

Geometry:

The area of a square is one-half the the square of the length of a diagonal

A line parallel to a side of a triangle that intersects the other two sides divides them proportionally.

The angle bisector of an angle divides the opposite side proportional to the other two sides.

The angle bisectors intersect at a point called the incenter, that is the center of the inscribed circle.

The length of the radius of the inscribed circle is equal to the area of the triangle divided by the semiperimeter.

The perpendicular bisectors meet at a point, called the circumcenter, that is the center of the circumscribed circle.

The altitudes meet at a point called the orthocenter.

The medians intersect at a point (called the centroid) that is two-thirds of the distance from each vertex to the midpoint of the opposite side.

The coordinates of the centroid can be found by averaging the coordinates of the vertices.

(Ceva's Theorem) In triangle ABC, with J on AB, K on BC, and L on CA, the segments AK, BL and CJ are concurrent (intersect in one point) if and only if $(AJ/JB)(BK/KC)(CL/LA) = 1$.

(Ceva's Theorem Trig Form) In triangle ABC, with J on AB, K on BC, and L on CA, the segments AK, BL and CJ are concurrent (intersect in one point) if and only if $(\sin(\text{BAK})/\sin(\text{KAC}))(\sin(\text{CBL})/\sin(\text{LBA}))(\sin(\text{ACJ})/\sin(\text{JCB})) = 1$.

The height of an equilateral triangle is $s\sqrt{3}/2$ (So the area is $s^2 \sqrt{3}/4$)

A regular hexagon inscribed in a circle can be divided into six congruent equilateral triangles, where the length of a side is equal to the radius of the circle. Therefore the area of a regular hexagon inscribed in a circle of radius r is: $A=r^2 3\sqrt{3}/2$.

The midpoint of the hypotenuse of a right triangle is equidistant to all three vertices.

When an altitude is drawn to the hypotenuse of a right triangle, three similar triangles are formed, and the following hold:

The square of the length of the altitude is equal to the product of the lengths of the segments of the hypotenuse.

When the length of each leg is squared it equals the product of the hypotenuse and the segment on the hypotenuse adjacent to that leg.

The sum of the exterior angles of a polygon, one at each vertex, is 360.

The interior angles of a regular polygon with n sides are $180-(360/n)$ each.

Two tangent segments drawn to a the same circle from the same point are equal.

The figure made by connecting a common external tangent, two radii, and a segment connecting the two centers is a trapezoid with two right angles. (Often you divide into a rectangle and a right triangle to solve for missing lengths.)

Two chords are have the same lengths if and only if they cut off equal length arcs.

Two chords are have the same lengths if and only if they are the same distance from the center.

A diameter drawn through a chord is perpendicular to the chord if and only if it bisects the chord and the arc.

The measure of an angle with its vertex at the center of a circle is equal to the measure of the intercepted arc.

The measure of an inscribed angle (that is an angle with its vertex on the circle) is one-half the intercepted arc.

If two inscribed angles intercept the same arc, then the angles have the same measure.

The measure of the angle made by a chord and a tangent is one-half the measure of the intercepted arc.

An angle inscribed in a semicircle is a right angle.

A triangle inscribed in a circle is a right triangle if and only if one of the sides is a diameter.

The measure of the angle formed by two intersecting chords is equal to one-half the sum of the measures of the intercepted arcs of the angle and its vertical angle.

The measure of the angle formed by two secants, two tangents, or a secant and a tangent drawn from a point outside a circle is equal to one-half the difference of the measures of the intercepted arcs.

If two chords intersect, the product of the lengths of the two parts of one chord is equal to the product of the lengths of the parts of the other chord.

When two secant segments are drawn to a circle from an external point, the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

When a secant segment and a tangent segment are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment is equal to the square of the length of the tangent segment.

In a cyclic quadrilaterals (If a quadrilateral is inscribed in a circle, it is called a cyclic quadrilateral), opposite angles are supplementary.(Hence, rectangles or squares are the only parallelograms that can be inscribed in a circle.)

(Ptolemy's Theorem) The product of the lengths of the diagonals is equal to the sum of the products of the lengths of the two pairs of opposite sides.

When a circle is circumscribed about an equilateral triangle, and chords are drawn from any point on the circle to the three vertices of the triangle, then the length of the longest chord is equal to the sum of the lengths of the other two chords.

Euler's formula for polyhedron: $f+v-e=2$ (The number of faces plus the number of vertices minus the number of edges equals 2).

There are only 5 regular polyhedron: tetrahedron (4 triangles), hexahedron (a cube, 6 squares), Octahedron (8 triangles), dodecahedron (12 pentagons), and icosahedron (20 triangles).

If the scale factor of two similar polygons is $a:b$, the ratio of their areas is $a^2:b^2$.

If the scale factor of two similar solids is $a:b$, the ratio of the lengths of two corresponding edges is $a:b$.

If the scale factor of two similar solids is $a:b$, the ratio of their surface areas is $a^2:b^2$.

If the scale factor of two similar solids is $a:b$, the ratio of their volumes is $a^3:b^3$.

Heron's Formula: If a triangle has sides of length a , b and c and $s=(a+b+c)/2$, then the area of the triangle is $\sqrt{s(s-a)(s-b)(s-c)}$.

If a rectangle is of a size that when a square is cut off of an end the remaining rectangle is similar to the original, the rectangle is said to be a "golden rectangle" and its sides are said to be in the "golden ratio". This ratio is $1:x$, where x is the positive root of $x^2-x-1=0$. ($x=(1+\sqrt{5})/2$)

(This is also the limit as k goes to infinity of two consecutive terms of the Fibonacci sequence)

The area of the triangle that has vertices with coordinates (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) can be found by the absolute value of the determinant of the matrix:

$$x_1 \ y_1 \ 1$$

$$x_2 \ y_2 \ 1$$

$$x_3 \ y_3 \ 1$$

This can be generalized to any polygon.

Combinatorics

(Multiplication Principle) If there are n choices for the first step of a two step process and m choices for the second step, the number of ways of doing the two step process is nm .

The number of arrangements of n objects is $n!$

The number of arrangements of r out of n objects is ${}_n P_r = n!/(n-r)!$

The number of arrangements of n objects in a circle is $(n-1)!$

The number of arrangements of n objects on a key ring is $(n-1)!/2$

The number of arrangements of n objects with r_1 of type 1, r_2 of type 2, ..., r_i of type i is $n!/(r_1!r_2!\dots r_i!)$

The number of ways of choosing n out of r objects is ${}_n C_r = n!/(n-r)!r!$

The number of distributions of n distinct objects in k distinct boxes is k^n .

The number of ways of distributing n identical objects in k distinct boxes is ${}_{(n+k-1)} C_n$.

The sum of the coefficients of the binomial expression $(x+y)^n$ is 2^n .

To find the sum of the coefficients of a power of any polynomial, replace the variables by 1.

When solving an equation for integer solutions, it is important to look for factoring. Important factoring forms:

$$a^2-b^2=(a-b)(a+b)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\dots+ab^{n-2}+b^{n-1})$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

If n is odd, $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \dots - ab^{n-2} + b^{n-1})$ (alternate signs)

Divisibility rules:

A number is divisible by 2 if and only if the last digit is divisible by 2.

A number is divisible by 3 if and only if the sum of the digits is divisible by 3.

A number is divisible by 4 if and only if the last 2 digits is a number divisible by 4.

A number is divisible by 5 if and only if the last digit is divisible by 5.

A number is divisible by 6 if and only if it is divisible by 2 and 3.

A number is divisible by 8 if and only if the last 3 digits is a number divisible by 8.

A number is divisible by 9 if and only if the sum of the digits is divisible by 9.

A number is divisible by 10^n if and only if the number ends in n zeros.

A number is divisible by 11 iff the sum of every other digit minus the sum of the rest of the digits is divisible by 11.

To find out if a number is divisible by seven, take the last digit, double it, and subtract it from the rest of the number.

Example: If you had 203, you would double the last digit to get six, and subtract that from 20 to get 14. If you get an answer divisible by 7 (including zero), then the original number is divisible by seven. If you don't know the new number's divisibility, you can apply the rule again.

Number Theory:

The greatest common divisor is found by looking at the prime factorizations or using the Euclidean algorithm.

The least common multiple of a and b is found by looking at the prime factorizations or $(ab)/\text{gcd}(a,b)$.

Two numbers are said to be relatively prime in the greatest common factor is 1.

If $\text{gcd}(a, b) = d$, then there exist integers x and y so that $ax + by = d$.

If d divides both a and b , then d divides $a+b$ and d divides $a-b$.

$a \equiv b \pmod{m}$ iff m divides $a-b$ iff a and b both have the same remainder when divided by m .

$a^{p-1} \equiv 1 \pmod{p}$ (a is not a multiple of p)

$a^{\phi(m)} \equiv 1 \pmod{m}$ ($\text{gcd}(a, m) = 1$)

If a probability experiment is repeated n times and the probability of success in one trial is p , then the probability of exactly r successes in the n trials is ${}_n C_r (p)^r (1-p)^{(n-r)}$.

Rules of Logarithms:

$\log_a(M) = y$ if and only if $M = a^y$

$\log_a(MN) = \log_a(M) + \log_a(N)$

$\log_a(M/N) = \log_a(M) - \log_a(N)$

$\log_a(M^p) = p \cdot \log_a(M)$

$\log_a(1) = 0$

$\log_a(a^p) = p$

$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$

The number of zeros at the end of $n!$ is determined by the number of 5's. To find this you do the following process: $n/5 = n_1$ and some remainder. Drop the remainder and compute $n_1/5 = n_2$ plus some remainder. Drop

the remainder and compute $n_2/5 = n_3$ plus some remainder, etc. The number of zeros is $n_1+n_2+n_3+n_4...$

The sum of any consecutive integers k through n , with n being the larger, simply use this equation:

$$(n+k)(n-k+1)$$

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