University of Gour Banga

(Established under West Bengal Act XXVI of 2007)

N.H.-34(Near Rabindra Bhawan), P.O.:Mokdumpur Dist.: Malda, West Bengal, Pin-732103

M.Sc. in Mathematics

Two Years (Four Semesters) Syllabus

Main Feature of the Syllabus

M.Sc. in Mathematics

Semeste	Paper	Paper Name	Marks	Time
I	MC 101	Abstract Algebra	50	2.00 Hr
	MC 102	Linear Algebra	50	2.00 Hr
	MC 103	Real Analysis	50	2.00 Hr
	MC 104	Ordinary Differential Equations & Special Functions	50	2.00 Hr
		Continuous Evaluation (Class Tests & Attendance)	(4 × 10) +	2.00 Hr
Total			**250**	
II	MC 201	Calculus of Several Variables	50	2.00 Hr
	MC 202	Complex Analysis	50	2.00 Hr
	MC 203	Topology	50	2.00 Hr
	MC 204	Partial Differential Equations	50	2.00 Hr
		C Programming & Seminar Presentation	30+20=50	2.00 Hr
Total			**250**	
III	MC 301	Functional Analysis	50	2.00 Hr
	MC 302	Classical Mechanics	50	2.00 Hr
	MS 303	Special Paper - 1(I)	50	2.00 Hr
	MS 304	Special Paper - 2(I)	50	2.00 Hr
		C Programming & Seminar Presentation	30+20=50	2.00 Hr
Total			**250**	
IV	MC 401	Numerical Analysis	50	2.00 Hr
	MC 402	Mathematical Methods	50	2.00 Hr
	MS 403	Special Paper - 1(II)	50	2.00 Hr
	MS 404	Special Paper - 2(II)	50	2.00 Hr
		Seminar Presentation & Grand Viva-Voce	30+20=50	2.00 Hr
Total			**250**	
Grand Total			**1000**	

Special Papers for Semester III & IV:

Two sets of Special Papers are to be chosen from the following list at the beginning of Semester-III. The Special Paper-1(II) and Special Paper-2(II) in Semester-IV refer to the corresponding part of the Special Paper-1(I) and Special Paper-2(I) of Semester -III respectively.

- ADVANCED TOPOLOGY I & II
- ADVANCED FUNCTIONAL ANALYSIS I & II
- ALGEBRAIC TOPOLOGY I & II
- DIFFERENTIAL GEOMETRY I & II
- ADVANCED REAL ANALYSIS I & II
- ADVANCED COMPLEX ANALYSIS I & II
- MEASURE AND TOPOLOGY I & II
- OPERATOR ALGEBRA I & II
- OPERATOR THEORY I & II
- TOPOLOGICAL GROUPS AND HARMONIC ANALYSIS I & II
- OPERATIONS RESEARCH I & II
- FLUID MECHANICS I & II

Detailed Syllabus

The duration of P.G. course of studies in Mathematics of University of GourBanga consists of two years with Semester-I, Semester-II, Semester-III and Semester-IV each of six months duration leading to Semester-I, Semester-II, Semester-III and Semester-IV examinations in Mathematics at the end of each semester. Syllabus for P.G. courses in Mathematics is hereby framed and revised following the guidelines of U.G.C. according to the following schemes and structures. All students admitted to P.G. course in Mathematics shall take courses of Semester-I, Semester-II, Semester-III and Semester-IV.

Scheme:

Total Marks = 1000 with 250 Marks in each semester comprising of four papers in each semester with 50 marks in each paper. Out of the total marks 20% Marks is allotted for Continuous Evaluations, Seminar Presentations and Viva-Voce etc.

In Semester-III and Semester-IV, the first two papers are common and the next two papers are special/optional papers.

In Semester-III and Semester-IV, the Department will offer a cluster of special/optional papers and the students will have to choose two according to the norms to be decided by the Department in each year. At the end of Semester-II the students will opt for the special papers and the norms of the distribution of students in each special paper will be decided by the Department according to their percentage of marks in Honours. However, the distribution of reserved candidates to the special papers will be done equally, as far as practicable, to all the special papers.

The papers of Seminar presentation in Semester-III and IV will be related to the papers prescribed in syllabus where the students will deliver a seminar presentation containing 30 marks (Semester III and IV)and 20 (Semester IV) marks for Grand Viva-Voce and the topic of the seminars will be decided by the Department and number of students will be evenly distributed to the faculty members as far as practicable.

Also the evaluation of all the written examination will be done by the internal examiners only. The Internal Assessment Tests (Continuous Evaluations) will be taken by the Department and all the internal members will evaluate the test on the respective papers/topics. It should be noted that some special papers may be included in future as per discretion of the Department (subject to approval of the authority) †.

SEMESTER I

Duration: 6 Months (Including Examinations) Total Marks: 250, Total No. of Lectures: 50 Hours per paper

Semesters	Papers	Topics	Marks
Semester I	MC 101	Abstract Algebra	50
	MC 102	Linear Algebra	50
	MC 103	Real Analysis	50
	MC 104	Ordinary Differential Equations & Special Functions	50
		Continuous Evaluation (Class Tests & Attendance)	$(4 \times 10) + 10 = 50$

MC 101

ABSTRACT ALGEBRA(50 MARKS/ 50 LECTURES)

Groups:

Review of basic concepts of Group Theory: Lagrange's Theorem, Cyclic Groups, Permutation Groups and Groups of Symmetry: S_n, A_n, D_n, Conjugacy Classes, Index of a Subgroup, Divisible Abelian Groups.

Homomorphism of Groups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Cayley's Theorem.

Generalized Cayley's Theorem, Direct Product and Semi-Direct Product of Groups, Fundamental Theorem (Structure Theorem) of Finite Abelian Groups, Cauchy's Theorem, Group Action, Sylow Theorems and their applications. Solvable Groups (Definition and Examples only).

Rings:

Ideals and Homorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings. Divisibility Theory: Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss' Theorem, Eisenstein's criterion.

References:

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Gallian, J., Contemporary Abstract Algebra, Narosa, 2011.

Further Reading:

1. Roman, S., Fundamentals of Group Theory: An Advanced Approach, Birkhauser, 2012.
2. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc, 1997.
3. Rotman, J., The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.
4. Rotman, J., A First Course in Abstract Algebra, Prentice Hall, 2005.
5. Pinter, Charles. C., A Book of Abstract Algebra, McGraw Hill, 1982.
6. Herstein, I.N., Topics in Abstract Algebra, Wiley Eastern Limited.
7. Fraleigh, J.B., A First Course in Abstract Algebra, Narosa.
8. Jacobson, N., Basic Algebra, I & II, Hindusthan Publishing Corporation, India.
9. Hungerford, T.W., Algebra, Springer.
10. Artin, M., Algebra, Prentice Hall of India, 2007.
11. Goldhaber, J.K., Ehrlich, G., Algebra, The Macmillan Company, Collier-Macmillan Limited, London.
12. Gopalakrishnan, N.S., University Algebra, New Age International, 2005.

MC 102

LINEAR ALGEBRA (50 MARKS/ 50 LECTURES)

Review of Vector Spaces:

Vector spaces over a field, subspaces. Sum and direct sum of subspaces. Linear span. Linear dependence and independence. Basis. Finite dimensional spaces. Existence theorem for bases in the finite dimensional case. Invariance of the number of vectors in a basis, dimension. Existence of complementary subspace of any subspace of a finite dimensional vector space. Dimensions of sums of subspaces. Quotient space and its dimension.

Matrices and Linear Transformations:

Matrices and linear transformations, change of basis and similarity. Algebra of linear transformations. The rank-nullity theorem. Change of basis. Isomorphism Theorems. Dual space. Bi-dual space and natural isomorphism. Adjoint of linear transformations. Eigenvalues and eigenvectors of linear transformations. Determinants. Characteristic and minimal polynomials of linear transformations, Cayley-Hamilton Theorem. Annihilators. Diagonalization of operators. Invariant subspaces and decomposition of operators. Canonical forms.

Inner Product Spaces:

Inner product spaces. Cauchy-Schwartz inequality. Orthogonal vectors and orthogonal complements. Orthonormal sets and bases. Bessel's inequality. Gram-Schmidt orthogonalization method. Hermitian, SelfAdjoint, Unitary, and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms, real quadratic forms.

References:

1. Friedberg, S.H., Insel, A.J. and Spence, L.J., Linear Algebra, Prentice Hall of India, Fourth Edition, 2004.
2. Kumaresan, S., Linear Algebra, A Geometric Approach, Prentice Hall of India, Fourth Printing, 2003.

Further Reading:

1. Artin, M., Algebra, Prentice Hall of India, 2007.
2. Halmos, P.R., Finite Dimensional Vector Spaces, Springer, 2013.
3. Roman, S., Advanced Linear Algebra, Springer, 2007.
4. Curtis, C.W., Linear Algebra: An Introductory Approach, Springer (SIE), 2009.
5. Hoffman, K. and Kunze, R., Linear Algebra, Prentice Hall of India.
6. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
7. Apostol, T.M., Calculus Vol. I & II, John Wiley and Sons, 2011.

MC 103

REAL ANALYSIS (50 MARKS/ 50 LECTURES)

Bounded Variation

Functions of Bounded Variation and their properties, Riemann Stieltjes integrals and its properties, Absolutely Continuous Functions.

The Lebesgue Measure

Lebesgue Measure: (Lebesgue) Outer measure and measure on R, Measurable sets form an σ-algebra, Borel sets, Borelσ-algebra, open sets, closed sets are measurable, Existence of a non-measurable set, Measure space, Measurable Function and its properties, Borel measurable functions, Concept of Almost Everywhere (a.e.), sets of measure zero, Steinhaus Theorem, Sequence of measurable functions, Egorov's Theorem, Applications of Lusin Theorem.

The Lebesgue Integral

Simple and Step Functions, Lebesgue integral of simple and step functions, Lebesgue integral of a bounded function over a set of finite measure, Bounded Convergence Theorem, Lebesgue integral of non-negative function, Fatou's Lemma, Monotone Convergence Theorem. The General Lebesgue integral: Lebesgue Integral of an arbitrary Measurable Function, Lebesgue Integrable functions. Dominated Convergence Theorem.

Convergence in Measure. Riemann Integral as Lebesgue Integral. Product measure spaces, Fubini's Theorem (applications only).

References:

1. Apostol, T.M., Mathematical Analysis, Narosa Publishing House, 2002.
2. Royden, H.L., Real Analysis, 3rd Edition, Macmillan, New York & London, 1988.
3. Aliprantis, C.D., Burkinshaw, O., Principles of Real Analysis, Third Edition, Harcourt Asia Pte Ltd.,1998.

Further Reading:

1. Halmos, P.R., Measure Theory, Springer, 2007.
2. Rudin, W., Principles of Mathematical Analysis, Tata McGraw Hill, 2001.
3. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. Tao, T., An Introduction to Measure Theory, American Mathematical Society.
5. Kolmogorov, A.N., Fomin, S.V., Measures, Lebesgue Integrals, and Hilbert Space, Academic Press,New York & London, 1961.
6. Rana, I.K., An introduction to Measure and Integration, Second Edition, Narosa.
7. Barra, G.D., Measure Theory and Integration, Woodhead Pub.
8. Kingman, J.F.C. and Taylor, S.J., Introduction to Measure and Probability, Cambridge UniversityPress, 1966.
9. Cohn, D.L., Measure Theory, Birkhauser, 2013.

10. Wheeden, R.L. and Zygmund, A., Measure and Integral, Monographs and Textbooks in Pure andApplied Mathematics, 1977.

11. Sohrab, H.H., Basic Real Analysis, Birkhauser, 2003.

MC 104

ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS (50 MARKS / 50 LECTURES)

Ordinary Differential Equations:

Existence and Uniqueness

First order ODE, Initial value problems, Existence theorem, Uniqueness, basic theorems, Ascoli Arzela theorem (statement only), Theorem on convergence of solution of initial value problems, Picard-Lindelof theorem (statement only), Peano's existence theorem (statement only) and corollaries.

Higher Order Linear ODE

Higher order linear ODE, fundamental solutions, Wronskian, variation of parameters.

Boundary Value Problems for Second Order Equations

Ordinary Differential Equations of the Strum-Liouville type and their properties, Application to Boundary Value Problems, Eigenvalues and Eigenfunctions, Orthogonality theorem, Expansion theorem. Green's function for Ordinary Differential Equations, Application to Boundary Value Problems.

Special Functions:

Singularities

Fundamental System of Integrals, Singularity of a Linear Differential Equation. Solution in the neighbourhood of a singularity, Regular Integral, Equation of Fuchsian type, Series solution by Frobenius method.

Hypergeometric Equation

Hypergeometric Functions, Series Solution near zero, one, and infinity. Integral Formula, Diffrentiation of Hypergeometric Function.

Legendre Equation

Legendre Functions, Generating Function, Legendre Functions of First & Second kind, Laplace Integral, Orthogonal Properties of Legendre Polynomials, Rodrigue's Formula.

Bessel Equation

Bessel's Functions, Series Solution, Generating Function, Integral Representation of Bessel's Functions, Hankel Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

References:

1. Simmons, G.F., Differential Equations, Tata McGraw Hill.
2. Agarwal, Ravi P. and O' Regan D., An Introduction to Ordinary Differential Equations, Springer, 2000.

Further Reading:

1. Codington, E.A and Levinson, N., Theory of Ordinary Differential Equation, McGraw Hill.
2. Ince, E.L., Ordinary Differential Equation, Dover.
3. Estham, M.S.P., Theory of Ordinary Differential Equations, Van Nostrand Reinhold Compa.Ny, 1970.
4. Piaggio, H.T.H., An Elementary Treatise On Differential Equations And Their Applications, G. BellAnd Sons, Ltd, 1949.
5. Hartman, P., Ordinary Differential Equations, SIAM, 2002.
6. Zill, D. G., Cullen, M.R., Differential Equations with Boundary Value Problems, Brooks/Cole, 2009.

SEMESTER II

Duration: 6 Months (Including Examinations) Total Marks: 250, Total No. of Lectures: 50 Hours per paper

Semesters	Papers	Topics	Marks
Semester II	MC 201	Calculus of Several Variables	50
	MC 202	Complex Analysis	50
	MC 203	Topology	50
	MC 204	Partial Differential Equations	50
		Continuous Evaluation(Class Tests & Attendance)	$(4 \times 10) + 10 = 50$

MC 201

CALCULUS OF SEVERAL VARIABLES (50 MARKS/ 50 LECTURES)

R^n as a normed linear space, and $L(\mathbb{R}^n, \mathbb{R}^m)$ as a normed linear space. Limits and continuity of functions from R^n to R^m. The derivative at a point of a function from R^n to R^m as a linear transformation. The tangent space and linear approximation. The chain rule.

Partial derivatives and higher order partial derivatives and their continuity. Sufficient conditions for differentiability. Comparison between the differentiability of a function from R^2 to R^2 and from C to C. Examples of discontinuous and non-differentiable functions whose partial derivatives exist. C^1 maps. Euler's theorem. Sufficient condition for equality of mixed partial derivatives. Proofs of the Inverse Function Theorem, the Implicit Function Theorem, and the Rank Theorem. Jacobians. The Hessian and the real quadratic form associated with it. Extrema of real-valued functions of several variables. Proof of the necessity of the Lagrange multiplier condition for constrained extrema.

Riemann Integral of real-valued functions on Euclidean spaces, measure zero sets, Fubini's Theorem. Partition of unity, change of variables. Stokes' Theorem and Divergence Theorem for integrals.

References:

1. Spivak, M., Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus,Addison-Wesley, 1995.
2. Munkres, J.R., Analysis on Manifolds, Addison-Wesley, 1991.

Further Reading:

1. Apostol, T.M., Mathematical Analysis, Narosa Publishing House, 2002.
2. Apostol, T.M., Calculus Vol I & II, John Wiley & sons, 2011.
3. Fleming, W., Functions of Several Variables, 2nd Edition, Springer-Verlag, 1977.
4. Kaplan, W., Advanced Calculus, Pearson, 2002.
5. Ghorpade, S.R. and Limaye, B.V., A Course in Multivariable Calculus and Analysis, Springer, 2009.

MC 202

COMPLEX ANALYSIS (50 MARKS/50 LECTURES)

Complex Numbers:

Complex Plane, Stereographic Projection.

Complex Differentiation:

Derivative of a complex function, Comparison between differentiability in the real and complex senses, Comparison between the real and complex differentiability via R-linear and C-linear maps, Cauchy-Riemann equations, Necessary and sufficient criterion for complex differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

Complex Functions and Conformality:

Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Conformal maps, Mobius Transformations.

Complex Integration:

The complex integral (over piecewise C^1 curves), Cauchy's Theorem and Integral Formula, Power series representation of analytic functions. The difference between Real Analytic functions and C^∞-functions over R. Real Analyticity vs. Complex Analyticity. Morera's Theorem, Goursat's Theorem, Liouville's Theorem, Fundamental Theorem of Algebra, Zeros of analytic functions, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications, Schwarz's Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy's Theorem for simply connected domains.

Singularities:

Definitions and Classification of singularities of complex functions, Isolated singularities, Uniform convergence of sequences and series. Laurent series, Casorati-Weierstrass Theorem, Poles, Residues, Residue Theorem and its applications to contour integrals, Meromorphic functions, Applications of Argument Principle, Applications of Rouche's Theorem.

References:

1. Conway, J.B., Functions of One Complex Variable, Second Edition, Narosa Publishing House, 1973.
2. Marsden, J.E. and Hoffman, M.J., Basic Complex Analysis, Third Edition, W. H. Freeman and Company, New York, 1999.

Further Reading:

1. Sarason, D., Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
2. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
3. Rudin, W., Real and Complex Analysis, McGraw- Hill Book Co., 1966.
4. Hille, E., Analytic Function Theory (2 vols.), Gonn& Co., 1959.
5. Gamelin, T.W., Complex Analysis, Springer, 2001.
6. Bak, J. and Newman, D.J., Complex Analysis, Springer, 2010.
7. Ponnusamy, S., Foundations of Complex Analysis, Narosa, 2008.

MC 203

TOPOLOGY (50 MARKS/50 LECTURES)

Review of Metric Spaces:

Examples and its properties.

Topological Spaces and Continuous Functions:

Topology on a set, Examples of Topologies (Topological Spaces): Discrete Topology, Indiscrete Topology, Finite Complement Topology, Countable Complement Topology, Topologies on the Real Line :R`, R*K*, R with usual Topology etc., Finer and Coarser Topologies, Basis and Sub basis for a topology. Product topology on X × Y, Subspace Topology.

Interior Points, Limit Points, Derived Set, Boundary of a set, Closed Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology.

Continuous Functions, Rules for Constructing Continuous Functions: Inclusion Map, Composition, by restricting the domain, by restricting/expanding the range, Pasting Lemma, Open maps, Closed maps and Homeomorphisms, Embedding of a Topological Space into another Topological Space (examples only).

(Infinite) Product Topology: Sub basis for product Topology defined by Projection Maps, Box Topology, Metric Topology.

Connectedness and Compactness:

Connected and Path Connected Spaces: Definitions, Examples and its simple properties, Connected subsets of the real line, Introduction to Components and Path Components, Local Connectedness.

Compact Spaces, Compact subsets of the real line, Heine-Borel Theorem.

References:

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

Further Reading:

1. Dugundji, J., Topology, Allyn and Bacon, 1966.
2. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
3. Kumaresan, S., Topology of Metric Spaces, Narosa Publishing House, 2010.
4. Kelley, J.L., General Topology, Van Nostrand Reinhold Co., New York, 1995.
5. Young, J.G., Topology, Addison-Wesley Reading, 1961.
6. Willard, S., General Topology, Dover.
7. Engelking, R., General Topology, Polish Scientific Pub.
8. Sierpinski, W., Introduction to General Topology, The University of Toronto Press, Canada.
9. Kuratowski, K., General Topology, Vol. I, Academic Press, New York and London.

MC 204

PARTIAL DIFFERENTIAL EQUATIONS (50 MARKS/ 50 LECTURES)

First Order P.D.E.

Formation and solution of PDE, Integral surfaces, Cauchy Problem order equation, Orthogonal surfaces, First order non-linear PDE, Characteristics, Compatible system, Charpit's method. Classification and canonical forms of PDE.

Second Order Linear P.D.E.

Classification, reduction to normal form; Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.

Elliptic Differential Equations

Derivation of Laplace and Poisson equation, Boundary Value Problem, Separation of Variables, Dirichlets's Problem and Neumann Problem for a rectangle, Interior and Exterior Dirichlets's problems for a circle, Interior Neumann problem for a circle, Solution of Laplace equation in Cylindrical and spherical coordinates, Examples.

Parabolic Differential Equations

Formation and solution of Diffusion equation, Dirac, Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and spherical coordinates, Examples.

Hyperbolic Differential Equations

Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D'Alembert's solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two- dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel's Principle, Examples.

Green's Function

Green's function for Laplace Equation, methods of Images, Eigenfunction Method, Green's function for the wave and Diffusion equations. Laplace Transform method: Solution of Diffusion and Wave equation by Laplace Transform.

References:

1. Sneddon, I.N., Elements of Partial Differential Equations, McGraw Hill.

Further Reading:

1. Williams, W.E., Partial Differential Equations.
2. Miller, F.H., Partial Differential Equations.
3. Petrovsky, I.G., Lectures on Partial Differential Equations.
4. Courant & Hilbert, Methods of Mathematical Physics Vol-II.
5. Rao, K.S., Introduction to Partial Differential Equations, Prentice Hall.

SEMESTER III

Duration: 6 Months (Including Examinations) Total Marks: 250, Total No. of Lectures: 50 Hours per paper

Semesters	Papers	Topics	Marks
Semester III	MC 301	Functional Analysis	50
	MC 302	Classical Mechanics	50
	MS 303	Special Paper - 1(I)	50
	MS 304	Special Paper - 2(I)	50
		C Programming & Seminar Presentation	30 + 20 = 50

MC 301

FUNCTIONAL ANALYSIS (50 MARKS/50 LECTURES)

Banach Spaces

Normed Linear Spaces and its properties, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Riesz Lemma. Bounded Linear Transformations. Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Necessary and sufficient conditions for Bounded (Continuous) and Unbounded (Discontinuous) Linear functionals in terms of their kernel. Hyperplane, Necessary and sufficient conditions for a subspace to be hyperplane. Applications of Hahn-Banach Theorem, Dual Space, Examples of Reflexive Banach Spaces. L^p-Spaces and their properties.

Hilbert Spaces

Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel’s Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval’s Identity, Orthogonal Complement and Projections. Riesz Representation Theorem for Hilbert Spaces, Adjoint of an Operator on a Hilbert Space with examples, Reflexivity of Hilbert Spaces, Definitions and examples of Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators and Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators.

References:

1. Limaye, B.V., Functional Analysis, Wiley Eastern Ltd, 1981.
2. Kreyszig, E., Introductory Functional Analysis and its Applications, John Wiley and Sons, New York,1978.

Further Reading:

1. Brown, A. and Pearcy, C., Introduction to Operator Theory I: Elements of Functional Analysis,Springer-Verlag New York, 1977.
2. Suhubi, E.S., Functional Analysis, Springer, New Delhi, 2009.
3. Aliprantis, C.D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.,1998.
4. Ponnusamy, S., Foundations of Functional Analysis, Narosa, 2011.
5. Goffman, C., Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
6. Bachman, G., Narici, L., Functional Analysis, Academic Press, 1966.
7. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.
8. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
9. Conway, J.B., A Course in Functional Analysis, Springer Verlag, New York, 1990.
10. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.

MC 302

CLASSICAL MECHANICS (50 MARKS /50 LECTURES)

Dynamical systems, Generalized coordinates, Degrees of freedom, Principle of virtual work. D'Alembert's principle. Unilateral and bilateral constraints. holonomic and non-holonomic system. Lagrange's equations for holonomic systems. Lagrange's equation for impulsive forces and for systems involving dissipatative forces. Conservation theorems. Hamilton's principle and principle of least action. Hamilton's canonical equations. Canonical transformation with different generating functions. Lagrange and Poisson brackets and their properties. Hamilton-Jacobi equations and separation of variables. Routh's equations Poisson's identity. Jacobi-Poisson Theorem. Brachistochrone problem. Configuration space and system point.

Variation of functional, Necessary and sufficient conditions for extrema, Euler-Lagrange's equations and its Applications:Geodesic, minimum surface of revolution, Brachistochrone problem and other boundary value problems in ordinary and partial differential equations.

References:

1. Goldstein, H., Classical Mechanics, Dover.
2. Arnold, V.I.(Vogtmann, K., Weinstein, A.), Mathematical Methods of Classical Mechanics, Springer(GTM), 1989.

Further Reading:

1. Rana, N.C. and Jog, P.S., Classical Mechanics, Tata McGraw Hill.
2. Louis, N.H.and Finch, J.D., Analytical Mechanics.

3. Ramsay, A.S., Dynamics, Part-II.

SPECIAL PAPERS:

MS 303

Special Paper - 1(I) (50 MARKS/ 50 LECTURES)

MS 304

Special Paper - 2(I) (50 MARKS/ 50 LECTURES)

SEMESTER IV

Duration: 6 Months (Including Examinations) Total Marks: 250, Total No. of Lectures: 50 Hours per paper

Semesters	Papers	Topics	Marks
Semester IV	MC 401	Numerical Analysis	50
	MC 402	Mathematical Methods	50
	MS 403	Special Paper - 1(II)	50
	MS 404	Special Paper - 2(II)	50
		Seminar Presentation & Grand Viva-Voce	30 + 20 = 50

MC 401

NUMERICAL ANALYSIS (50 MARKS/ 50 LECTURES)

Numerical Solution of System of Linear Equations

Triangular factorization methods, Matrix inversion method, Iterative methods- Jacobi method, Gauss Jacobi method, Gauss-Seidel method, Successive over relaxation (SOR) method and convergence condition of Iterative methods, Rate of convergence of methods.

Solution of Non-linear Equations

Iteration methods: Tchebyshev method, Multipoint method, Modified Newton-Raphson method (for real roots simple or repeated), Rate of convergence of all iteration methods.

System of Non-linear Equations

Newton's Method, Quasi-Newton's method.

Numerical Solution of Initial Value Problem for ODE

First order Equation: Runge-Kutta methods, Multistep predictor-corrector methods, Convergence and stability.

Two Point Boundary Value Problem for ODE

Finite difference method, Shooting Method.

Numerical Solution of PDE by Finite Difference Method

Parabolic equation in one dimension (Heat equation), Explicit finite difference method, Implicit CrankNickolson method, Hyperbolic equation in one-space dimension (Wave equation)- Finite difference method, Convergence and Stability.

References:

1. Jain, M.K., Iyenger, S.R.K. and Jain, R.K., Numerical Methods for Scientific and Engineering Computation, New Age International.

Further Reading:

1. Atkinson, K.E., An Introduction to Numerical Analysis, John Wiley & Sons, 1989.
2. Smith, G.D., Numerical Solution of Partial Differential Equations.
3. Berzin&Zhidnov, Computing methods.
4. Isacson& Keller, Analysis of Numerical methods.
5. Ralston & Rabinowitz, A First Course in Numerical Analysis.
6. Jain, M.K., Numerical Solution of Differential Equations.
7. Fox, L., Numerical Solution of Ordinary and Partial Differential Equations, Oxford Univ. Press.

MC 402

MATHEMATICAL METHODS(50 MARKS /50 LECTURES)

Laplace Transform

Laplace transform, properties of Laplace transform, inversion formula of Laplace transform (Bromwich formula), Convolution theorem, Application to ordinary and partial differential equations.

Fourier Transform

Properties of Fourier transform, Inversion formula, Convolution, Parseval's relation, Multiple Fourier transform, Bessel's inequality, Application of transform to Heat, Wave and Laplace equations.

Hankel Transform

Hankel transform, Inversion formula of Hankel transform, Parseval relation, Finite Hankel transform, Application to partial differential equations.

Integral Equation

Basic concepts, Volterra integral equations, Relationship between linear differential equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equation of first kind, Abel's integral equation, Fredholm integral equations, Fredholm equations of the second kind, the method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Eigen values and Eigen functions of a Fredholm alternative, Construction of Green's function for Boundary Value Problems, Singular integral equations.

References:

1. Courant & Hilbert, Methods of Mathematical Physics,Vol-I, II.

Further Reading:

1. Sneddon, I.N., The Uses of Integral Transforms, McGraw Hill.
2. Tranter, C.J., Integral Transforms in Mathematical Physics, John Wiley & sons.
3. Sneddon, I.N., Fourier Transform, Dover.
4. Lovitt, W.V., Linear Integral Equations, Dover.
5. Tricomi, F.G., Integral Equations, Dover.
6. Andrews, L. and Shivamoggi, V.K., Integral Transforms for Engineers, SPIE Press.
7. Debnath, L. and Bhatta, D., Integral Transforms and Their Applications, CRC Press.
8. Davics, B., Integral Transforms and Their Applications, Springer.
9. Pinkus, A. and Zafrany, S., Fourier Series and integral transforms, Cambridge University Press.

SPECIAL PAPERS:

MS 403

Special Paper 1(II)(50 MARKS /50 LECTURES)

MS 404

Special Paper 2(II)(50 MARKS /50 LECTURES)

SPECIAL PAPERS FOR SEMESTER III & IV:

ADVANCED TOPOLOGY I (50 Marks)

Compactness, Limit point compactness, sequentially compact spaces, countably compact spaces. Locally compact spaces.

Countability Axioms, The Separation Axioms, Lindelof spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze Extension Theorem.

Tychonoff Theorem &Compactification:Tychonoff Theorem, Completely Regular spaces, Local Compactness, One-point compactification, Stone-Cech Compactification.

Metrization: UrysohnMetrization Theorem, Topological Imbedding, Imbedding Theorem of a regular space with countable base in R^n, Partitions of Unity, Topological *m*-Manifolds, Imbedding Theorem of a compact *m*-manifold in R^n.

Local Finiteness, Nagata-Smirnov Metrization Theorem, Paracompactness, Stone's Theorem, Local Metrizability, Smirnov Metrization Theorem. Uniform Spaces.

References:

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

2. Dugundji, J., Topology, Allyn and Bacon, 1966.
3. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963. Kelley, J.L.,General Topology, Van Nostrand Reinhold Co., New York, 1995.
4. Bourbaki, N., TopologieGnrale.
5. Hocking, J. and Young, G., Topology, Addison-Wesley Reading, 1961.
6. Steen, L. and Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York,1970.

ADVANCED TOPOLOGY II (50 Marks)

Nets and Filters: Directed Sets, Nets and Sub-nets, Convergence of a net, Ultranets, Partially Ordered Sets and Filters, Convergence of a filter, Ultrafilters, Basis and Subbase of a filter, Nets and Filters in Topology.

Complete Metric Spaces & Function Spaces: Complete Metric Spaces, Baire Category Theorem, The Peano Space-Filling Curve, Hahn-Mazurkiewicz Theorem (statement only). Compactness in Metric Spaces, Equicontinuity.

Pointwise and Compact Convergence, The Compact-Open Topology, Stone-Weierstrass Theorem, Ascoli's Theorem, Baire Spaces, A Nowhere Differentiable Function.

An Introduction to Dimension Theory, Topological notion of (Lebesgue)dimension.

References:

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. Dugundji, J., Topology, Allyn and Bacon, 1966.
3. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963. Kelley, J.L.,General Topology, Van Nostrand Reinhold Co., New York, 1995.
4. Bourbaki, N., TopologieGnrale.
5. Hocking, J. and Young, G., Topology, Addison-Wesley Reading, 1961.
6. Steen, L. and Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York,1970.

ADVANCED FUNCTIONAL ANALYSIS I (50 Marks)

Normed linear spaces, Banach spaces. Stone-Weierstrass Theorem, Ascoli-Arzela Theorem.

Bounded linear operators. Dual of a normed linear space. Hahn-Banach theorem, Computing the dual of well-knownBanach spaces.

Weak and weak* topologies, Banach-Alaoglu Theorem. The double dual.

L^p-spaces, Completeness and other Properties. Riesz representation theorem for the space $C[0,1]$.

Linear Topological Spaces, Locally Convex Spaces and their Characterization in terms of a family of Seminorms.

References:

1. Rudin, W., Real and complex analysis, McGraw-Hill, 1987.
2. Rudin, W., Functional analysis, McGraw-Hill, 1991.
3. Conway, J.B., A course in functional analysis, GTM (96), Springer-Verlag, 1990.
4. Yosida, K., Functional analysis, Springer-Verlag, 2004.
5. Katznelson, Y., An introduction to harmonic analysis, Dover Publications, 1976.
6. Stein, E.M. and Shakrachi, R., Fourier Analysis: An Introduction, Princeton Lectures in Analysis.
7. Hernez, E. and Weiss, G., A first course on wavelets, Studies in Advanced Mathematics, CRC Press,1996.
8. Kelley, J.L. and Namioka, I., Linear Topological Spaces, D.VanNostrand Company, 1963.
9. Aliprantis, C.D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.,1998.
10. Goffman, C. and Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi,1987.
11. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.

ADVANCED FUNCTIONAL ANALYSIS II (50 Marks)

Krein-Milman Theorem and its Applications, Uniform Convexity, Strict Convexity and their Applications.

Fourier and Fourier-Stieltjes' series, summability kernels, convergence tests.

Fourier transforms, the Schwartz space, Fourier Inversion and Plancherel theorem. Maximal functions and boundedness of Hilbert transform. Statement of Paley-Wiener Theorem. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem (discussion without proof). Introduction to wavelets and multi-resolution analysis.

References:

1. Rudin, W., Real and complex analysis, McGraw-Hill, 1987.
2. Rudin, W., Functional analysis, McGraw-Hill, 1991.
3. Conway, J.B., A course in functional analysis, GTM (96), Springer-Verlag, 1990.
4. Yosida, K., Functional analysis, Springer-Verlag, 2004.
5. Katznelson, Y., An introduction to harmonic analysis, Dover Publications, 1976.
6. Stein, E.M. and Shakrachi, R., Fourier Analysis: An Introduction, Princeton Lectures in Analysis.
7. Hernez, E. and Weiss, G., A first course on wavelets, Studies in Advanced Mathematics, CRC Press,1996.
8. Kelley, J.L. and Namioka, I., Linear Topological Spaces, D.VanNostrand Company, 1963.
9. Aliprantis, C.D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.,1998.

10. Goffman, C. and Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi,1987.

11. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.

ALGEBRAIC TOPOLOGY I (50 Marks)

The Fundamental Group and Covering Spaces: Homotopy of paths, Fundamental Group, Covering Spaces, Fundamental Group of the Circle, Fundamental Group of the Punctured Plane.

Special Van Kampen Theorem, Fundamental Group of S^n, Seifert - Van Kampen Theorem (statement and applications).

Fundamental Group of Surfaces. Essential and Inessential Maps, Borsuk - Ulam Theorem for S^2.

Fundamental Theorem of Algebra, Vector Fields and Fixed Points, Brouwer's Fixed-Point Theorem for the disc, Homotopy Type, Deformation Retract, Strong Deformation Retract.

Covering spaces, lifting properties, Universal cover, classification of covering spaces, Deck transformations.

Jordan Separation Theorem.

References:

1. Hatcher, A., Algebraic Topology, Cambridge University Press, 2002.
2. Massey, W.S., A basic course in algebraic topology, GTM (127), Springer, 1991.
3. Munkres, J.R., Elements of algebraic topology, Addison-Wesley, 1984.
4. Greenberg, M. J., Lectures on algebraic topology, Benjamin, 1967.
5. Singer, I.M. and Thorpe, J.A., Lecture notes on elementary topology and geometry, UTM, Springer.
6. Spanier, E., Algebraic Topology, Springer-Verlag, 1982.

ALGEBRAIC TOPOLOGY II (50 Marks)

Categories and functors. Singular homology groups, axiomatic properties, Mayer-Vietoris sequence, homology with coefficients, statement of universal coefficient theorem for homology, simple computation of homology groups.

CW-complexes and Cellular homology, comparison with singular theory, Simplicial complex and simplicial homology as a special case of Cellular homology, Relationship between fundamental group and first homology group. Computation of homology of projective spaces.

Definition of singular cohomology, axiomatic properties, statement of universal coefficient theorem for cohomology. Betti numbers and Euler characterisitic. Cup and cap product, Poincare duality. Cross product and statement of Kunneth theorem. Degree of maps with applications to spheres.

References:

1. Hatcher, A., Algebraic Topology, Cambridge University Press, 2002.
2. Massey, W.S., A basic course in algebraic topology, GTM (127), Springer, 1991.
3. Munkres, J.R., Elements of algebraic topology, Addison-Wesley, 1984.
4. Greenberg, M. J., Lectures on algebraic topology, Benjamin, 1967.
5. Singer, I.M. and Thorpe, J.A., Lecture notes on elementary topology and geometry, UTM, Springer.
6. Spanier, E., Algebraic Topology, Springer-Verlag, 1982.

DIFFERENTIAL GEOMETRY I (50 Marks)

Calculus on Euclidean Spaces

Euclidean Spaces, Tangent Vectors, Directional Derivatives, Curves in R^3, 1-forms, Differential Forms, Mappings.

Frame Fields

Dot Product, Curves, Frenet Formulae, Arbitrary Speed Curves, Covariant Derivatives, Frame Fields, Connection Forms, The Structural Equations.

Euclidean Geometry

Isometries of R^3, Tangent Map of an Isometry, Orientation, Euclidean Geometry, Congruence of Curves.

Calculus on a Surface

Surfaces in R^3, Patch Computations, Differentiable Functions and Tangent Vectors, Differentiable Forms on a Surface, Mappings of Surfaces, Integration of Forms, Topological Properties of Surfaces, Manifolds.

References:

1. O'Neill, Barret, Elementary Differential Geometry, Elsevier Academic Press, 2006.
2. Pressley, A., Elementary Differential Geometry, Springer, 2004.

DIFFERENTIAL GEOMETRY II (50 Marks)

Tensor Algebra

Finite Dimensional Real Linear Spaces, Their Subspaces and Dual Spaces. Summation Convention, Change of Bases, Contravariant and Covariant Vectors. Multilinear Functionals, Tensor Spaces, Algebra of Tensors. Symmetric and Skew-Symmetric Tensors. Exterior Algebra.

Manifolds

Definition and Examples. Differentiable Curves. Submanifolds. Tangents. Differential of a Map.

Vector Analysis on Manifolds

Vector and Tensor Fields, Integral Curves and Flows, Lie Bracket. One Parameter Group of Transformations. Exponential Maps.

Linear Connections

Linear Connections, Their Torsion and Curvature.

Riemannian Manifolds

Riemannian Manifolds. Curvature Tensor, Ricci Tensor, Scalar Curvature, Sectional Curvature.

References:

1. Bishop, Richard L. and Goldberg, Samuel I., Tensor Analysis on Manifolds, Macmillan, 1968.
2. Hicks, Noel J., Notes on Differential Geometry, Van Nostrand. 1965.
3. Kumaresan, S., Differential Geometry and Lie Groups, Hindusthan Book Agency, 2002.
4. Boothby, William M., An Introduction to Differentiable Manifolds and Riemannian Geometry, 1975.

ADVANCED REAL ANALYSIS I (50 Marks)

Ordinal Numbers

Order type, well ordered sets, transfinite induction, ordinal numbers, comparability of ordinal numbers, Arithmetic of ordinal numbers. First uncountable ordinal (Ω).

Descriptive properties of sets

Perfect sets, every closed set is the union of a perfect set and a finite or denumerable set. Nowhere dense set. First category, second category and residual sets. In a complete metric space X every subset of X is residual in X if and only if it contains a dense G_δ-set. Borel sets of order type $\alpha(<\Omega)$ and its properties.

Functions of special classes

Baire class functions of order type $\alpha(<\Omega)$ and its properties. Relation Between Baire functions and Borel sets.

Continuity

Lower and upper semi-continuous functions with their properties. Absolute continuity and Lusin (N) condition. Lebesgue density point of a set and Lebesgue density theorem, Approximate continuity and its simple properties.

Derivative

The Vitali-covering theorem, Dini's derivatives and its properties. Derivative of a monotone function, Determining a function by its derivative. Lebesgue point.

References:

1. Bruckner, A., Bruckner, J.B. and Thomson, J.B., Real Analysis.
2. Goffman, C., Real Functions.
3. Jeffrey, R.L., The Theory of Functions of a Real Variable.
4. Natanson, I.P., Theory of Functions of a Real Variable, Vol. I & II.
5. Hobson, E.W., Theory of Functions of a Real Variable, Vol. I & II.

6. Royden, H.L., Real Analysis.
7. Munroe, M.E., Introduction to Measure and Integration.
8. Lee, P.Y., Lanzhou Lectures on Henstock Integration.
9. Das, A.G., Generalized Riemann Integral.

ADVANCED REAL ANALYSIS II (50 Marks)

Fourier Series

Trigonometric series, Fourier series, Dirichlet's kernel, pointwise convergence-Dini's test, Jordan test, convergence of Ces`aro means-Fej´er's theorem, Lebesgue-Fej´er's theorem, Riemann's theorem. Cantor's uniqueness theorem.

Integration on R

Henstock integral: Gauge functions, δ-finite partition, Cousin lemma, definition of Henstock integral and examples, Saks-Henstock lemma, Linearity property, Fundamental theorem. Relation of Henstock integral with Newton, Riemann and Lebesgue integrals. Absolute integrability of Henstock integral, Monotone and Dominated Convergence theorem of Henstock integral.

General Measure and Integration

Measure space, measurable functions, integration of non-negative function, convergence theorems, Fatou's lemma, Signed measure, positive and negative sets. Hahn and Jordan decomposition theorems. Absolute continuous and singular measures, Radon-Nikodym theorem and its consequences.

References:

1. Bruckner, A., Bruckner, J.B. and Thomson, J.B., Real Analysis.
2. Goffman, C., Real Functions.
3. Jeffrey, R.L., The Theory of Functions of a Real Variable.
4. Natanson, I.P., Theory of Functions of a Real Variable, Vol. I & II.
5. Hobson, E.W., Theory of Functions of a Real Variable, Vol. I & II.
6. Royden, H.L., Real Analysis.
7. Munroe, M.E., Introduction to Measure and Integration.
8. Lee, P.Y., Lanzhou Lectures on Henstock Integration.
9. Das, A.G., Generalized Riemann Integral.

ADVANCED COMPLEX ANALYSIS I (50 Marks)

The Functions $M(r)$, $A(r)$, Hadamard Theorem on Growth of $\log M(r)$, Schwarz Inequality, Borel-Caratheodory Inequality.

Entire Functions, Growth of an Entire Function, Order and Type and their Representations in terms of the Taylor Coefficients, Distribution of Zeros, Schottky's Theorem (no proof), Picard's Little Theorem, Weierstrass Factor Theorem, The Exponent of Convergence of Zeros, Hadamard Factorization Theorem, Canonical Product, Borel's First Theorem, Borel's Second Theorem (statement only).

Analytic Continuation, Natural Boundary, Analytic Element, Global Analytic Function, Concept of Analytic Manifolds, Multiple Valued Conditions, Branch Points and Branch Cut, Riemann Surfaces.

References:

1. Conway, J.B., Functions of one complex variable, Second Edition, Narosa Publishing House.
2. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
3. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. Hille, E., Analytic Function Theory (2 vols.), Gonn& Co., 1959.
5. Titchmarsh, E.C., The Theory of Functions, Oxford University Press, London.
6. Markusevich, A.I., Theory of Functions of a Complex Variable, Vol. I, II, III.
7. Copson, E.T., An Introduction to the Theory of Functions of a Complex Variable.
8. Hayman, W.K., Meromorphic Functions.
9. Kaplan, W., An Introduction to Analytic Functions.

ADVANCED COMPLEX ANALYSIS II (50 Marks)

Harmonic Functions, Characterization of Harmonic Functions by Mean-Value Property, Poisson's Integral Formula, Dirichlet Problem for a Disc.

Doubly Periodic Functions, Weierstrass Elliptic Functions.

Meromorphic Functions, Expnasions, Definition of the functions $m(r,a)$, $N(r,a)$ and $T(r)$. Nevanlinna's First Fundamental Theorem, Cartan's Identity and Convexity Theorems, Order of Growth, Order of a Meromorphic Function, Comparative Growth of $\log M(r)$ and $T(r)$, Nevanlinna's Second Fundamental Theorem, Estimation of $S(r)$ (statement only), Nevanlinna's Theory of Deficient Values, Upper Bound of the Sum of
Deficiencies.

References:

1. Conway, J.B., Functions of one complex variable, Second Edition, Narosa Publishing House.
2. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
3. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. Hille, E., Analytic Function Theory (2 vols.), Gonn& Co., 1959.
5. Titchmarsh, E.C., The Theory of Functions, Oxford University Press, London.
6. Markusevich, A.I., Theory of Functions of a Complex Variable, Vol. I, II, III.
7. Copson, E.T., An Introduction to the Theory of Functions of a Complex Variable.

8. Hayman, W.K., Meromorphic Functions.

9. Kaplan, W., An Introduction to Analytic Functions.

MEASURE AND TOPOLOGY I (50 Marks)

General Measure Spaces:

Their properties and construction. Measure and Measurable sets.

Integration:

Signed Measure, Hahn Decomposition Theorem, Mutually Singular Meausres, Radon-Nikodym Theorem, Lebesgue Decomposition, Riesz Representation Theorem, Extension Theorem (Caratheodory).

Product Measure, Fubini's Theorem, Differentiation and Integration, Decomposition into Absolutely Continuous and Singular Parts.

Measure on Locally Compact Spaces:

Borel Sets, Baire Sets, Baire Sandwich Theorem, Borel and Baire Measure, Regularity of Measures, Regular Borel Extension of a Baire Measure, Completion, Continuous Functions with Compact Support, Integration of Continuous Functions with Compact Support, Riesz-Markoff Theorem.

References:

1. Royden, H.L., Real Analysis, 3rd Edition, Macmillan, New York & London, 1988.

2. Halmos, P.R., Measure Theory, Van Nostrand, Princeton, 1950.

3. Yeh, J., Real Analysis, Theory of Measure and Integration, World Scientific, 2006.

4. Oxtoby, J.C., Measure and Category, Springer Verlag, 1980.

5. Berberian, S.K., Measure and Integration, Chelsea Publishing Company, NY, 1965.

6. Barra, G.de, Measure Theory and Integration, Wiley Eastern Ltd., 1981.

7. Rana, I.K., An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.

8. Bartle, R.G., The Elements of Integration, John Wiley and Sons, Inc., NY, 1966.

9. Kingman, J.F.C. and Taylor, S.J., Introduction to Measure and Probability, Cambridge UniversityPress, 1966.

10. Cohn, D.L., Measure Theory, Birkhauser, 2013.

11. Wheeden, R.L. and Zygmund, A., Measure and Integral, Monographs and Textbooks in Pure andApplied Mathematics, 1977.

12. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.

13. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.

MEASURE AND TOPOLOGY II (50 Marks)

Measure and Category:

Measure and Category on Real Line, Countable and Uncountable Sets, Sets of First Category, Null Sets, Theorems of Cantor, Baire and Borel, Application of Category Method to Nowhere Differentiable Functions, Liouville Numbers, Measure and Category of the Set of Liouville Numbers.

Banach Category Theorem, Statement of Theorems of Marczewski and Sikorski, Cardinals of Measure Zero, Decomposition into a Null set and a Set of First Category. Similarities between the Classes of Sets of Measure Zero and of First Category.

The Principle of Duality:

Examples of Duality, Lusin Sets and their Duals, Extended Principle of Duality.

Category Measure Spaces:

Spaces in which Category and Measure Agree, Topologies generated by Lower Densities, The Lebesgue Density Topology.

References:

1. Royden, H.L., Real Analysis, 3rd Edition, Macmillan, New York & London, 1988.
2. Halmos, P.R., Measure Theory, Van Nostrand, Princeton, 1950.
3. Yeh, J., Real Analysis, Theory of Measure and Integration, World Scientific, 2006.
4. Oxtoby, J.C., Measure and Category, Springer Verlag, 1980.
5. Berberian, S.K., Measure and Integration, Chelsea Publishing Company, NY, 1965.
6. Barra, G.de, Measure Theory and Integration, Wiley Eastern Ltd., 1981.
7. Rana, I.K., An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.
8. Bartle, R.G., The Elements of Integration, John Wiley and Sons, Inc., NY, 1966.
9. Kingman, J.F.C. and Taylor, S.J., Introduction to Measure and Probability, Cambridge UniversityPress, 1966.
10. Cohn, D.L., Measure Theory, Birkhauser, 2013.
11. Wheeden, R.L. and Zygmund, A., Measure and Integral, Monographs and Textbooks in Pure andApplied Mathematics, 1977.
12. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
13. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.

OPERATOR ALGEBRA I (50 Marks)

Banach Algebra:

Definition of Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an elements, The spectral radius, Gelfand formula, Multiplicative linear functionals and the maximal ideal space, Gleason-Kahane-Zelazko Theorem, The Gelfand Transforms, The Spectral mapping theorem, Isometric Gelfand Transform.

C^*- Algebras:

Definition of C^*-Algebras and examples, Self-adjoint, Unitary, Normal, Positive and Projection elements in C^*-Algebras, Commutative C^*-Algebras, C^*-Homomorphisms, Representation of commutative C^*-Algebras, subalgebras and the spectrum, The Spectral Theorem, Positive linear functionals in C^*-algebras, States and the GNS construction.

References:

1. Conway, J.B., A Course in Functional Analysis, Springer Verlag, New York, 1990.
2. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
3. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.
4. Bonsall and Duncan, Complete Normed algebras, Springer-Verlag.
5. Kadison and Ringrose, Fundamentals of operator theory, Vol. I and II, Academic press.
6. Rickart, General theory of Banach Algebras, : Van Nostrand.
7. Arveson, W., An invitation to C*-Algebras, Springer-Verlag.
8. Palmer, Banach Algebras and the general theory of C*-algebras, Cambridge University Press.

OPERATOR ALGEBRA II (50 Marks)

Von Neumann Algebras:

Von Neumann algebras, Monotone sequence of operators, Range Projections, The Commutant, The Double Commutant theorem, The Kaplansky Density theorem, L^∞ as Von Neumann Algebra, Maximal Abelian Algebras, Abelian Von Neumann Algebras, Cyclic and separating vectors, Representation of Abelian Von Neumann Algebras, The L^∞ functional calculus, Connectedness of the unitary group, The projection lattice, Kaplansky's formula, The centre of a Von Neumann Algebra, Various types of projections.

References:

1. Conway, J.B., A Course in Functional Analysis, Springer Verlag, New York, 1990.
2. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
3. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.
4. Bonsall and Duncan, Complete Normed algebras, Springer-Verlag.
5. Kadison and Ringrose, Fundamentals of operator theory, Vol. I and II, Academic press.
6. Rickart, General theory of Banach Algebras, : Van Nostrand.
7. Arveson, W., An invitation to C*-Algebras, Springer-Verlag.
8. Palmer, Banach Algebras and the general theory of C*-algebras, Cambridge University Press.

OPERATOR THEORY I (50 Marks)

Bounded linear Operators:

Resolvent set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Spectral properties of a bounded linear operator, Spectral mapping theorem for polynomials.

Banach Algebra:

Definition of normed and Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an element, The spectral radius.

Compact linear operators:

Spectral properties of compact linear operators on a normed linear space, Operator equations involving compact linear operators, Fredholm alternative theorem, Fredholm alternative for integral equations. Spectral theorem for compact normal operators.

References:

1. Brown, A. and Pearcy, C., Introduction to Operator Theory, I, II, Springer-Verlag, 1977.
2. Coway, J.B., A Course in Functional Analysis, Springer, 1990.
3. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
4. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.
5. Kreyszig, E., Introductory Functional Analysis with Applications, John Wiley and sons.
6. Bachman, G. and Narici, L., Functional Analysis, Dover Publications.
7. Taylor, A. and Lay, D., Introduction to Functional Analysis, John Wiley and Sons.
8. Dunford, N. and Schwartz, J.T., Linear Operators 3, John Wiley and Sons.
9. Halmos, P.R., Introduction to Hilbert space and the theory of Spectral Multiplicity, Chelsea PublishingCo., N.Y.

OPERATOR THEORY II (50 Marks)

Self-adjoint operators:

Spectral properties of bounded self-adjoint linear operators on a complex Hilbert space, Positive operators, Square root of a positive operator, Projection operators, Spectral family of a bounded self-adjoint linear operator and its properties, Spectral theorem for a bounded self-adjoint linear operator.

Normal Operators:

Spectral properties for bounded normal operators, Spectral theorem for bounded normal operators.

Unbounded linear operators in Hilbert space:

Hellinger-Toeplitz theorem, Symmetric and self-adjoint operators, Closed linear operators, Spectrum of an unbounded self-adjoint linear operator, Cayley Transformation of an operator, Spectral theorem for unitary and self-adjoint operators, Multiplication operator and differentiation operator, Application to Quantum Mechanics.

References:

1. Brown, A. and Pearcy, C., Introduction to Operator Theory, I, II, Springer-Verlag, 1977.
2. Coway, J.B., A Course in Functional Analysis, Springer, 1990.
3. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
4. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.
5. Kreyszig, E., Introductory Functional Analysis with Applications, John Wiley and sons.
6. Bachman, G. and Narici, L., Functional Analysis, Dover Publications.
7. Taylor, A. and Lay, D., Introduction to Functional Analysis, John Wiley and Sons.
8. Dunford, N. and Schwartz, J.T., Linear Operators 3, John Wiley and Sons.
9. Halmos, P.R., Introduction to Hilbert space and the theory of Spectral Multiplicity, Chelsea PublishingCo., N.Y.

TOPOLOGICAL GROUPS AND HARMONIC ANALYSIS I (50 Marks)

Definition of a topological group and its basic properties, Subgroups and Quotient Groups, products, fundamental systems of neighbourhoods, open subgroups. First, Second, Third Isomorphism Theorems, Properties of Topological Groups involving Connectedness and Compactness. Separation Axioms in topological groups, Compact and Locally Compact Topological Groups.

Preliminaries of Lebesgue Measure, Measurable Function, Integration, Product Measure, Fubini's Theorem, Signed Measure, Hahn Decomposition Theorem, Mutually Singular Measure, Radon Nikodym Theorem, Lebesgue Decomposition, Decomposition into Absolutely Continuous and Singular parts, Differentiation and Integration. Baire and Borel Sets, Borel and Baire Measures on Locally Compact Spaces, Regularity of Measure, Integration of Continuous Functions with compact Support, Statement of Riesz-Markoff Theorem.

References:

1. Hewitt, E. and Ross, K., Abstract Harmonic Analysis, Vol I, II, Academic Press, NY, 1963.
2. Bachman, G., Elements of Abstract Harmonic Analysis, Academic Press, NY and London, 1964.
3. Rudin, W., Fourier Analysis on Groups, McGraw Hill Publishing Co. Ltd.
4. Loomis, L., An Introduction to Abstract Harmonic Analysis, Van Nostrand, N.J., 1953.
5. Goldberg, R.R., Fourier Transforms, Cambridge University Press, London & NY, 1961.

6. Folland, G.B., Real Analysis, John Wiley and sons, 1984.
7. Humphreys, J.E., Introduction to Lie algebras and representation theory, GTM (9), Springer.
8. Bagchi, S.C., Madan, S., Sitaram, A. and Tiwari, U.B., A first course on representation theory andlinear Lie groups, University Press.
9. Lang, S., *SL*(2,R), GTM (105), Springer.
10. Knapp, W., Representation theory of semisimple groups. An overview based on examples, PrincetonMathematical Series (36), Princeton University Press.

TOPOLOGICAL GROUPS AND HARMONIC ANALYSIS II (50 Marks)

The Haar Integral, Haar Measure on Locally Compact Groups, Convolutions of Functions and Measures (without proof).

Group actions on topological spaces, the space *X/G* in the topological as also in the analytical case assuming regularity conditions of the group action.

Haar Measure on R,T,Z and some simple matrix groups.

Elements of Representation Theory, Unitary Representations of Locally Compact Groups, Invariant subspaces and irreducibility, Schur's lemma. Compact groups: Unitarity of finite dimensional representations, Peter-Weyl theory, Representations of *SU*(2,C), Representation of a finite group.

References:

1. Hewitt, E. and Ross, K., Abstract Harmonic Analysis, Vol I,II, Academic Press, NY, 1963.
2. Bachman, G., Elements of Abstract Harmonic Analysis, Academic Press, NY and London, 1964.
3. Rudin, W., Fourier Analysis on Groups, McGraw Hill Publishing Co. Ltd.
4. Loomis, L., An Introduction to Abstract Harmonic Analysis, Van Nostrand, N.J., 1953.
5. Goldberg, R.R., Fourier Transforms, Cambridge University Press, London & NY, 1961.
6. Folland, G.B., Real Analysis, John Wiley and sons, 1984.
7. Humphreys, J.E., Introduction to Lie algebras and representation theory, GTM (9), Springer.
8. Bagchi, S.C., Madan, S., Sitaram, A. and Tiwari, U.B., A first course on representation theory andlinear Lie groups, University Press.
9. Lang, S., *SL*(2,R), GTM (105), Springer.
10. Knapp, W., Representation theory of semisimple groups. An overview based on examples, PrincetonMathematical Series (36), Princeton University Press.

OPERATIONS RESEARCH I (50 Marks)

Dynamic Programming:

Introduction, Nature of dynamic programming, Deterministic processes, Non-Sequential discrete optimization, Allocation problems, Assortment problems, Sequential discrete optimization, Long-term planning problem, Multi-stage decision process, Application of Dynamic Programming in production scheduling and routing problems.

Sequencing:

Problems with n jobs two machines, n-jobs three machines and n-jobs, m-machines.

Inventory control:

Inventory control -Deterministic including price breaks and Multi-item with constraints, -Probabilistic (with and without lead time), Fuzzy and Dynamic inventory models.

Network:

PERT and CPM: Introduction, Basic difference between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM Network components and precedence relationships, Critical path analysis, Probability in PERT analysis.

Replacement and Maintenance Models:

Introduction, Failure Mechanism of items, Replacement of items deteriorates with time, Replacement policy for equipments when value of money changes with constant rate during the period, Replacement of items that fail completely individual replacement policy and group replacement policy, other replacement problems staffing problem, equipment renewal problem.

1. Ronald, V. Hartley, Operations Research A Managerial Emphasis Goodyear Publishing Company Inc., 1976, California.
2. Beveridge and Scheehter, Optimization Theory and Practice, McGraw Hill Kogakusha, Tokyo, 1970.
3. Gross and Harris, Queueing Theory, John Wiley
4. Johnson L.A., Montgomery, Operations Research in Production Planning, Scheduling &InventoryControl, John Wiley, 1974.

OPERATIONS RESEARCH II (50 Marks)

Queuing Theory:

Basic Structures of queuing models, Poisson queues M/M/1, M/M/C for finite and infinite queue length, Non-Poisson queue -M/G/1, Machine-Maintenance (steady state).

Reliability:

Concept, Reliability Definition, System Reliability, System Failure rate, Reliability of the Systems connected in Series or / and parallel.

Information Theory:

Introduction, Communication Processes memory less channel, the channel matrix, Probability relation in a channel, noiseless channel. A Measure of information- Properties of Entropy function, Measure of Other information quantities marginal and joint entropies, conditional entropies, expected mutual information, Axiom for an Entropy function, properties of Entropy function. Channel capacity, efficiency and redundancy. Encoding Objectives of Encoding. Shannon Fano Encoding Procedure, Necessary and sufficient Condition for Noiseless Encoding.

Simulation:

Introduction, Steps of simulation process, Advantages and disadvantages of simulation, Stochastic simulation and random numbers Monte Carlo simulation, Random number, Generation, Simulation of Inventory Problems, Simulation of Queuing problems, Role of computers in Simulation, Applications of Simulations.

References:

1. Ronald, V. Hartley, Operations Research A Managerial Emphasis Goodyear Publishing Company Inc., 1976, California.
2. Beveridge and Scheehter, Optimization Theory and Practice, McGraw Hill Kogakusha, Tokyo, 1970.
3. Gross and Harris, Queueing Theory, John Wiley
4. Johnson L.A., Montgomery, Operations Research in Production Planning, Scheduling &InventoryControl, John Wiley, 1974.

FLUID MECHANICS I (50 Marks)

Viscous incompressible fluid flow:

Field equations, Boundary conditions, Reynold’s number, Vorticity equation, Circulation, Flow through parallel plates, Flow through pipes of circular and elliptic cross sections.

Inviscid Compressible Fluid:

Field equations, Circulation, Propagation of small disturbance. Mach number and cone, Bernoulli’s equation. Irrotational motion, Velocity potential. Bernoulli’s equation in terms of Mach number. Pressure, density, temperature in terms of Mach number, Critical conditions. Steady channel flow, Area-velocity relation. Mass flow through a converging nozzle. Flow through a de-Laval nozzle. Normal shock waves, Governing equations and the solution. Entropy change.

Vortex Motion:

Vortex line, Vortex tube, Properties of the vortex, Strength of the vortex, Rectilinear vortices, Velocity component, centre of vortices. A case of two vortex filaments, vortex pair. Vortex doublet. Image of vortex filament with respect to a plane. An infinite single row of parallel rectilinear vortices of same strength. Two infinite row of parallel rectilinear vortices, Karman’s vortex street. Rectilinear vortex with circular section. Rankine’s combine vortex. Rectilinear vortices with elliptic section.

References:

1. Prandt, L., Essential of fluid dynamics, Springer, 2004.
2. White, F.M., Viscous Fluid Flow, McGraw Hill, 1991.
3. Panton, R.L., Incompressible Flow, John Wiley and Sons, 1984.
4. Rosenhead, L., Laminar Boundary Layer, Dover, 1988.
5. Sherman, F.S., Viscous Flow (McGraw Hill).

6. Pai, S.I., Viscous Flow Theory, D.VanNostrand, 1997.
7. Schlichting, H., Boundary Layer Theory, Springer, 2001.
8. Chorlton, F., Text Book of Fluid Dynamics, CBS Publ.
9. Love, A.E., A treatise on mathematical theory of elasticity, McGraw Hill Book Co., 1956.
10. Kondepudi, D. and Prigogine, I., Modern thermodynamics, John Wiley and Sons, Inc., 1998.
11. Landau, L.M. and Lifshitz, E.M., Fluid Mechanics, Butterworth Heinemann, 2005.

FLUID MECHANICS II (50 Marks)

Irrotational Motion in Two Dimensions:

General motion of a cylinder in two dimensions. Motion of a cylinder in a uniform stream, Liquid streaming past a fixed circular cylinder and two coaxial cylinders. Equations of motion of a circular cylinder. Circulation about a moving cylinder. Conjugate function. Elliptic cylinder. Liquid streaming past a fixed elliptic cylinder. Elliptic cylinder rotating in an infinite mass of liquid at rest at infinity. Circulation about an elliptic cylinder. Kinetic energy. Blasius theorem and its application. Kutta and Joukowski theorem, D'Alemberts paradox. Application of conformal mapping.

Viscous Flow:

Navier-Stokes equations, Vorticity and circulation in viscous fluids. Reynolds number, Boundary conditions, Flow of a viscous fluid with free surface on an inclined plane. Flow between parallel plates. Flow through pipes of circular, elliptic section under constant pressure gradient. Laminar flow between concentric rotating cylinder. Steady motion of a viscous fluid due to a slowly rotating sphere. Unsteady motion of a flat plate. Pulsatile flow between parallel surfaces. Prandtls concept of boundary layer. Boundary layer flow along a flat plate. Momentum and energy integral equation for the boundary layer. Von Karman Pohlhousen method. Turbulence, Calculation of Turbulent BL.

References:

1. Prandt, L., Essential of fluid dynamics, Springer, 2004.
2. White, F.M., Viscous Fluid Flow, McGraw Hill, 1991.
3. Panton, R.L., Incompressible Flow, John Wiley and Sons, 1984.
4. Rosenhead, L., Laminar Boundary Layer, Dover, 1988.
5. Sherman, F.S., Viscous Flow (McGraw Hill).
6. Pai, S.I., Viscous Flow Theory, D.VanNostrand, 1997.
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9. Love, A.E., A treatise on mathematical theory of elasticity, McGraw Hill Book Co., 1956.
10. Kondepudi, D. and Prigogine, I., Modern thermodynamics, John Wiley and Sons, Inc., 1998.
11. Landau, L.M. and Lifshitz, E.M., Fluid Mechanics, Butterworth Heinemann, 2005.

†In case of Special Papers, the course instructor(s) will have the liberty to frame the syllabus, however it should be presented before the Department Council before starting the course and should be duly accepted. Although the course structure and syllabus describes a general outline of the intended courses, yet the respective faculty members shall have the freedom for the orientation of the course taught by him/her and also to improve the actual study material depending on his/her requirements keeping in view of the level of awareness of the students in a particular year/semester. If time permits, any course instructor will enjoy the liberty to extend the syllabus in a manageable amount as an experimental basis. After the course is over, the Department Council, if finds suitable, refer this case to the Board of Studies (PG) in Mathematics so that the portion can be included in the syllabus as an optional part. Any future instructor of the course will have the liberty to include/exclude that portion.