 42,800 \, \text{kJ/kg} \), \( \rho_{\text{ref}} = 0.98 \), \( \eta_s = 0.98 \)
- \( \pi_s = 0.96, \, \varepsilon_c = 0.89, \, \varepsilon_i = 0.91, \, \eta_l = 0.99, \, \eta_u = 0.98 \)
- \( P_0/P_\infty = 1, \, T_s = 1670 \, \text{K}, \, \eta_s = 8 \) and 24

Note: thrust for real engine falls faster at high \( M_0 \) than \( f \)

**Fig. 7.7a** Specific thrust for two compressor pressure ratios.

**Fig. 7.7b** Fuel/air ratio for two compressor pressure ratios.

Source: Example 7.4

"Elements of Propulsion: Gas Turbines and Rockets" by Jack D. Mattingly

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### Effect of Speed on Real Turbojet

**R-1, Example 7.4**

- $S$ increases exponentially since thrust for real engine falls to zero at high $M_0$ before $f$ does.
- Thermal efficiency at high $M_0$ is zero for real engine but increases in ideal engine.

#### Effects of Component Losses on Performance

<table>
<thead>
<tr>
<th>Performance Parameter</th>
<th>Ideal Turbojet</th>
<th>Real Turbojet</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Thrust, $F/m_0$</strong></td>
<td>Maximum $F/m_0$ occur at a higher $\pi_c$; goes to zero at high $M_0$</td>
<td>Similar variation; Similar magnitude; Maximum $F/m_0$ occur at a lower $\pi_c$ (for same $M_0$); goes to zero at high $M_0$ with lower $\pi_c$</td>
</tr>
<tr>
<td><strong>Thrust Specific Fuel Consumption, $S$</strong></td>
<td>Lower $S$; decreases with increasing $\pi_c$</td>
<td>Higher $S$; decreases to an minimum, then increases with increasing $\pi_c$; increases with velocity, and rapidly towards infinity at high $M_0$</td>
</tr>
<tr>
<td><strong>Fuel/Air Ratio, $f$</strong></td>
<td>Lower $f$</td>
<td>Higher $f$ due to increase in $c_p$, combustor inefficiency and higher combustor exit massflow ($m_0 + m_f$)</td>
</tr>
<tr>
<td><strong>Propulsive Efficiency, $\eta_p$</strong></td>
<td>Lower $\eta_p$</td>
<td>Slightly higher $\eta_p$ due to lower exhaust velocity</td>
</tr>
<tr>
<td><strong>Thermal Efficiency, $\eta_T$</strong></td>
<td>Higher $\eta_T$</td>
<td>Lower $\eta_T$; goes to zero at high $M_0$ for engine with high $\pi_c$</td>
</tr>
<tr>
<td><strong>Overall Efficiency, $\eta_O$</strong></td>
<td>Higher $\eta_O$</td>
<td>Lower $\eta_O$ due to lower $\eta_T$</td>
</tr>
</tbody>
</table>

*Fig. 7.7c Specific fuel consumption for two compressor pressure ratios.*

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"Elements of Propulsion: Gas Turbines and Rockets" by Jack D. Mattingly
Effects of Afterburning

- Afterburner increases both specific thrust and fuel consumption
- Magnitude of increases depend on $\pi_c$
- $\pi_c$ is important in gas turbine engine design
- Mission type dictates $\pi_c$ choice

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Real Turbojet with Afterburner

- Afterburner increases both specific thrust and fuel consumption
- Magnitude of increases depend on $\pi_c$
- $\pi_c$ is important in gas turbine engine design
- Mission type dictates $\pi_c$ choice
Summary

• Concept and methodology of parametric cycle analysis of a real turbojet, without and with afterburning
  • Effects of assumptions and losses on engine performance
  • Effects of afterburning on turbojet output

Reflection Question

• Perform a parametric cycle analysis on a turbojet engine, using the input from Example 7.1 (pages 387 – 389) of the reference, “Elements of Propulsion: Gas Turbines and Rockets” by Jack D. Mattingly.