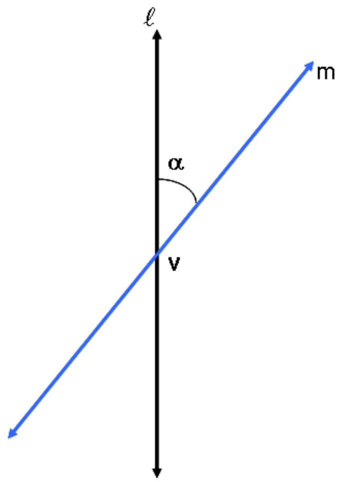
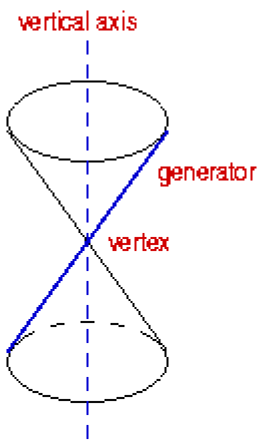
Class-XI
Mathematics
Conic Sections
Chapter-11
Chapter Notes

Key Concepts

1. Let λ be a fixed vertical line and m be another line intersecting it at a fixed point V and inclined to it at and angle α

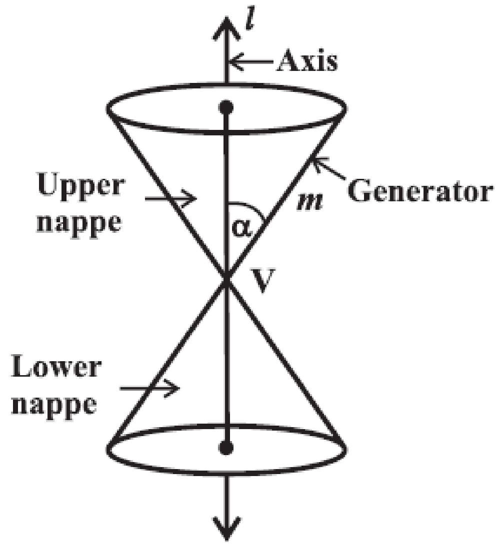


On rotating the line m around the line λ in such a way that the angle α remains constant a surface is generated is a double-napped right circular hollow cone.



2. The point V is called the vertex; the line λ is the axis of the cone. The rotating line m is called a generator of the cone. The vertex separates the

cone into two parts called nappes.



3. The sections obtained by cutting a double napped cone with a plane are called conic sections. Conic sections are of two types (i) degenerate conics (ii) non degenerate conics.

4. If the cone is cut at its vertex by the plane then degenerate conics are obtained.

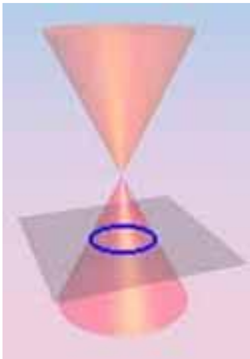
5. If the cone is cut at the nappes by the plane then non degenerate conics are obtained.

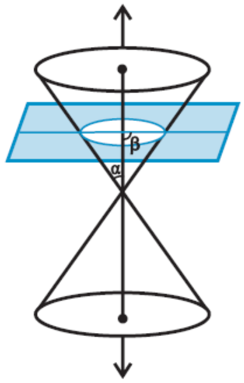
6. Degenerate conics are point, line and double lines.

7. Circle, parabola, ellipse and hyperbola are degenerate conics.

8. When the plane cuts the nappes (other than the vertex) of the cone, degenerate conics are obtained.

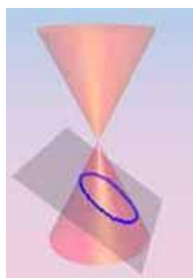
(a) When $\beta = 90^\circ$, the section is a circle.

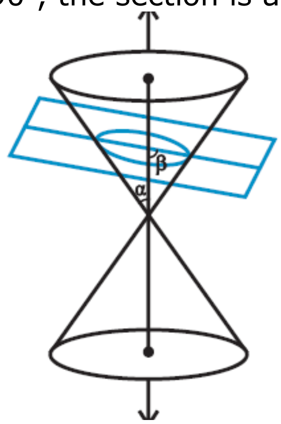




The plane cuts the cone horizontally.

(b) When $\alpha < \beta < 90^\circ$, the section is an ellipse.

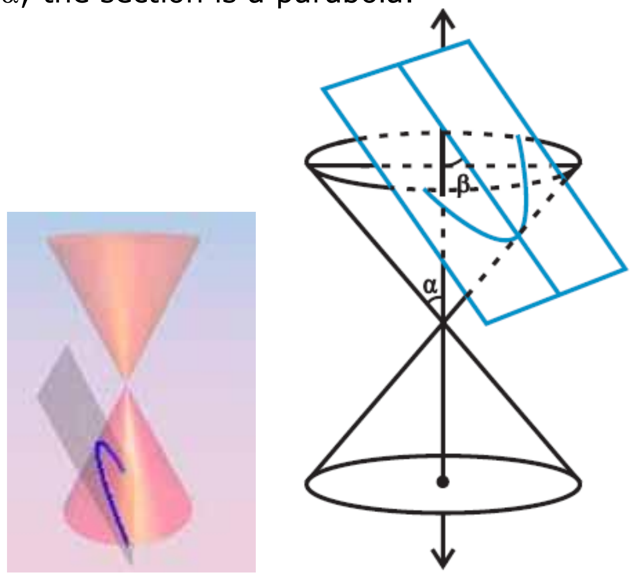




Ellipse

The plane cuts one part of the cone in an inclined manner

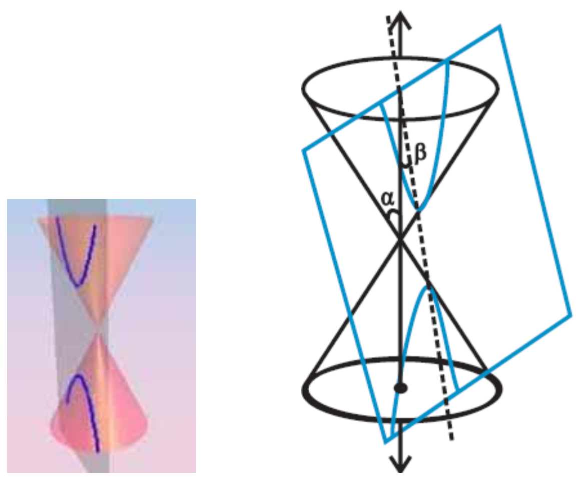
(c) When $\beta = \alpha$; the section is a parabola.



Parabola

The plane cuts the cone in such a way that it is parallel to a generator

(d) When $0 \leq \beta < \alpha$; the plane cuts through both the nappes the curve of intersection is a hyperbola.

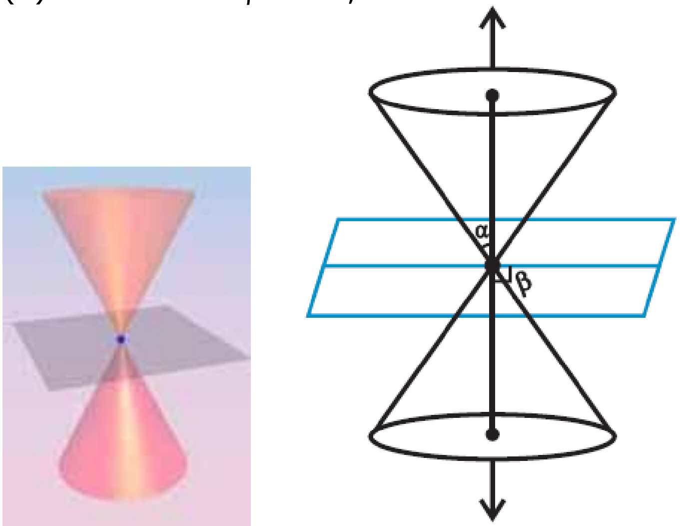


Hyperbola

The plane cuts both parts of the cone.

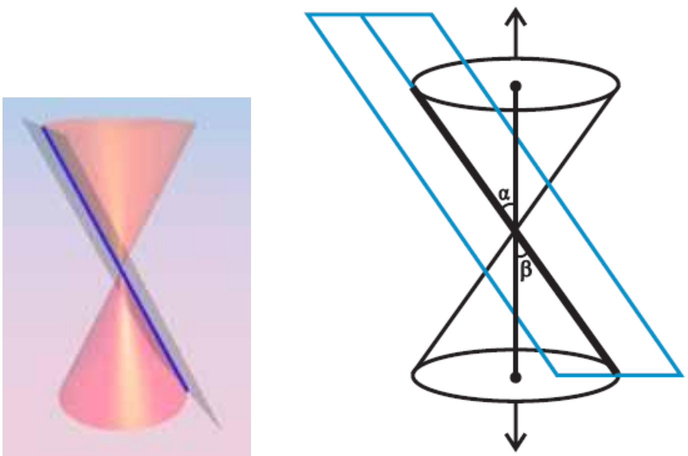
9. When the plane cuts at the vertex of the cone, we have the following different cases:

(a) When $\alpha < \beta \leq 90^0$, then the section is a point.



Point degenerated case of a circle.

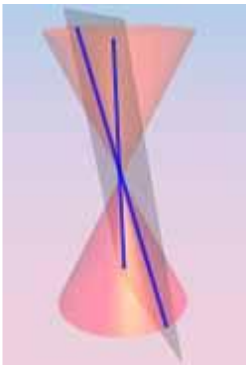
(b) When $\beta = \alpha$, the plane contains a generator of the cone and the section is a straight line.



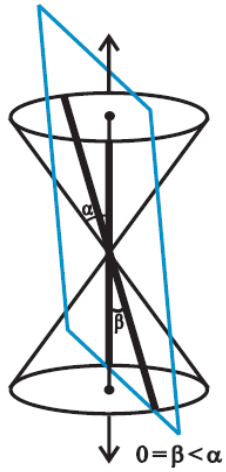
Line

It is the degenerated case of parabola.

(c) When $0 \leq \beta < \alpha$, the section is a pair of intersecting straight lines . It is the degenerated case of a hyperbola.

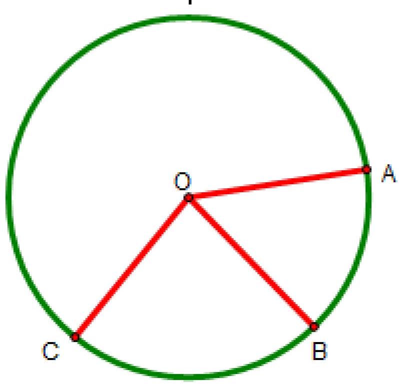


Double Line



10. A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

11. The fixed point is called the centre of the circle and the distance from the centre to a point on the circle is called the radius of the circle.



In the circle O is the centre and O A = OB = OC are the radii.

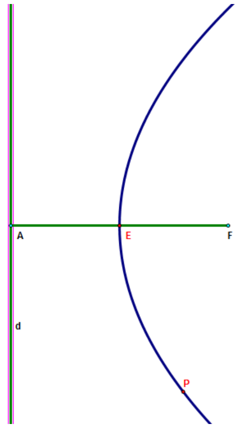
12. If the centre of a circle is (h, k) and the radius is r, then the equation of the circle is given by $(x - h)^2 + (y - k)^2 = r^2$

13. A circle with radius of length zero is a point circle.

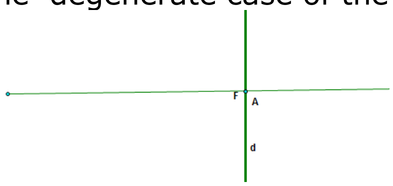
14. If the centre of a circle is at origin and the radius is r, then the equation of the circle is given by $x^2 + y^2 = r^2$

15. A **parabola** is the locus of a point, which moves in a plane in such a way that its distance from a fixed point (not on the line) in the plane is equal to

its distance from a fixed straight line in the same plane.



15. If the fixed point is on the fixed line then the set of points which are equidistant from the line and focus will be straight line which passes through the fixed point focus and perpendicular to the given line. This straight line is the degenerate case of the parabola.

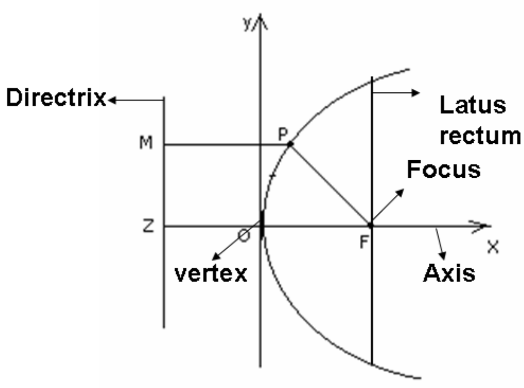


16. The fixed line is called the directrix of the parabola and the fixed point F is called the focus.

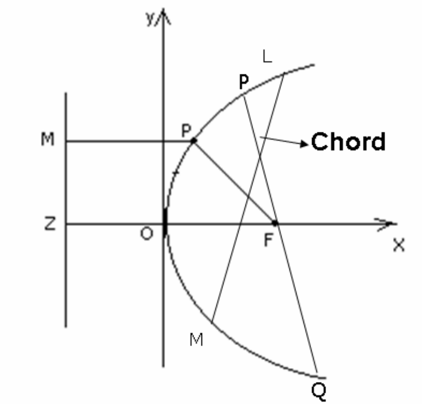
17. Para' means 'for' and 'bola' means throwing. The path taken by the trajectory of a rocket artillery etc are parabolic.
One of nature's best known approximations to parabolas is the path taken by a body projected upward and obliquely to the pull of gravity, as in the parabolic trajectory of a golf ball.

18. A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola

with the axis is called the vertex of the parabola.

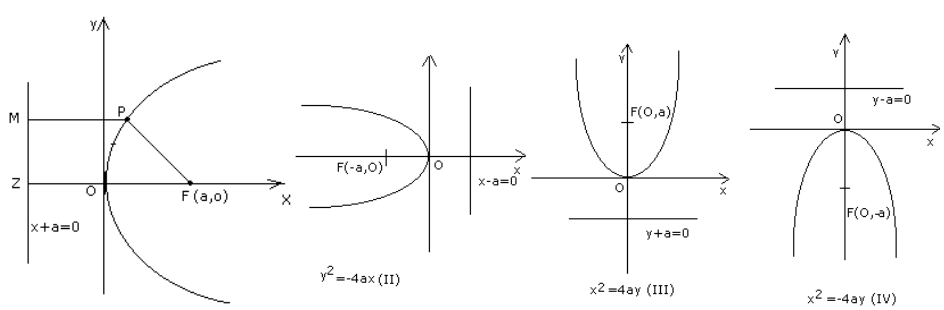


19. A chord of a parabola is the line segment joining any two points on the parabola. If the chord passes through the focus it is focal chord. LM and PQ are both chords but PQ is focal chord.



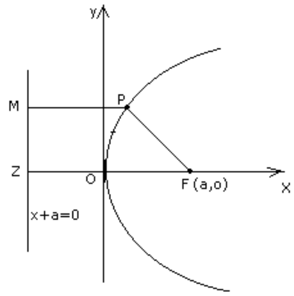
20. The chord which passes through the focus is called focal chord. Focal chord perpendicular to the axis is called the **latus rectum** of the parabola.

21. The equation of a parabola is simplest if the vertex is at the origin and the axis of symmetry is along the x-axis or y-axis. The four possible such orientations of parabola are shown below:



22. In terms of loci, the conic sections can be defined as follows: Given a line Z and a point F not on Z a conic is the locus of a point P such that the distance from P to F divided by the distance from P to Z is a constant. i.e PF/PM = e, a constant called eccentricity.

In case of parabola eccentricity e =1.

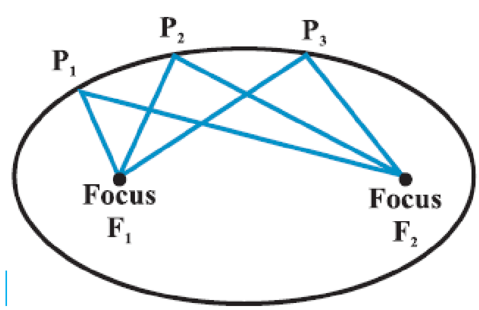


23. Parabola is symmetric with respect to its axis. If the equation has a y^2 term, than the axis of symmetry is along the x-axis and if the equation has an x^2 term, then the axis of symmetry is along the y-axis.

24. When the axis of symmetry is along the x-axis the parabola opens to the
   (a) Right if the coefficient of x is positive,
   (b) Left if the coefficient of x is negative.

25. When the axis of symmetry is along the y-axis the parabola opens
   (c) Upwards if the coefficient of y is positive.
   (d) Downwards if the coefficient of y is negative.

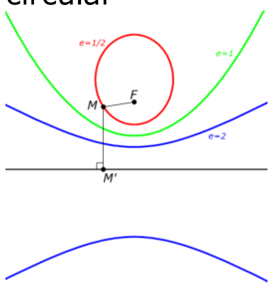
26. An ellipse is the set of all points in a plane, the sum of whose distance from two fixed points in the plane is a constant. These two fixed points are called the *foci.* For instance, if F_1 and F_2 are the foci and P_1, P_2, P_3 are the points on the ellipse then



$P_1F_1+P_1F_2=P_2F_1+P_2F_2=P_3F_1+P_3F_2$ is a constant and this constant is more than the distance between the two foci.

27. An ellipse is the locus of a point that moves in such a way that its distance from a fixed point (called focus) bears a constant ratio, to its distance from a fixed line (called directrix). The ratio e is called the eccentricity of the ellipse. For an ellipse e <1.

28. The eccentricity is a measure of the flatness of the ellipse. The eccentricity of a conic section is a measure of how far it deviates from being circular



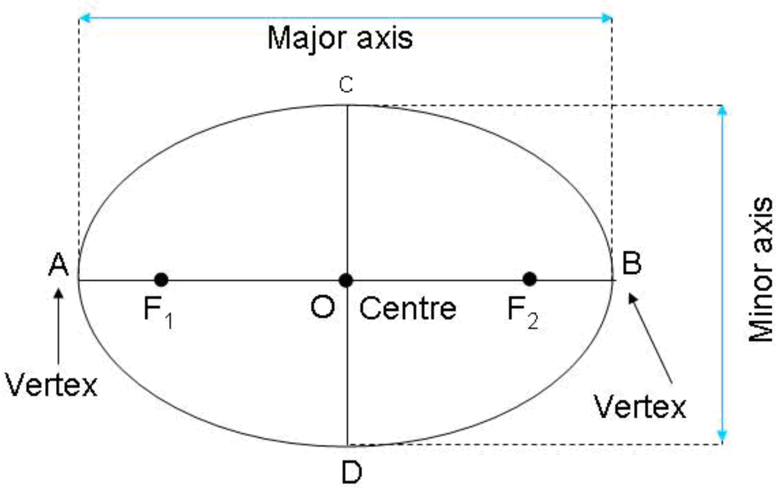
29. Terms associated with ellipse

(a) The mid point of the line segment joining the foci is called the **centre** of the ellipse. In the figure O is the centre of ellipse. For the simplest ellipse the centre is at origin.

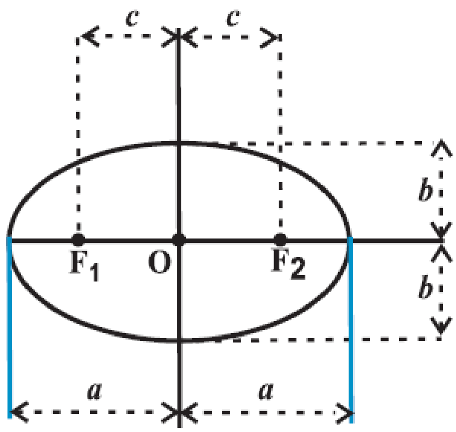
(b) The line segment through the foci of the ellipse is called the **major axis** and the line segment through the centre and perpendicular to the major axis is called the **minor axis. In the figure below AB and** In case of simplest ellipse the two axes are along the coordinate axes. Two axes intersect at the centre of ellipse.

(c) Major axes represent longer section of parabola and the foci lies on major axes.

(d) The end points of the major axis are called the **vertices** of the ellipse.

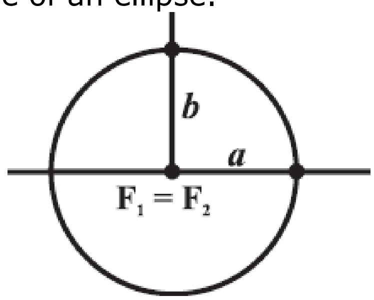


30. If the distance from each vertex on the major axis to the centre be a, then the length of the major axis is 2a. Similarly, if the distance of each vertex on minor axis to the centre is b, the length of the minor axis is 2b. Finally, the distance from each focus to the centre is c. So, distance between foci is 2c.

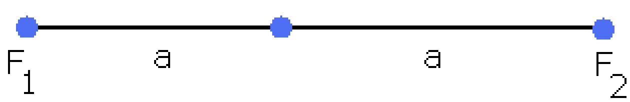


31. Semi major axis a, semi minor axis b and distance of focus from centre c are connected by the relation $a^2 = b^2 + c^2$ or $c^2 = a^2 - b^2$

32. In the equation $c^2 = a^2 - b^2$, if a is fixed and c vary from 0 to a, then resulting ellipses will vary in shape.
Case (i) When c = 0, both foci merge together with the centre of the ellipse and $a^2 = b^2$, i.e., a = b, and so the ellipse becomes circle .Thus circle is a special case of an ellipse.

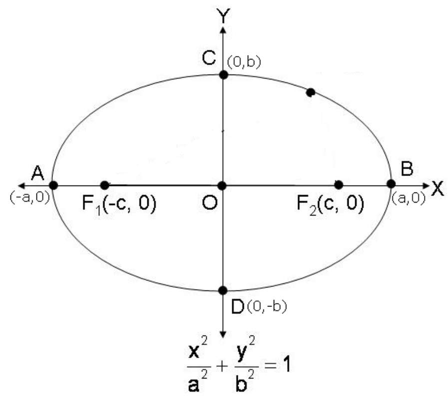


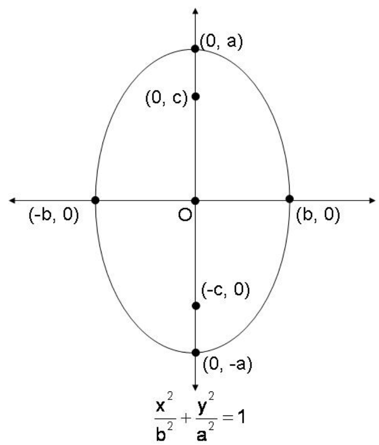
Case (ii) When c = a, then b = 0. The ellipse reduces to the line segment F_1F_2 joining the two foci.



33. The eccentricity of an ellipse is the ratio of the distances form the centre of the ellipse to one of the foci and to one of the vertices of the ellipse. Eccentricity is denoted by e i.e., $e = \frac{c}{a}$.

34. The standard form of ellipses having centre at the origin and the major and minor axis as coordinate axes. There are two possible orientations:



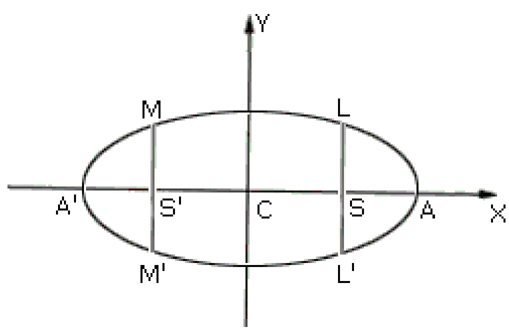


35. Ellipse is symmetric with respect to both the coordinate axes and across the origin. Since if (x, y) is a point on the ellipse, then (- x, y), (x, -y) and (-x, y) are also points on the ellipse.

36. Since the ellipse is symmetric across the y-axis. It follows that another point $F_2(-c,0)$ may be considered as a focus, corresponding to another directrix. Thus every ellipse has two foci and two directrices.

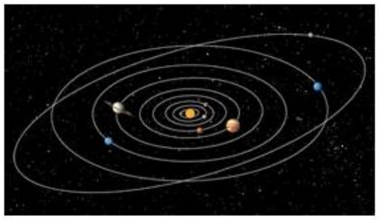
37. The foci always lie on the major axis. The major axis can be determined by finding the intercepts on the axes of symmetry. That is, major axis is along the x-axis if the coefficient of x^2 has the larger denominator and it is along the y-axis if the coefficient of y^2 has the larger denominator.

38. Lines perpendicular to the major axis A'A through the foci F1 and F2 respectively are called latus rectum. Lines LL’ and MM’ are latus rectum.



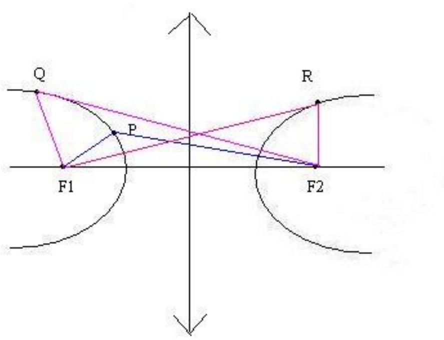
39. The sum of focal distances of any point on an ellipse is a constant and is equal to the major axis.

40. Conic ellipse can be seen in the physical world. The orbital of planets is elliptical.



Apart from this one can see an ellipse at many places since every circle, viewed obliquely, appears elliptical.
If the glass of water is seen from top or if it is held straight it appears to be circular but if it is tilt it will be elliptical.

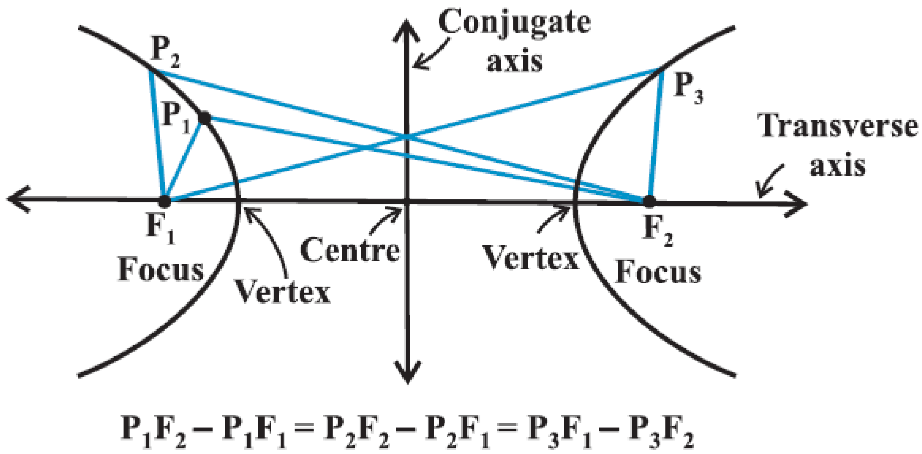
41. A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant. The two fixed points are called the foci of the hyperbola.



(Distance to F_1) − (distance to F_2) = constant

42. A hyperbola is the locus of a point in the plane which moves in such a way that its distance from a fixed point in the plane bears a constant ratio, e > 1, to its distance from a fixed line in the plane. The fixed point is called **focus**, the fixed line is called **directrix** and the constant ratio e is called the **eccentricity** of the hyperbola.

43. Terms associated with hyperbola



(a) The mid-point of the line segment joining the foci is called the centre of the hyperbola.

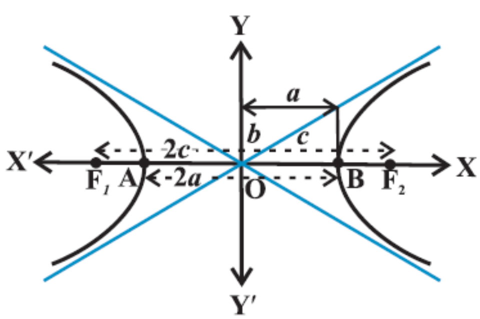
(b) The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is conjugate axis.

(c)The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.

44. The hyperbola is perfectly symmetrical about the centre O.

45. Let the distance of each focus from the centre be c, and the distance of each vertex from the centre be a.
Then, $F_1F_2 = 2c$ and AB = 2a
If the point P is taken at A or B then $PF_2 - PF_1 = 2a$

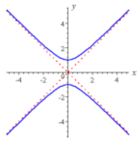
46. If the distance between two foci is 2c, between two vertices is 2a i.e length of the transverse axis is 2a , length of conjugate axis is 2b then a,b,c are connected as $c^2 = a^2 + b^2$



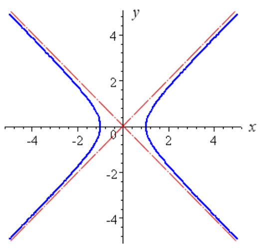
47. The ratio $e = \frac{c}{a}$ is called the eccentricity of the hyperbola. From the shape of the hyperbola, we can see that the distance of focus from

origin, c is always greater than or equal to the distance of the vertex from the centre, so c is always greater than or equal to a.
Since $c \geq a$, the eccentricity is never less then one.

48. The simplest hyperbola is the one in which the two axes lie along the axes and centre is at origin. Two possible orientations of hyperbola are



"**north-south**" opening hyperbola.



East-West Opening Hyperbola

49. A hyperbola in which a = b called an equilateral hyperbola.

50. Hyperbola is symmetric with respect to both the axes, since if (x, y) is a point on the hyperbola. (-x, y), (x, -y) and (-x, -y) are also points on the hyperbola.

51. The foci are always on the transverse axis. Denominator of positive term gives the transverse axis.

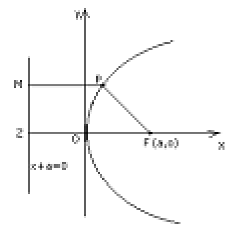
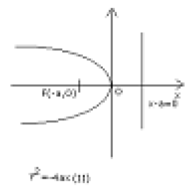
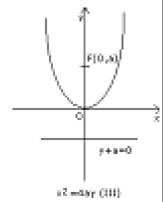
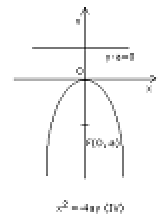
52. Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

Key Formulae

1. The equation of a circle with centre (h, k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.

2. If the centre of the circle is the origin O(0, 0), then the equation of the circle reduces to $x^2 + y^2 = r^2$

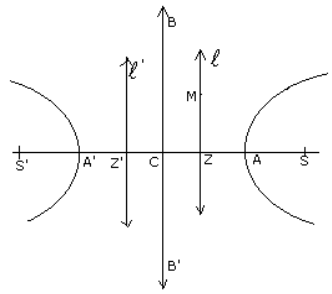
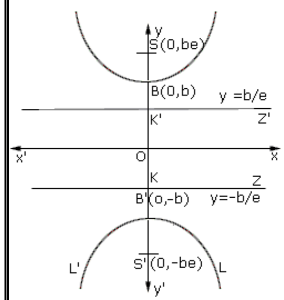
3.

	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
				
Coordinates of vertex	(0,0)	(0,0)	(0,0)	(0,0)
Coordinates of focus	(a,0)	(-a,0)	(0, a)	(0, -a)
Equation of the directrix	x = -a	x = a	y = -a	y = a
Equation of the axis	y = 0	y = 0	x = 0	x = 0
Length of the Latus Rectum	4a	4a	4a	4a

4.

	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a<b$
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	(a, 0) and (-a, 0)	(0, +b) and (0, -b)
Coordinates of foci	(ae, 0) and (-ae, 0)	(0, be) and (0, -be)
Length of the major axis	2a	2b
Length of the minor axis	2b	2a
Equation of the major axis	y = 0	x = 0
Equation of the minor axis	x = 0	y = 0
Equations of the directrices	$x=\frac{a}{e}$ and $x=-\frac{a}{e}$	$y=\frac{b}{e}$ and $y=-\frac{b}{e}$
Eccentricity	$e=\frac{c}{a}=\sqrt{1-\frac{b^2}{a^2}}$	$e=\frac{c}{b}=\sqrt{1-\frac{a^2}{b^2}}$
Length of the latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

5.

	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
		
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	(a, 0) and (-a, 0)	(0, b) and (0, -b)
Coordinates of foci	$(\pm ae, 0)$	$(0, \pm be)$
Length of the transverse axis	2a	2b
Length of the conjugate axis	2b	2a
Equations of the directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Eccentricity	$e = \frac{c}{a} = \sqrt{1 + \frac{b^2}{a^2}}$	$e = \frac{c}{b} = \sqrt{1 + \frac{a^2}{b^2}}$
Length of the latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Equation of the transverse axis	y = 0	x = 0
Equation of the conjugate axis	x = 0	y = 0