Trigonometric Identities & Formulas

Tutorial Services – Mission del Paso Campus

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin \boldsymbol{x} = \cos \boldsymbol{x} \tan \boldsymbol{x} \qquad \cos \boldsymbol{x} = \sin \boldsymbol{x} \cot \boldsymbol{x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

Pythagorean Identities in Radical Form

$$\sin x = \pm\sqrt{1 - \cos^2 x}$$

$$\tan x = \pm\sqrt{\sec^2 x - 1}$$

$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Odd-Even Identities

Also called negative angle identities

Sin $(-x)$ = -sin x Csc $(-x)$ = -csc x

Cos $(-x)$ = cos x Sec $(-x)$ = sec x

Tan $(-x)$ = -tan x Cot $(-x)$ = -cot x

Phase Shift $= \frac{-c}{b}$

Period $= \frac{2\pi}{b}$

Sum and Difference Formulas/Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$
$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$
$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

How to Find Reference Angles

Step 1: Determine which quadrant the angle is in

Step 2: Use the appropriate formula

Quad I	=	is the angle itself		
Quad II	=	$180 - \theta$	or	$\pi - \theta$
Quad III	=	$\theta - 180$	or	$\theta - \pi$
Quad IV	=	$360 - \theta$	or	$2\pi - \theta$

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Ratio or Quotient Identities

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

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Sum and Difference Formulas - Identities

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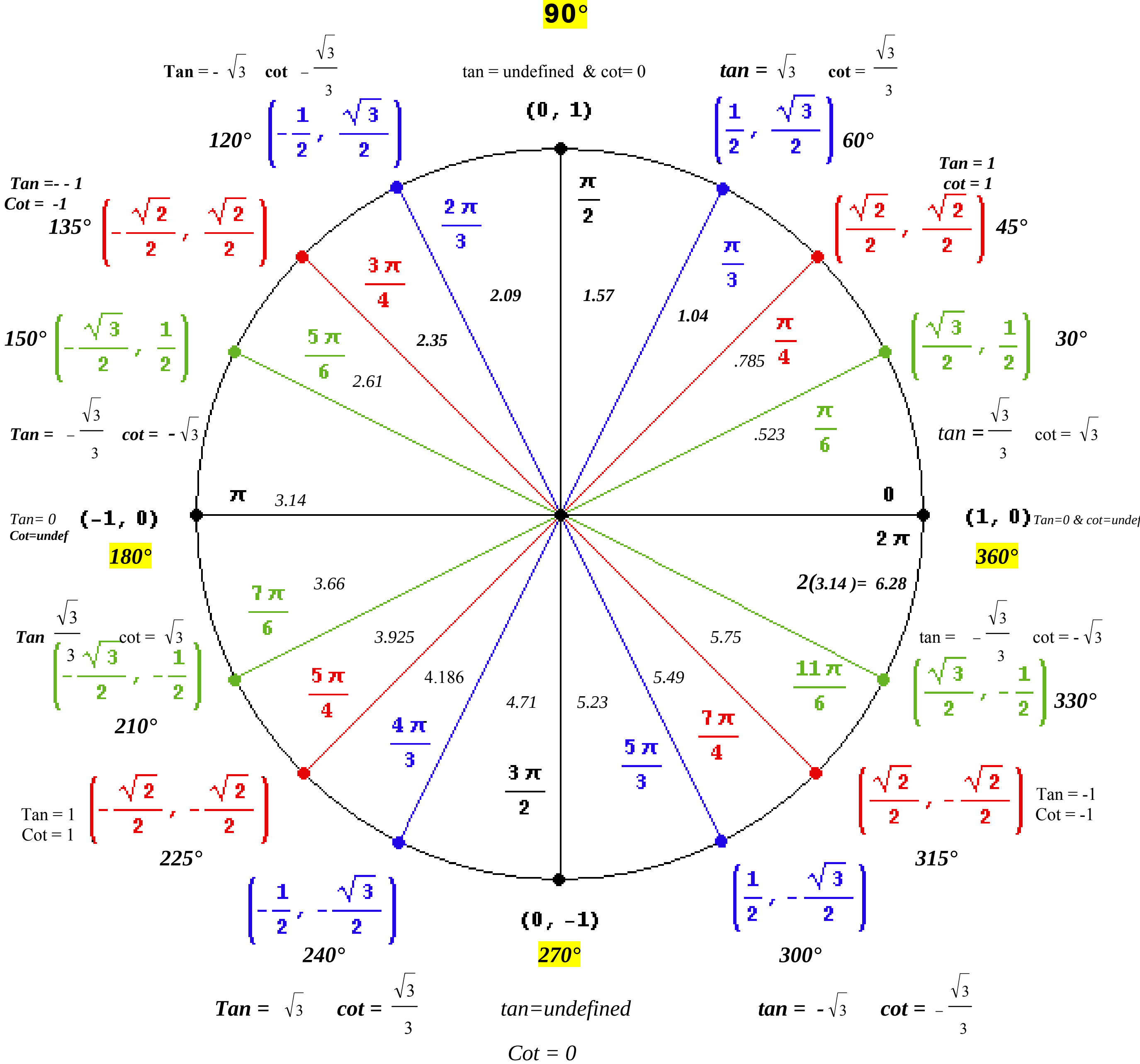
The Unit Circle



Definition of Trigonometric Functions concerning the Unit Circle

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \qquad \csc\theta = \frac{hyp}{opp} = \frac{r}{y}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r} \qquad \sec\theta = \frac{hyp}{adj} = \frac{r}{x}$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} \qquad \cot\theta = \frac{adj}{opp} = \frac{x}{y}$$

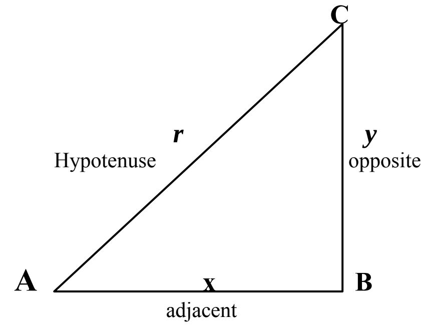
Right Triangle Definitions of Trigonometric Functions

Note: sin & cos are complementary angles, so are tan & cot and sec & cos, and the sum of complementary angles is 90 degrees.

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ $\csc\theta = \frac{hyp}{opp} = \frac{r}{y}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ $\sec\theta = \frac{hyp}{adj} = \frac{r}{x}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$ $\cot\theta = \frac{adj}{opp} = \frac{x}{y}$



Adjacent = is the side adjacent to the angle in consideration. So if we are considering Angle A, then the adjacent side is CB

Trigonometric Values of Special Angles

Degrees	0°	30°	45°	60°	90°	180°	270°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sinθ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tanθ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined

To Convert *Degrees to Radians*, Multiply by $\frac{\pi \text{ rad}}{180 \deg}$

To Convert *Radians to Degrees*, Multiply by $\frac{180 \deg}{\pi \text{ rad}}$

Vocabulary

- Cotangent Angles - are two angles with the same terminal side
- Reference Angle - is an acute angle formed by terminal side of angle(α) with x-axis

Double Angle Identities

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Half Angle Identities

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$$

Power Reducing Formulas

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Product-to-Sum Formulas

$$\sin u\sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$

$$\cos u\cos v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$

$$\sin u\cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$

$$\cos u\sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Law of Sines

Solving Oblique Triangles using sine: AAS, ASA, SSA, SSS, SAS

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

Cosine: SAS, SSS

Standard Form

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = b^2 + a^2 - 2ab\cos C$$

Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Finding the Area of non-90degree Triangles

Area of an Oblique Triangle

$$area = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Heron's Formula

Step 1: Find "s" $\quad s = \dfrac{(a+b+c)}{2}$

Step 2: Use the formula $\quad area = \sqrt{s(s-a)(s-b)(s-c)}$