BANK PO

QUANTITATIVE APTITUDE

UPSC PATHSHALA

HCF and LCM

What will be covered in this chapter?

- Definition of Multiples and Factor, Co Prime **Numbers**
- **Highest Common Factor**
- Methods of finding Highest Common Factor
- Finding HCF of more than two numbers
- Least Common Multiple
- Methods of finding Least common Multiple
- Finding HCF and LCM of Fractions
- Finding HCF and LCM of Decimal Fractions
- Finding Product of two numbers from HCF and **LCM**
- Comparing fractions from LCM

1. DEFINITION

Factor

If a number x divides another number y exactly, then we can say x is the factor of y.

Example: Factors of 12 are 1, 2, 3, 4, 6, and 12.

Multiples

On the other hand, multiple of a number is its table itself.

Example: Multiples of 4 are 4, 8, 12, 16,

Co-primes

Co-primes are those numbers that have only 1 as their common factor. There is no other common factor bewteen them

Example: 4 and 7, 14 and 15

Consider yourself as the leader of a group who has to distribute packets of chips to your group members. Now you have 501 packets of biscuits and 410 packets of chips. You, being a fair leader, want each group member to get the same number of packets of biscuits and chips, else what if a fight breaks out among the group members. But you see that the number of packets of biscuits and chips is different, so how many people in the group can you ensure that they get the same number? You will have to use a simple maths formula, that of calculating the Highest Common Factor of 501 and 410.

2. Highest Common Factor

Highest Common Factor (H.C.F.) or Greatest Common Divisor (G.C.D.) or Greatest Common Measure

The H.C.F. of two or more than two numbers is the greatest number that divides each of the numbers exactly.

3. Methods of calculating the H.C.F. of numbers

• Factorization Method: In this method, all the given numbers are expressed as the product of their prime factors & then calculating the product of its least powers of common prime factors will give H.C.F.

Example:

Find HCF of 36 and 84

Step 1

Expressing 36 as product of prime factors -

 $36 = 2^2 \times 3^2$

Expressing 84 as product of prime factors-

 $84 = 2^2 \times 3 \times 7$

Step 2

HCF = Product of least powers of common prime factors

 $HCF = 2^2 \times 3 = 12$

• Division Method: Divide the larger by the smaller one then; divide the divisor by the remainder. Repeat this process of dividing the preceding number by the remainder till zero is obtained as remainder. The last divisor is our H.C.F.

4. Finding HCF of two or more numbers

To find HCF of more than two numbers, first, find the HCF of two numbers. Then find HCF of the first two numbers and the third number. If HCF of three numbers 'a', 'b', and 'c' has to be found out, first find HCF of 'a' and 'b', let us suppose the HCF of 'a' and 'b' is 'd', the final HCF will be HCF of 'd' and 'c'.

Example:

Find HCF of 513, 1134, and 1215

Step 1

Divide large number by the smaller one-

So HCF of 1134 and 1215 is 81 Step 2 Now find the HCF of 81 and 513

HCF of 513, 1134 and 1215 is 27 which is the last divisor.

Remember the racing competitions in school when there was a competition on who is the fastest runner? Sometimes we used to compete with our friends also and make it a game. Now imagine you and your two other friends,let's name them 'A' and 'B' for reference, start running from the school canteen to the playground and back to the canteen. You completed this round in 3 minutes while your friend A took 4 minutes and B took 2 minutes. Now you all took different times to complete the round, but you all being friends would meet as well after the rounds, so after how much time will you meet at the point where you started i.e canteen? The answer to this question is simple, you have to calculate the 'Least Common Factor' of 3, 4, and 2!

5. Least Common Multiple (L.C.M.)

L.C.M is the least number which is exactly divisible by each one of the given numbers is called their L.C.M.

6. Methods of calculating the L.C.M. of numbers

• Factorization Method: Expressing each one of the given numbers into the product of its prime factors. Then, a product of the highest powers of all the factors gives L.C.M.

Example:

Find LCM of 24, 36 and 40

Step 1

Express each number as a product of its prime factors

$$24 = 2 \times 2 \times 2 \times 3 = 2^{3} \times 3$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^{2} \times 3^{2}$$

$$40 = 2 \times 2 \times 2 \times 5 = 2^{3} \times 5$$

Step 2

Product of its highest powers

$$LCM = 2^3 X 3^2 X 5 = 360$$

 Division Method (short-cut): Arrange the given numbers in a row in any order & divide them by the smallest number which divides at least two of the given numbers exactly and carries forward the numbers which are not divisible. Repeat the same process till no number is further divisible except no 1. The product of the divisors and the undivided numbers is the L.C.M. of the given numbers.

Example:

Find LCM of 16, 24, 36 and 54

Step 1

2	16	24	36	54
2	8	12	18	27
2	4	6	9	27
3	2	3	9	27
3	2	1	3	9
	2	1	1	3

Step 2

LCM = Product of divisors and undivided numbers

 $= 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3$

= 432

7. H.C.F. and L.C.M. of Fractions

- H.C.F. = H.C.F. of Numerators/L.C.M. of Denominators
- L.C.M. = L.C.M. of Numerators/H.C.F. of Denominators

Example:)

Find HCF and LCM of the following fractions 2/3, 8/9, 16/81, 10/27.

HCF of fractions = HCF of 2,8,16,10 / LCM of 3,9,81,27

= 2/81

LCM of fractions = LCM of 2,8,16,10 / HCF of 3,9,81,27

= 80/3

8. H.C.F. and L.C.M. of Decimal numbers

H.C.F and L.C.M of decimal numbers can be calculated by converting the decimal numbers into fractions & then following the same approach of finding the H.C.F and L.C.M of fractions as given above.

Example:

Find HCF and LCM of 0.63, 1.05 and 2.1

Step 1

Convert decimals into fractions 63/100, 105/100, 21/10

Step 2

Finding HCF of fractions = HCF of numerators/ LCM of denominators
= HCF of 63, 105, 21/ LCM of 100, 100,10
= 21/ 100
= 0.21

Finding LCM of fractions = LCM of numerators / HCF of denominators

= LCM of 63, 105, 21 / HCF of 100, 100,10

= 630/100

= .63

9. Product of two numbers from HCF and LCM

Product of two numbers = Product of HCF and LCM

Example:

The product of two numbers is 1320 and their HCF is 6, find the LCM.

1320 = 6 X LCM

1320 / 6 = LCM

LCM = 220

10. Comparision of fractions

Find the LCM of the denominators of the given fractions. Multiply both numerator and denominator by the same number in order to convert the denominator into an LCM number. Compare the numerators, the fraction with the greatest numerator is the greatest fraction.

Example:)

Which of the following fraction is the greatest? 17/18, 31/36, 43/45, 59/60

Step 1

Step 2

Multiply both numerator and denominator by the same number to convert the denominator into LCM.

```
17/18 = 17 X 10 / 18 X 10 = 170/180
31/36 = 31 X 5 / 36 X 5 = 155/180
43/45 = 43 X 4 / 45 X 4 = 172/ 180
59/60 = 59 X 3 / 60 X 3 = 177 /180
```

Step 3

Find the greatest numerator amongst the fractions

Thus since 177 is the greeted numerator, 59/60 is the greatest fraction.

Problems

Type 1

Reducing a fraction to lowest terms

Q1. Reduce 18/27 to lowest term

Ans. Step 1 - Find the HCF of 18,27

HCF of 18 and 27 is 9

Step 2- Divide both the numerator and denominator by HCF

18/27 = 18/9

27/9

= 2/3



It has to be remembered that whenever a fraction has to be reduced to the lowest terms, the 'HCF' of the numerator and denominator has to be found out and not the 'LCM' After finding the HCF, both 'numerator' and 'denominator' have to be 'divided' with the HCF to get the fraction in lowest terms.

Type 2

Finding the greatest possible length

Q2. Find the greatest possible length which can be used to measure exactly the lengths of 126 cm, 162 cm, and 198 cm.

Ans. Step 1

Find the HCF of 126, 162 and 189.

HCF = 18 cm

Pitfalls /



The greatest possible length is always the HCF and not the LCM.

If length is given as a combination of different units eg 7 m 35 cm, then convert into one unit, 735 cm. Calculating HCF when lengths are given in different units will give the wrong answer.

Type 3

Finding the greatest number

Q3. Find the greatest number which on dividing 1657 and 2037 leaves remainder 6 and 5 respectively.

Ans. Step 1

Subtract the remainder from the respective number

1657-6 = 1651

2037 - 5 = 2032

Step 2

Find the HCF of the new numbers so obtained after subtraction if remainder

HCF of 1651 and 2032

Required number =127





The remainder has to be subtracted from the numbers given before finding the HCF.

Type 4

Finding the largest number when the remainder is same

Q4. Find the largest number which divides 62, 132, and 237 when the same remainder is left in each case.

Ans. Step 1

Subtract three numbers from each other

132-62 = 70

237-62 = 175

237-132= 105

Step 2

Find the HCF of the new numbers so obtained

HCF of 175,105,70 = 35

Required number is 35



When no specific remainder is given but mentioned that remainders left are the same, then the numbers have to be subtracted from each other. No remainder has to be assumed in this case.

Type 5

Finding the least number exactly divisible by given numbers

Q5. Find the least number exactly divisible by 15, 20, 24, 32, 36

Ans. Step 1

Find the LCM of 15,20,24,32,36

LCM = 1440

Required number is 1440





The least number exactly divisible is always the LCM, it should not be confused with calculating HCF.

Type 6

Finding the least number when the remainder is given

Q6. Find the least number which when divided by 6,7,8,9, and 12 leaves the same remainder 1 in each case.

Ans. Step1

Find the LCM of 6,7,8,9, 12

LCM 6,7,8,9 12 = 504

Step 2

Add the remainder to the LCM so obtained

Required number = 504 + 1 = 505

Pitfalls /x



It should be remembered that the remainder given has to be added to the LCM, only LCM is not the answer.

Type 7

Finding the largest and smallest number of a particular number of digits

Q7. Find the largest number of 4 digits exactly divisible by 12,15,18,27.

Ans. Step 1

Find the largest 4 digit number

It is 9999

Step 2

Find LCM of 12, 15, 18 27

LCM of 12,15,18, 27 = 540

Step 3

Subtract LCM so obtained from 9999

9999-540= 9720

Required number is 9720





LCM has to be 'subtracted' when the largest number has to be found out.

Q8. Find the smallest number of five digits exactly divisible by 16,24,36,54

Ans. Step 1

Find the smallest five digit number

It is 10000

Step 2

Find LCM of 16,24,36,54

LCM of 16,24,36,54 = 432

Step 3

Divide 10000 by 432 and find the remainder

Remainder = 64

Step 4

Add the difference of LCM and remainder in 10000

Required number = 10000 + (432-64) = 10368

Pitfalls /k



When the smallest number has to be found the 'Difference' of LCM and remainder has to be 'added'.

Type 8

Finding the least number when remainders are given

Q 9. Find the least numbers which when divided by 20,25,35,40 leave 14,19,29,34 as remainder.

Ans. Step 1

Subtract remainders from the respective numbers

20-14 = 6

25-19=6

35-29=6

40-34=6

Step 2

Find the LCM of the numbers

LCM of 20, 25, 35, 40 = 1440

Step 3

Subtract the remainder from the LCM so obtained to get the required number Required number 1440 - 6 = 1394

Pitfalls /K



Remainders have to be subtracted from the numbers first, the L.C.M of the new numbers has to be found out. The difference obtained after subtracting remainders from respective numbers has to be then again subtracted from the LCM to get the required number.

Type 9

Miscellaneous

Q10. Traffic lights at different road crossings change after 48 sec, 72 sec, and 108 sec respectively. If they all change simultaneously at 8:20:00 hours, then at what time will they again change simultaneously.

Ans. Step 1

Find the LCM of 48,72, 108

LCM of 48, 72, 108 = 432 seconds

Step 2

Convert 432 seconds into minutes

432 seconds = 7 minutes 12 seconds

Step 3

Add the time obtained to the time when they change simultaneously

8:20:00 + 7 minutes 32 seconds = 8:27:12

Thus next simultaneous change will be at 8:27:12 hours



Units of measurement should be the same, if crossing time is given in seconds, it should be seconds for all three lights.

Whenever it is asked to find out about the simultaneous meetings of events, LCM always has to be found out.

Practise Questions

LEVEL 1

Q1. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, and 12
seconds respectively. In 30 minutes, how many times do they toll together?
A.8
B.11
C.13
D.16
E.None of these
Q2. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15, and 18
is:
A.6
B.98
C.180
D.364
E.None of these
Q3. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62,
98, and 130 as remainders respectively, is:
A.11115
B.15110
C.15130
D.15310
E.None of these
Q4. The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275,
then the other is:
A.269
B.275
C. 308
D.310
E.None of these

Q5. A, B, and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point? A.15 minutes 15 seconds B.42 minutes 30 seconds C.42 minutes D.46 minutes 12 seconds E. None of these
Q6. The L.C.M. of two numbers is 48. The numbers are in the ratio 2: 3. The sum of the number is: A.30 B.22 C.40 D.60 E.None of these
Q7. The least number, which when divided by 12, 15, 20, and 54 leaves in each case a remainder of 8 is: A.534 B.486 C.544 D.548 E. None of these
Q8. The product of the two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is: A. 124 B.100 C.111 D.175 E. None of these

Q9. The product of the two numbers is 2028 and their H.C.F. is 13. The number of such
pairs is:
A.1
B.2
C.3
D.5
E. None of these
Q10. The least number which should be added to 2497 so that the sum is exactly divisible
by 5, 6, 4, and 3 is:
A.10
B.14
C.23
D.30
E.None of these

Answers and Solutions

1. Answer - D (16)

Explanation: L.c.M. of 2, 4, 6, 8, 10, 12 is 120.So, the bells will toll together after every 120 seconds, i.e, 2 minutes. In 30 minutes, they will toll together 30/2 +1 16

2. Answer D (364)

Explanation: L.C.M. of 6, 9, 15, and 18 is 90.

Let the required number be 90k + 4, which is a multiple of 7.

Least value of k for which (90k + 4) is divisible by 7 is = 4.

Required number $(90 \times 4) + 4364$.

3. Answer- B (15110)

Explanation Here (48 - 38) = 10, (60 - 50) = 10, (72 - 62) = 10, (108 - 98) 10 & (140 - 130) = 10.

Required number = (L.C.M. of 48, 60, 72, 108, 140) - 10 15120- 10 = 15110

4. Answer- c (308)

Explanation: Other number = $[11 \times 7700]/275 = 308$

5. Answer - D (46 minutes 12 seconds)

Explanation: L.C.M. of 252, 308 and 198 = 2772.So, A, B, and C will again meet at the starting point in 2772 see i.e., 46 min. 12 sec

6. Answer c (40)

Explanation: Let the numbers be 2x and 3x.

Then, their L.C.M. = 6x.

So, 6x = 48 or x = 8.

The numbers are 16 and 24.

Hence, required sum = (16 + 24) = 40.

7. Answer- D (548)

Explanation: Required number = (L.C.M. of 12, 15, 20, 54) +8

540 + 8 = 548.

Answer c (111)

Explanation: Let the numbers be 37a and 37b.

Then, $37a \times 37b = 4107$

ab = 3.

Now, co-primes with product 3 are (1, 3).

So, the required numbers are $(37 \times 1, 37 \times 3)$ i.e., (37, 111).

Greater number = 111.

8. Answer B (2)

Explanation: Let the numbers 13a and 13b.

Then, $13a \times 13b = 2028$

ab = 12.

Now, the co-primes with product 12 are (1, 12) and (3, 4).

9. Answer-c (23)

Explanation: L.C.M. of 5, 6, 4 and 3=60. On dividing 2497 by 60, the remainder is

37. Number to be added = (60-37) = 23

LEVEL 2

20 | Quantitative Aptitude

Q1. A rectangular courtyard 140cm long 525cm wide is to be paved exactly with square tiles, all of the same sizes. What is the largest size of the tile which could be used for the purpose? A)64cm B)35cm C)21cm D)28cm E)None of these
Q2. Two containers contain 50 and 125 liters of water respectively. Find the maximum capacity of a container which can measure the water in each container an exact number of times(in liters) A)25 B)11 c)12 D)15 E)None of these
Q3. Two baskets contain 183 and 242 Apples respectively, which are distributed in equal numbers among children. Find the largest number of apples that can be given, so that 3 apples are leftover from the first basket and 2 from the second. A)45 B)40 C)60 D)56 E)None of these
Q4. A person has 3 bars whose lengths are 12,16,24m respectively. He wants to cut the longest possible pieces, all of the same lengths from each of the 3 bars, what is the length of each piece, if he cuts it without any wastage A)12m B)20m

C)6m
D)4m
E)None of these
Q5. Four bells commence tolling together and toll at the intervals of 3,9,12,15 seconds
resp. In 60 minutes how many times they will toll together.
A)20
B)21
C)24
D)30
E)None of these
Q6.In a seminar, the number of participants in Technology, Economics, and Science is
150, 90, and 180 respectively. Find the minimum number of rooms required, wherein each
room the same number of participants are to be seated and all of them being in the same
subject.
A)27
B)32
c)30
D)25
E)None of these
Q7. Naren, Suraj, and Praveen start running around a circular stadium and complete
one round in 12s, 9s, and 15s respectively. In how much time will they meet again at the
starting point?
A)2m
B)2.30m
C)3m
D)3.30m
E)None of these

Q8. Three boxes of lengths 60m,30m, and 45m are to be cut into pieces of equal length.
What is the greatest possible length of each piece?
A)15m
B)18m
C)5m
D)3m
E)None of these
Q9. In a college, all the students can stand in a row, so that each row has 9, 7, and 12
students. Find the least number of students in the school?
A)145
B)265
c)186
D)252
E)None of these
Q10. The HCF of 3 different no is 17, Which of the following cannot be their LCM?
A)540
B)289
c)340
D)425
E)None of these

Answers and Solutions

1. Answer B)35 cm

Solution:

 $30 \text{ cm} = 4 \times 5 \times 7$

 $525 \text{ cm} = 5 \times 5 \times 3 \times 7$

Hence common factors are 5 and 7

Hence LCM = $5 \times 7 = 35$

2. Answer A)25

Solution:

HCF of 50 and 125 = 25

3. Answer - C)60

Solution:

183-3 = 180

142-2= 240

HCF of 240, 180 = 60

4. Answer D)4 m

Solution:

HCF of 12, 16, 24 = 4m

5. Answer - B)21

Solution:

LCM of 3, 9, 12, 15 = 180s = 3m

In 60m= 60/3= 20=> 20+1 = 21

6. Answer C)30

Solution:

HCF of 150, 90 and 180 = 30

No of participants can be seated in each room= 30

7. Answer - C)3 m

Solution: LCM of 12, 9, 15 =180s= 3m

8. Answer- A)15 m

Solution: HCF of 60, 30 and 45m= 15m

9. Answer D)252

Solution: LCM of 9, 7 and 12= 252

10. Answer- A)540

Solution:540 cannot be divisible by 13

LEVEL 3

- Q1. Find the least number which when divided by 2, 3, 4, and 5 leaves a remainder 3. But when divided by 9 leaves no remainder?
- a) 33
- b) 63
- c) 81
- D) 123
- e) None of these
- **Q2.** Find the 4-digit smallest number which when divided by 12, 15, 25, 30 leaves no remainder?
- a) 1020
- b) 1120
- c) 1200
- d) 1800
- e) None of these

Q3. Find the least number which when divided by 12, 27, and 35 leaves 6 as a remainder?
a. 3774
b. 3780
c. 3786
d. 4786
e. None of these
Q4. The HCF and LCM of the two numbers are 84 and 840 respectively. If the first number
is 168, find the second one
a) 420
b) 360
c) 210
d) 480
e) None of these
Q5. Find the last number which when divided by 6, 8, 15 and 30 leaves the remainder 2, 4, 11 and 26 respectively?
a) 124
b) 116
b) 120
d) 134
e) None of these
Q6. HCF and LCM of two numbers are 5 and 275 respectively and the sum of these two
numbers is 80. Find the sum of the reciprocals of these numbers
a) 16/125
b) 32/275
c) 32/125
d) 16/275
e) None of these

Q7. Three buckets contain balloons filled with water. First bucket contains 243 balloons. Second contains 304 balloons and the last bucket contains 127 balloons. Find the largest number of balloons that can be given equally to the children such that 3, 4 and 7 balloons are left in the first, second and third bucket respectively?
a) 20
b) 30
C) 40 d) 60
•
e) None of these
Q8. Riya, Anil and Rishi start running around a circular stadium and complete one round in 15s, 12s and 21s respectively. In how much time will they meet again at the starting point?
a) 6min
b) 7min
d) 8min
d) 9min
e) None of these
Q9. In a college all the students are made to stand in four rows. 4 rows contain 12,8, 22, 30
students respectively. Find the least number of students in the college?
a) 3360
b) 3630
c) 3960
d) 3990
e) None of these
Q10. Find the greatest number that will divide 427 and 900 leaving the remainders 3 and 8
respectively?
a) 2
b) 4
c) 8
d) 12
e) None of these

Answers and Solutions

1. Answer b) 63

Explanation: LCM of 2, 3, 4 and 5 is 30, let number be 30k + 3

put k = 2, we get 63 which is divisible by 9

2. Answer c) 1200

Explanation: LCM of 12, 15, 25 and 30 is 300 least number of 4-digit divided by 300 is 1200

3. Answer-c) 3786

Explanation: Number = LCM (12, 17, 35) + 6 = 3780 + 6 = 3786

4. Answer a) 420

Explanation:

LCM*HCF = a*b 840 *84= 168*b, b = 420

5. Answer b) 116

Explanation : LCM (6, 8, 15, 30) - 4 = 120 - 4 = 116

6. Answer d) 16/275

Explanation: a *b = 5*275 and a+b = 80 (a+b)/(a*b) = 80/(5*275) = 16/275

7. Answer-d) 60

Explanation: HCF (240, 300, 120) = 60

8. Answer b) 7min

Explanation: LCM (15, 12, 21) = 420 second = 7 minutes

9. Answer c) 3960

Explanation: LCM (12, 8, 22, 30) = 3960

10. Answer b) 4

Explanation: HCF (427 - 3, 900 - 8) (424, 892) = 4