

MOTION IN 1 – DIMENSION (Rectilinear Motion & Relative motion)

MECHANICS

Kinematics

The word kinematics means 'science of motion' branch of the mechanics which deals with study of motion without going into the cause of motion, i.e. force, torque etc.

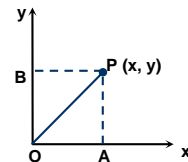
Dynamics (or Kinetics)

It is branch of mechanics which is concerned about the causes (i.e. force, torque) that cause motion of bodies.

Position

If a particle is restricted to move along a given straight line (assumed along x-axis), its position is represented by the x-coordinate relative to a fixed origin.

If the particle moves in a plane (let x-y plane) its position is completely known when the x and y coordinates of its position are known with respect to the given coordinate axes ox and oy.



Rest : If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

Motion : If a body changes its position as time passes with respect to frame of reference, it is said to be in motion

Types of motion :

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
Ex. (i) Motion of car on a straight road. (ii) Motion of freely falling body.	Ex. (i) Motion of car on a circular tum. (ii) Motion of billiards ball.	Ex. (i) Motion of flying kite. (ii) Motion of flying insect.

DISTANCE AND SPEED

(a) Distance :

- (i) The total length of actual path traversed by the body between initial and final positions is called distance.
- (ii) It has no direction and is always positive.
- (iii) Distance covered by particle never decreases.
- (iv) Its SI unit is meter (m) and dimensional formula is $[M^0L^1T^0]$.

(b) Speed :

The rate of distance covered with time is called speed.

- (i) It is a scalar quantity having symbol v .
- (ii) Dimension : $[M^0L^1T^{-1}]$
- (iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

(c) Average speed :

- (i) It is defined as distance travelled by particle per unit time in a given interval of time.
- (ii) If S is the distance travelled by particle in time interval t , then average speed in that time interval is $\frac{S}{t}$.

(iii) Average speed = $\frac{\text{Total distance travelled}}{\text{Time taken}}$; $v_{av} = \frac{\Delta s}{\Delta t}$

(d) Instantaneous speed :

- (i) It is the speed of a particle at a particular instant of time. When we say "speed", it usually means instantaneous speed.

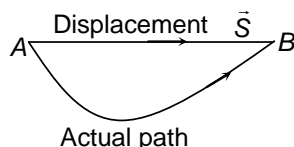
(ii) The instantaneous speed is average speed for infinitesimally small time interval (*i.e.*, $\Delta t \rightarrow 0$).

(iii) Instantaneous speed $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

DISPLACEMENT AND VELOCITY

(a) Displacement

- (i) The change in position of a body in a certain direction is known as displacement.
- (ii) The distance between the initial and final position is known as **magnitude of displacement**.
- (iii) Displacement of an object may be positive, negative or zero and it is independent of the path followed by the object.
- (iv) Its SI unit is meter and dimensional formula is $[M^0L^1T^0]$.
- (v) Displacement is a vector \vec{S} drawn from the initial position (A) to the final position (B)



(b) **Velocity** : Velocity is the rate of change of position vector.

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Unit : ms^{-1} (metre per second)

(c) **Average velocity** : It is defined as the ratio of displacement to time taken by the body

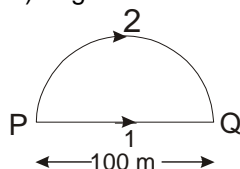
$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}} ; \quad \vec{v}_{\text{av}} = \frac{\Delta \vec{r}}{\Delta t}$$

(d) **Instantaneous velocity (v)** : It is the velocity of particle at any instant of time

$$\text{Mathematically, } v = \lim_{\Delta t \rightarrow 0} \langle v \rangle = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

(e) **Uniform velocity** : A particle with uniform velocity undergoes equal displacements in equal intervals of time however small the intervals may be.

Example 1: Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).



(a) Find the distance travelled by Ram and Shyam?

(b) Find the displacement of Ram and Shyam?

Solution :

(a) Distance travelled by Ram = 100 m

Distance travelled by Shyam = $\pi (50 \text{ m}) = 50\pi \text{ m}$

(b) Displacement of Ram = 100 m

Displacement of Shyam = 100 m

AVERAGE SPEED AND AVERAGE VELOCITY

(i) If a body covers s_1 distance with speed v_1 , s_2 with speed v_2 , then its average speed is

$$v_{\text{av}} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \frac{\sum s}{\sum \frac{s}{v}}$$

(ii) If a body covers first half distance with speed v_1 and next half with speed v_2 , then

$$\text{Average speed} = \frac{2v_1v_2}{v_1+v_2} \quad (\text{Harmonic mean})$$

(iii) If a body travels with uniform speed v_1 for time t_1 and with uniform speed v_2 for time t_2 , then average

$$\text{speed} = \frac{v_1t_1 + v_2t_2}{t_1 + t_2} = \frac{\sum vt}{\sum t}$$

If $t_1 = t_2 = \frac{T}{2}$ then $v_{av} = \frac{v_1 + v_2}{2}$ [T = time of journey] (Arithmetic mean)

(iv) If body covers first one third with speed v_1 , next one third with speed v_2 and remaining one third with

$$\text{speed } v_3 \text{ then } v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

(v) If a body moves from one point (A) to another point (B) with speed v_1 and returns back (from B to A)

$$\text{with speed } v_2 \text{ then average velocity is 0 but average speed} = \frac{2v_1v_2}{v_1 + v_2}$$

Example 2: In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find
(a) Average speed of Ram and Shyam?
(b) Average velocity of Ram and Shyam?

Sol. (a) Average speed of Ram = $\frac{100}{4}$ m/s = 25 m/s

$$\text{Average speed of Shyam} = \frac{50\pi}{5} \text{ m/s} = 10\pi \text{ m/s}$$

(b) Average velocity of Ram = $\frac{100}{4}$ m/s = 25 m/s

$$\text{Average velocity of Shyam} = \frac{100}{5} \text{ m/s} = 20 \text{ m/s}$$

ACCELERATION

Time rate of change of velocity is called **acceleration**.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Unit : ms^{-2} (metre per second²)

(a) **Average acceleration** : The time rate of change of velocity of an object is called acceleration of the object.

$$a_{av} = \frac{\text{change in velocity}}{\text{total time}}; \quad \vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

(b) **Instantaneous acceleration** : The acceleration at any instant is called **instantaneous acceleration**. Mathematically

$$a = \lim_{\Delta t \rightarrow 0} \langle a \rangle = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

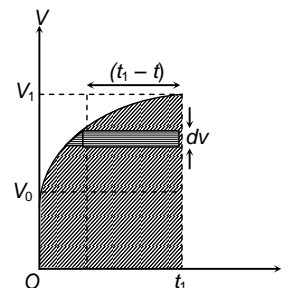
(c) **Uniform acceleration** : A particle with uniform acceleration undergoes equal changes in velocity in equal intervals of time, however small the intervals may be.

GRAPHICAL SOLUTION OF RECTILINEAR MOTION

(a) v-t Curve

The area under the v-t curve measures the change in position x .

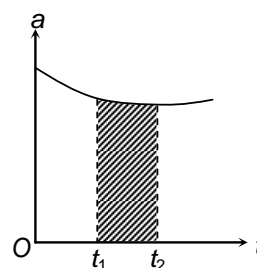
$$x_1 - x_0 = \text{area under v-t curve} = \int_0^{t_1} v dt$$



(b) a-t Curve

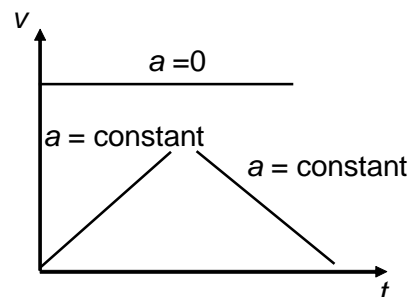
Area under the a –t curve measures the change in velocity

$$v_2 - v_1 = \text{area under (a –t) curve} = \int_{t_1}^{t_2} a \, dt$$



(c) Characteristics of v-t graph

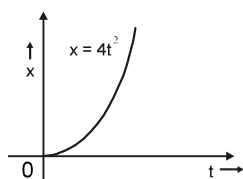
- (i) If the graph obtained is a line parallel to x-axis, the acceleration is zero.
- (ii) If the graph obtained is an oblique straight line of positive slope, the acceleration is uniform and if it is of negative slope, the retardation is constant.



(d) Graphical representation of motion

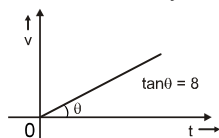
- (i) Slope of tangent to position time graph gives velocity.
- (ii) Slope of tangent to v–t curve gives acceleration.
- (iii) Area enclosed between v–t curve and time axis between an interval of time gives displacement.
- (iv) Slope of tangent to a–t curve gives rate of change of acceleration
- (v) Area enclosed between a–t curve and time axis between an interval of time gives change in velocity.

Example 3 : The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



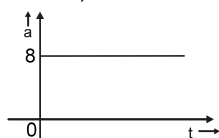
Solution : $x = 4t^2 \Rightarrow v = 8t$

Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.



$$a = \frac{dv}{dt} = 8$$

Hence, acceleration is constant throughout and is equal to 8.



EQUATIONS OF MOTION

General equations of motion :

$$v = \frac{dx}{dt} \Rightarrow dx = v dt \Rightarrow \int dx = \int v dt$$

$$a = \frac{dv}{dt} \Rightarrow dv = a dt \Rightarrow \int dv = \int a dt$$

$$a = \frac{v dv}{dx} \Rightarrow v dv = a dx \Rightarrow \int v dv = \int a dx$$

Equations of motion of a particle moving with uniform acceleration in straight line :

(i) $v = u + at$

(ii) $S = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2aS$

(iv) $S_{n^{\text{th}}} = u + \frac{1}{2}a(2n-1)$

(v) $x = x_0 + ut + \frac{1}{2}at^2$

Here

u = velocity of particle at $t = 0$

S = Displacement of particle between 0 to t

= $x - x_0$ (x_0 = position of particle at $t = 0$, x = position of particle at time t)

a = uniform acceleration

v = velocity of particle at time t

$S_{n^{\text{th}}}$ = Displacement of particle in n^{th} second

Motion under gravity

Whenever a particle is thrown up or down or released from a height, it falls freely under the effect of gravitational force of earth.

The equations of motion :

(i) $v = u + gt$

(ii) $h = h_0 + ut + \frac{1}{2}gt^2$ or $h - h_0 = s = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2g(h - h_0)$ or $v^2 = u^2 + 2gs$

(iv) $h_{n^{\text{th}}} = u + \frac{g}{2}(2n-1)$

where h = vertical displacement, $h_{n^{\text{th}}}$ = vertical displacement in n^{th} second

Example - 4 : A police inspector in a jeep is chasing a pickpocket on a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a . Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

Solution : Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \quad \dots (i)$$

During this interval the jeep travels a distance

$$s + d = vt \quad \dots (ii)$$

By (i) and (ii), $\frac{1}{2}at^2 + d = vt$

$$\text{or, } t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if t is real and positive.

This will be possible if $v^2 \geq 2ad$ or, $v \geq \sqrt{2ad}$.

RELATIVE MOTION IN ONE DIMENSION

- (i) If two bodies A and B are moving in straight line same direction with velocity V_A and V_B , then relative velocity of A with respect to B is $v_{AB} = v_A - v_B$. Similarly $v_{BA} = v_B - v_A$



- (ii) If two bodies A and B are moving in straight line in opposite direction then



$$v_{AB} = v_A + v_B \text{ (towards B)}$$

$$v_{BA} = v_B + v_A \text{ (towards A)}$$

$$v_{AB} = -v_{BA}$$

Same concept is used for acceleration also.

- (iii) If two cars A and B are moving in same direction with velocity v_A and v_B ($v_A > v_B$) when A is behind B at a distance d , driver in car A applies brake which causes retardation a in car A , then minimum value of d to avoid collision is $\frac{(v_A - v_B)^2}{2a}$ i.e., $d > \frac{(v_A - v_B)^2}{2a}$.

- (iv) A particle is dropped and another particle is thrown downward with initial velocity u , then

(a) Relative acceleration is always zero

(b) Relative velocity is always u .

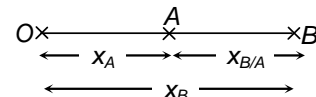
(c) Time at which their separation is x is $\frac{x}{u}$.

5. Two bodies are thrown upwards with same initial velocity with time gap τ . They will meet after a time t from projection of first body.

$$t = \frac{\tau}{2} + \frac{u}{g}$$

RELATIVE MOTION OF TWO PARTICLES

When two particles A and B move along the same straight line, denoting by $x_{B/A}$, the relative position coordinate of B with respect to A , we have $\vec{x}_B = \vec{x}_A + \vec{x}_{B/A}$



Denoting by $v_{B/A}$ and $a_{B/A}$ respectively, the relative velocity and the relative acceleration of B with respect to A , we have

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

DEPENDENT MOTIONS

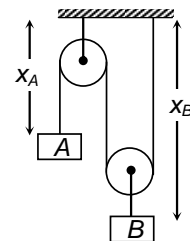
The position of a particle will depend upon the position of another or several other particles. The motions are then said to be dependent. In figure (a) the position of block B depends upon the position of block A

$$x_A + 2x_B = \text{length of rope}$$

Differentiating twice with respect to time t

$$v_A + 2v_B = 0$$

$$a_A + 2a_B = 0$$



Example - 5 : A body is released from a height and falls freely towards the earth. Exactly 1 sec later, another body is released. What is the distance between the bodies 2 sec after the release of the second body if $g = 9.8 \text{ m/s}^2$?

Solution : According to given problem, second body falls for 2 s, so that

$$h_2 = \frac{1}{2} g (2)^2$$

The first body has fallen for $2 + 1 = 3$ sec.

$$h_1 = \frac{1}{2} g (3)^2$$

∴ Separation between two bodies 2 s after the release of second body

$$d = h_1 - h_2 = \frac{1}{2} g(3^2 - 2^2) = 4.9 \times 5 = 24.5 \text{ m.}$$