UNIT & DIMENSIONS AND MEASUREMENT STRAIGHT LINES

PHYSICAL QUANTITIES

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

PHYSICAL QUANTITIES

- Fundamental or Basic Quantities
- Suplimentary Quantities
- Derived Quantities

Fundamental Units : These are the elementary quantities which covers the entire span of physics. Any other quantities can be derived from these. All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance, time and velocity cannot be chosen as basic quantities as V = d/t). An International Organization named CGPM : General Conference on weight and Measures, chose seven physical quantities as basic or fundamental.

Fundamental Units

	Length (L)	Time (T)	Mass (M)	Temperature (K)	Electric Current (A)	Luminous Intensity (Cd)	Amount of Substance (mol)
Unit / symbol →	metre (m)	second (s)	kilogram (kg)	Kelvin (K)	Ampere (A)	candela (Cd)	mole (mol)

These are the elementary quantities (in our planet) that's why chosen as basic quantities. In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.

i.e.,

(A) Area, (V) Velocity, (ρ) Density, (S) Sp.Heat Capacity, (R) Resistance, (Cd) Luminous, (mol) mole

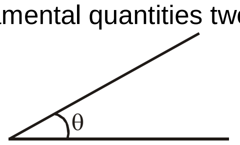
Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)

But (L) Length, (A) Area, (V) Velocity

cannot be used as basic quantities as Area = $(\text{Length})^2$ so they are not independent.

Supplementary quantities

Besides seven fundamental quantities two supplementary quantities are also defined. They are



(i) Plane angle – Unit = radian (rad)
(ii) Solid angle – Unit = Steradian (sr)

Derived quantities

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.

i.e., Momentum $P = mV = (m)\dfrac{\text{displacement}}{\text{time}} = \dfrac{ML}{T} \quad M^1 L^1 T^{-1}$

Here $[M^1 L^1 T^{-1}]$ is called dimensional formula of momentum , and we can say that momentum has

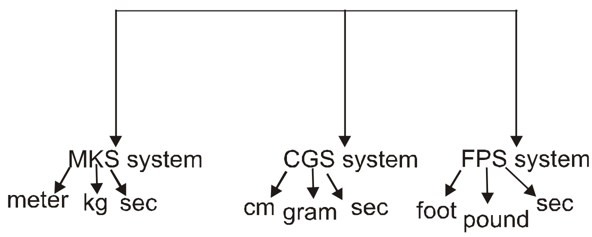
1 Dimension in M (mass)
1 Dimension in L (meter)

and –1 Dimension in T (time)
The representation of any quantity in terms of basic quantities (M,L,T....) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

Physical quantity	SI Unit			
	Name	Symbol	Expression in terms of other units	Expression in terms of SI base Units
Frequency	hertz	Hz	—	s^{-1}
Force	newton	N	—	$kg\ m\ s^{-2}$ or $kg\ m/s^2$
Pressure, stress	pascal	Pa	N/m^2 or Nm^{-2}	$kgm^{-1}s^{-2}$ or $kg/s^2\ m$
Energy, work, quantity of heat	joule	J	Nm	$kg\ m^2\ s^{-2}$ or $kg\ m^2/s^2$
Power, radiant flux	watt	W	J/s or Js^{-1}	$kg\ m^2\ s^{-3}$ or $kg\ m^2/s^3$
Quantity of electricity, Electric charge	coulomb	C	—	A-s
Electric potential, Potential difference, Electromotive force	volt	V	W/A or WA^{-1}	$Kg\ m^2s^{-3}\ A^{-1}$ or $kg\ m^2/s^3\ A$
Capacitance	farad	C	C/V	$A^2s^4\ kg^{-1}\ m^{-2}$
Electric resistance	ohm	Ω	V/A	$kg\ m^2\ s^{-3}\ A^{-2}$
Conductance	mho	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
magnetic flux	weber	Wb	Vs or J/A	$kg\ m^2\ s^{-2}\ A^{-1}$
Magnetic field, magnetic flux density, magnetic induction	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
Inductance	henry	H	Wb/A	$kg\ m^2\ s^{-2}\ A^{-2}$
Luminous flux, luminous Power	lumen	lm	—	cd /sr
Luminance	lux	lx	lm/m^2	m^{-2} cd sr^{-1}
Activity of a radio nuclide/radioactive source	becquerel	Bq		s^{-1}

OTHER CLASSIFICATION :

If a quality involves only length, mass and time (quantities in mechanics), then its unit can be written in MKS, CGS or FPS system.



For MKS system : In this system Length, mass and time are expressed in meter, kg and sec. respectively. It comes under SI system.
For CGS system : In this system ,Length, mass and time are expressed in cm, gram and sec. respectively.
For FPS system : In this system, length, mass and time are measured in foot, pound and sec. respectively.

Example 1: Find the unit of speed.

Solution : Speed = $\frac{\text{distance}}{\text{time}} = \frac{m}{s} = m/s = m s^{-1}$

Prefixes to the Power of 10:

The physical quantities whose magnitude is either too large or too small can be expressed more compactly by the use of certain SI prefixes.

Factor of 10	Prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

Example 2 : Fill in the blank by suitable conversion of units

$1\ kg\ m^2s^{-2}$ = ________ $g\ cm^2s^{-2}$

Solution : $1kg\ m^2s^{-2} = 1\times10^3\ g\ (10^2cm)^2s^{-2} = 10^7 g\ cm^2s^{-2}$

DIMENSIONS

The dimensions of a physical quantities are the powers to which the base quantities are raised to represent that quantity.

(a) Application of dimensional analysis

(i) In conversion of units from one system to other.

(ii) To check the dimensional correctness of a given physical relation.

(iii) To establish the relation among various physical quantities.

(iv) To find dimensions of physical constants or co-efficients.

(b) **Limitations of dimensional analysis**

(i) by this method the value of dimensionless constant cannot be calculated.

(ii) by this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.

(iii) if a physical quantity in mechanics depends on more than three factors, then relation among them cannot be established.

(c) **Dimensions of commonly used Physical Quantities**

S.No.	Physical Quantity (Mechanics)	SI Units	Dimensional formula
1.	Velocity = displacement/time	m/s	M^0LT^{-1}
2.	Acceleration = velocity/time	m/s^2	M^0LT^{-2}
3.	Force = mass × acceleration	$kg\text{-}m/s^2$ = Newton or N	MLT^{-2}
4.	Work = force × displacement	$kg\text{-}m^2/s^2$ = N-m = Joule or	ML^2T^{-2}
5.	Energy	J	ML^2T^{-2}
6.	Torque = force × perpendicular distance	N-m	ML^2T^{-2}
7.	Power = work/time	J/s or watt	ML^2T^{-3}

S.No.	Physical Quantity (Mechanics)	SI Units	Dimensional formula
8.	Momentum = mass × velocity	Kg-m/s	MLT^{-1}
9.	Impulse = force × time	Kg-m/s or N-s	MLT^{-1}
10.	Angle = arc/radius	radian or rad	$M^0L^0T^0$
11.	Strain = $\frac{\Delta L}{L}$ or $\frac{\Delta V}{V}$	no units	
12.	Stress = force/area	N/m^2	$ML^{-1}T^{-2}$
13.	Pressure = force/area	N/m^2	$ML^{-1}T^{-2}$
14.	Modulus of elasticity = stress/strain	N/m^2	$ML^{-1}T^{-2}$
15.	Frequency = 1/ time period	per sec or hertz (Hz)	$M^0L^0T^{-1}$
16.	Angular velocity = angle/time	rad/s	$M^0L^0T^{-1}$
17.	Moment of inertia = (mass) (distance) ^{2}	$kg\text{-}m^2$	ML^2T^0
18.	Surface tension = force/length	N/m	ML^0T^{-2}
19.	Gravitational constant	$N\text{-}m^2/kg^2$	$M^{-1}L^3T^{-2}$
20.	Thermodynamic temperature	kelvin (K)	$M^0L^0T^0K$
21.	Heat	joule	ML^2T^{-2}
22.	Specific heat	$Jkg^{-1}K^{-1}$	$M^0L^2T^{-2}K^{-1}$
23.	Latent heat	$J\ kg^{-1}$	$M^0L^2T^{-2}$
24.	Universal gas constant	$J\ mol^{-1}\ K^{-1}$	$ML^2T^{-2}K^{-1}mol^{-1}$
25.	Boltzmann's constant	JK^{-1}	$ML^2T^{-2}K^{-1}$
26.	Stefan's constant	$Js^{-1}m^{-2}\ K^{-4}$	$MT^{-3}K^{-4}$
27.	Planck's constant	Js	ML^2T^{-1}
28.	Solar constant	$J\ m^{-2}\ s^{-1}$	ML^0T^{-3}
29.	Thermal conductivity	$Js^{-1}m^{-1}\ K^{-1}$	$MLT^{-3}\ K^{-1}$
30.	Thermal resistance	$Kscal^{-1}$	$M^{-1}L^{-2}T^3K$
31.	Enthalpy	cal	ML^2T^{-2}
32.	Entropy	$cal\ K^{-1}$	$ML^2T^{-2}K^{-1}$

Example 3 : Check the accuracy of the relation $\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$, where ν is the frequency, ℓ is length, T is tension and m is mass per unit length of the string.

Solution : The given relation is

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$$

Writing the dimensions on either side, we get

$$LHS = \nu = [T^{-1}] = [M^0L^0T^{-1}]$$

$$\text{RHS} = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \left[T^{-1}\right]$$

As LHS = RHS

$\therefore$ Dimensionally the formula is correct.

SIGNIFICANT FIGURES

(a) The rules for determining the number of significant figures

(i) All the non-zero digits are significant.

(ii) All the zeros between two non-zero digits are significant.

(iii) If the number is less than 1, the zeros on the right of decimal point but to the left of 1st non-zero digit are not significant.

(iv) All the zeros to the right of the last non-zero digit (trailing zeros) in a number without a decimal point are not significant, unless they come from experiment.

(v) The trailing zeros in a number with a decimal point are significant.

(b) Significant figure in algebraic operation

(i) In multiplication or division, the number of significant digits in the final result should be equal to the number of significant digits in the quantity, which has the minimum number of significant digits.

(ii) In addition or subtraction the final result should retain as many decimal places as are there in the number with the least decimal place.

Example 4 : Add and subtract 428.5 and 17.23 with due regards to significant figures

Solution : We have

	428.50		428.50
	17.23		17.23
Sum	445.73	Difference	411.27

Rounding off the results of the above sum and difference to the first decimal,

We have correct sum 445.7 and correct difference 411.3.

ERROR ANALYSIS IN EXPERIMENTS

(a) Errors in sum or difference

Let $X = A \pm B$

Maximum absolute error in X is, $\Delta X = \pm(\Delta A + \Delta B)$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in individual quantities.

(b) Errors in product

Let, X = AB

Maximum possible value of $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

(c) Errors in division

Let, $X = \frac{A}{B}$

The maximum value of $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

(d) Errors in quantity raised to some power

Let $X = \frac{A^n}{B^m}$

Maximum value of $\frac{\Delta X}{X} = \pm\left(n\frac{\Delta A}{A} + m\frac{\Delta B}{B}\right)$

Example 5 : The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limit.

Solution : Here, ℓ = (10.5 ± 0.2) cm

b = (5.2 ± 0.1) cm

P = 2(ℓ + b) = 2 (10.5 + 5.2) = 31.4 cm

ΔP = ± 2 (Δℓ + Δb) = ± 0.6

Hence perimeter = (31.4 ± 0.6) cm.
