

Senior Secondary School Certificate Examination

July 2017 (Compartment)

Marking Scheme — Mathematics 65(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question (s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.
8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS**SECTION A**

$$1. \quad A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \quad \frac{1}{2}$$

$$A + A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \quad \frac{1}{2}$$

$$2. \quad y = x \log x$$

$$\Rightarrow \frac{dy}{dx} = 1 + \log x \quad 1$$

$$3. \quad \int_0^\pi \cos^5 x \, dx = 0 \quad 1$$

$$4. \quad AB: \frac{x-3}{1} = \frac{y+2}{2} = \frac{z-5/2}{2}$$

$$\text{DR's of required line} < 1, 2, 2 > \quad 1$$

SECTION B

$$5. \quad \frac{dx}{dt} = -3 \text{ cm/min}, \frac{dy}{dt} = 2 \text{ cm/min} \quad \frac{1}{2}$$

$$A = x \cdot y$$

$$\frac{dA}{dt} = x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt} \quad 1$$

$$= 2 \text{ cm}^2/\text{min} \quad \frac{1}{2}$$

i.e. Area is increasing at the rate of $2 \text{ cm}^2/\text{min}$.

6. $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$

$$= \tan^{-1}\left[\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}\right] \quad \frac{1}{2}$$

$$= \tan^{-1}\left[\frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}}\right] \quad \frac{1}{2}$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) \quad \frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2} \quad \frac{1}{2}$$

7. $A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = O$ 1

Pre-multiplying (or Post multiplying) by A^{-1} , we get

$$A^{-1} = \frac{1}{7}(A - 3I) = \begin{bmatrix} 2/7 & 3/7 \\ -1/7 & -5/7 \end{bmatrix} \quad 1$$

8. Cartesian equation of required line is

$$\frac{x-3}{2} = \frac{y+7}{-1} = \frac{z+4}{3} \quad 1$$

Vector equation of required line

$$\vec{r} = (3\hat{i} - 7\hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k}) \quad 1$$

9. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$$f'(x) = 12x^2 - 12x - 72$$

$$\frac{1}{2}$$

$$f'(x) = 0 \Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 3$$

$$\frac{1}{2}$$

Disjoint intervals are $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$

$$\frac{1}{2}$$

$f(x)$ is strictly increasing on $(-\infty, -2) \cup (3, \infty)$

$f(x)$ is strictly decreasing on $(-2, 3)$

$$\frac{1}{2}$$

10. $I = 3 \int \frac{dx}{\sqrt{5 - 4x - x^2}}$

$$= 3 \int \frac{dx}{\sqrt{(3)^2 - (x + 2)^2}}$$

$$1$$

$$= 3 \sin^{-1} \frac{x + 2}{3} + C$$

$$1$$

11. Let amount invested in bond B_1 is Rs. x and in bond B_2 is Rs. y

L.P.P. is Maximum $Z = \frac{8}{100}x + \frac{9y}{100}$

$$\frac{1}{2}$$

subject to

$$\left. \begin{array}{l} x \geq 20000 \\ y \leq 35000 \\ x + y \leq 75000 \\ x \geq y \\ x, y \geq 0 \end{array} \right\}$$

$$1 \frac{1}{2}$$

12. A and B are independent events

$$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{1}{8} \quad \frac{1}{2}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8} \quad 1$$

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{5}{8} = \frac{3}{8} \quad \frac{1}{2}$$

SECTION C

13. $x^2 = y$ (say) 1

$$\frac{y}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4} \quad 1$$

$$\text{Solving we get, } A = -\frac{1}{3}, B = \frac{4}{3}$$

$$\begin{aligned} \therefore \int \frac{x^2}{(x^2+1)(x^2+4)} dx &= -\frac{1}{3} \int \frac{dx}{(x^2+1)} + \frac{4}{3} \int \frac{dx}{x^2+4} \\ &= -\frac{1}{3} \tan^{-1} x + \frac{2}{3} \tan^{-1} \frac{x}{2} + C \quad 2 \end{aligned}$$

14. $R_1 \rightarrow R_1 - R_2 - R_3$ 1

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \quad 1$$

$$= 2c[ab + b^2 - bc] - 2b[bc - c^2 - ac]$$

$$= 4abc \quad 2$$

OR

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad 1$$

$$\Rightarrow A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \quad 1$$

$$\Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad 1$$

15. $\cot^{-1} x - \cot^{-1} (x+2) = \frac{\pi}{4}, x > 0$

$$\Rightarrow \tan^{-1} \frac{1}{x} - \tan^{-1} \frac{1}{x+2} = \frac{\pi}{4} \quad 1$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{1}{x} - \frac{1}{x+2}}{1 + \frac{1}{x(x+2)}} \right] = \frac{\pi}{4} \quad 1$$

$$\Rightarrow \frac{2}{x^2 + 2x + 1} = 1 \Rightarrow x^2 + 2x - 1 = 0 \quad 1$$

$$\Rightarrow x = \sqrt{2} - 1 \quad 1$$

OR

$$\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5}$$

$$= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{3}{4} \quad 1$$

$$= \tan^{-1} \left[\frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{5}{16}} \right] \quad 1$$

$$= \tan^{-1} \frac{56}{33} = \sin^{-1} \frac{56}{65} \quad 1+1$$

16. $y = (x \cos x)^x + (\sin x)^{\cos x}$

Let $u = (x \cos x)^x$

$$\Rightarrow \log u = x(\log x + \log \cos x) \quad \frac{1}{2}$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \left[\frac{1}{x} - \tan x \right] + \log (x \cos x)$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x [1 - x \tan x + \log (x \cos x)] \quad \dots(i) \quad 1$$

$v = (\sin x)^{\cos x}$

$$\log v = \cos x \log \sin x \quad \frac{1}{2}$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \cos x \cdot \cot x + \log \sin x \cdot (-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} \left[\frac{\cos^2 x}{\sin x} - \sin x \log \sin x \right] \quad \dots(ii) \quad 1$$

$$\therefore \frac{dy}{dx} = (x \cos x)^x [1 - x \tan x + \log(x \cos x)] + (\sin x)^{\cos x} \left[\frac{\cos^2 x}{\sin x} - \sin x \log \sin x \right] \quad 1$$

OR

$y = a (\sin t - t \cos t)$

$$\frac{dy}{dt} = a[\cos t + t \sin t - \cos t] \quad 1$$

$= a t \sin t$

$x = a[\cos t + t \sin t]$

$$\frac{dx}{dt} = a[-\sin t + t \cos t + \sin t] \quad 1$$

$= a t \cos t$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t \quad \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx}$$

$$= \frac{\sec^2 t}{a t \cos t} = \frac{1}{a t} \sec^3 t \quad 1$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=\frac{\pi}{4}} = \frac{8\sqrt{2}}{\pi a} \quad \frac{1}{2}$$

17. Given differential equation can be written as

$$\frac{dx}{dy} = \frac{2x e^{x/y} - y}{2y e^{x/y}} \quad \frac{1}{2}$$

Put $x = vy$

$$\frac{dx}{dy} = v + y \frac{dv}{dy} \quad 1$$

$$v + y \cdot \frac{dv}{dy} = v - \frac{1}{2e^v}$$

$$\int e^v dv = - \int \frac{dy}{2y} \quad 1$$

$$e^v = -\frac{1}{2} \log |y| + C$$

$$\Rightarrow e^{x/y} = -\frac{1}{2} \log |y| + C \quad 1$$

when $x = 0, y = 1$, we get $C = 1$

$$\therefore e^{x/y} = 1 - \frac{1}{2} \log |y| \quad \frac{1}{2}$$

$$18. I = \int_0^\pi \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx \quad 1$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx \quad 1$$

$$= 2\pi \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$$

$$= -2\pi \int_1^0 \frac{dt}{1+t^2}$$

1

put $\cos x = t, -\sin x dx = dt$

$$= 2\pi [\tan^{-1} t]_0^1$$

$$= \frac{\pi^2}{2}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

1

$$19. \quad |\vec{a} \times \vec{b}| = 1$$

1

$$\Rightarrow |\vec{a}| |\vec{b}| \sin \theta = 1$$

1

$$\Rightarrow \sin \theta = \frac{1}{\sqrt{2}}$$

1

$$\Rightarrow \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

1

$$20. \quad \text{Unit vector perpendicular to } \vec{a} \text{ and } \vec{b}$$

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

1

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix} = 18\hat{i} - 18\hat{j} + 9\hat{k}$$

1

$$|\vec{a} \times \vec{b}| = 27$$

$$\therefore \hat{n} = \frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$$

1

$$\text{Required vector} = 2\hat{i} - 2\hat{j} + \hat{k}$$

1

21. Let A worked for x days and B worked for y days

$$\text{Minimise } z = 225x + 300y$$

1

subject to constraints

$$\left. \begin{aligned} 9x + 15y &\geq 90 \Rightarrow 3x + 5y \geq 30 \\ 6x + 6y &\geq 48 \Rightarrow x + y \geq 8 \end{aligned} \right\}$$

2

$$x, y \geq 0$$

Value: Any relevant value

1

22. Let P = probability of doublet

$$P = \frac{1}{6}, q = \frac{5}{6}$$

 $\frac{1}{2}$

| | | | | |
|--------|-------------------|------------------|------------------|-----------------|
| x | 0 | 1 | 2 | 3 |
| P(x): | $\frac{125}{216}$ | $\frac{75}{216}$ | $\frac{15}{216}$ | $\frac{1}{216}$ |
| xP(x): | 0 | $\frac{75}{216}$ | $\frac{30}{216}$ | $\frac{3}{216}$ |

 $\frac{1}{2}$

2

 $\frac{1}{2}$

$$\text{Mean} = \sum xP(x) = \frac{108}{216} = \frac{1}{2}$$

 $\frac{1}{2}$

23. Let H_1 be the event that bolt is manufactured by machine A

H_2 be the event that bolt is manufactured by machine B

H_3 be the event that bolt is manufactured by machine C

and E be the event that bolt selected is defective

$$P(H_1) = \frac{25}{100}, P(H_2) = \frac{35}{100}, P(H_3) = \frac{40}{100}$$

1

$$P(E/H_1) = \frac{5}{100}, P(E/H_2) = \frac{4}{100}, P(E/H_3) = \frac{2}{100}$$

1

Reqd prob. is

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2) + P(H_3) \cdot P(E/H_3)}$$

1

$$= \frac{28}{69}$$

1

65(B)
SECTION D

24. $AB = 8I$

1

$$\Rightarrow A^{-1} = \frac{1}{8}B = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$1\frac{1}{2}$

Given equation in matrix form is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

1

$$\Rightarrow AX = C$$

$$\Rightarrow X = A^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

1

$$\Rightarrow x = 3, y = -2, z = -1$$

$1\frac{1}{2}$

OR

$$A^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}, A^3 = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$1\frac{1}{2} + 1\frac{1}{2}$

$$A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 11 \\ 0 & 0 & 11 \end{bmatrix}$$

$1\frac{1}{2}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$1\frac{1}{2}$

25. $f: A \rightarrow A$

Let $x_1, x_2 \in A$ such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1 - 2}{x_1 - 1} = \frac{x_2 - 2}{x_2 - 1}$$

$$\Rightarrow x_1 = x_2$$

$\Rightarrow f$ is one-one

$$\text{Now } y = \frac{x-2}{x-1} \Rightarrow x-2 = xy - y$$

$$\Rightarrow x(y-1) = y-2$$

$$\Rightarrow x = \frac{y-2}{y-1}$$

For each $y \in A = \mathbb{R} - \{1\}$, there exists $x \in A$

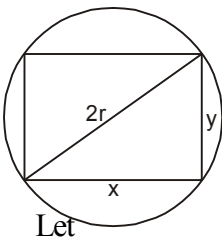
Thus f is onto. Hence f is bijective

$$\text{and } f^{-1}(x) = \frac{x-2}{x-1}$$

$$(i) \quad f^{-1}(x) = \frac{5}{6} \Rightarrow \frac{x-2}{x-1} = \frac{5}{6} \Rightarrow x = 7$$

$$(ii) \quad f^{-1}(2) = 0$$

26.



Let x, y respectively be the sides of rectangle $\therefore y = \sqrt{4r^2 - x^2} \dots (1)$

$$A = xy$$

$$Z = A^2 = 4x^2r^2 - x^4$$

$$\frac{dZ}{dx} = 8r^2x - 4x^3$$

$$\frac{dZ}{dx} = 0 \Rightarrow 4x(2r^2 - x^2) = 0$$

$$\Rightarrow x = \sqrt{2}r$$

$$\frac{d^2Z}{dr^2} = 8r^2 - 12x^2$$

$$\left. \frac{d^2Z}{dx^2} \right|_{x=\sqrt{2}r} = -16r^2 < 0$$

1

$$\Rightarrow \text{Area is maximum when } x = \sqrt{2}r$$

$$\therefore y = \sqrt{2}r \quad (\text{From (i)})$$

$$\text{i.e. } x = y$$

Hence, Area is maximum when rectangle is a square

1

27. Equation of plane passing through $(-1, 3, 2)$

$$a(x+1) + b(y-3) + c(z-2) = 0 \quad \dots(i)$$

1

Required plane is perpendicular to $x + 2y + 3z = 5$

$$\text{and } 3x + 3y + z = 0$$

$$\therefore \left. \begin{array}{l} a + 2b + 3c = 0 \\ 3a + 3b + c = 0 \end{array} \right\}$$

1+1

$$\Rightarrow \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3}$$

1

$$\therefore \text{Equation (i)} \Rightarrow$$

$$7x - 8y + 3z + 25 = 0$$

1

Vector Equation of plane is

$$\vec{r} \cdot (7\hat{i} - 8\hat{j} + 3\hat{k}) = -25$$

1

OR

Equation of plane passing through $(1, 1, 4)$, $(3, -1, 2)$ and $(4, 1, -2)$ is

$$\begin{vmatrix} x-1 & y-1 & z-4 \\ 2 & -2 & -2 \\ 3 & 0 & -6 \end{vmatrix} = 0$$

2

$$\Rightarrow 2x + y + z = 7 \quad \dots(i)$$

Equation of line passing through $(3, -4, -5)$ and $(2, -3, 1)$

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k \quad 1$$

$$\Rightarrow x = 3 - k, y = k - 4, z = 6k - 5 \quad \dots(ii) \quad 1$$

$(3 - k, k - 4, 6k - 5)$ lies on (i)

$$6 - 2k + k - 4 + 6k - 5 - 7 = 0$$

$$\Rightarrow k = 2 \quad 1$$

Eqn (ii) \Rightarrow point of intersection is $(1, -2, 7)$ 1

28. $\frac{dy}{dx} + y \cdot \cot x = 4x \operatorname{cosec} x \quad \dots(i)$

Here $P = \cot x$, I.F. $= e^{\int \cot x \cdot dx} = e^{\log \sin x} = \sin x$ 1 $\frac{1}{2}$

Hence the solution is

$$y \sin x = \int 4x \cdot dx \quad 2$$

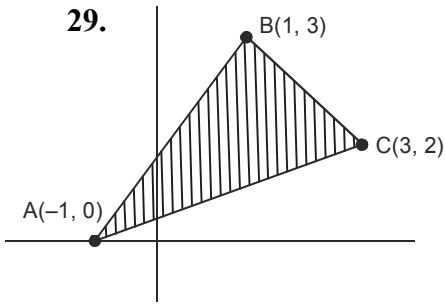
$$\Rightarrow y \sin x = 2x^2 + C \quad 1 \frac{1}{2}$$

When $x = \frac{\pi}{2}$, $y = 0$, $C = -\frac{\pi^2}{2}$

\therefore Required solution is

$$y \sin x = 2x^2 - \frac{\pi^2}{2} \quad 1$$

29.



Equation of AB: $y = \frac{3}{2}(x + 1)$

Equation of BC: $y = -\frac{1}{2}x + \frac{7}{2}$

Equation of AC: $y = \frac{1}{2}(x + 1)$

$$1 \frac{1}{2}$$

Required area

$$= \frac{3}{2} \int_{-1}^1 (x + 1) dx - \frac{1}{2} \int_1^3 (x - 7) dx - \frac{1}{2} \int_{-1}^3 (x + 1) dx$$

$$1 \frac{1}{2}$$

$$= \frac{3}{2} \left[\frac{x^2}{2} + x \right]_{-1}^1 - \frac{1}{2} \left[\frac{x^2}{2} - 7x \right]_1^3 - \frac{1}{2} \left[\frac{x^2}{2} + x \right]_{-1}^3$$

$$1 \frac{1}{2}$$

$$= 3 + 5 - 4$$

$$= 4 \text{ sq. units}$$

$$1 \frac{1}{2}$$

OR

$$\int_1^3 (3x^2 + e^{2x}) dx$$

$$h = \frac{2}{n}, \text{ as } n \rightarrow \infty, h \rightarrow 0$$

$$\int_1^3 f(x) dx = \lim_{\substack{h \rightarrow 0 \\ n \rightarrow \infty}} h [f(x) + f(1+h) + f(1+2h) + \dots + f(1+n)]$$

$$\int_1^3 (3x^2 + e^{2x}) dx = \lim_{\substack{h \rightarrow 0 \\ n \rightarrow \infty}} 3h^3 \Sigma(n-1)^2 + 6h^2 \cdot \Sigma(n-1) + 3nh + he^2 [1 + e^{2h} + e^{4h} + \dots + f(1-n)]$$

2

$$= \lim_{\substack{h \rightarrow 0 \\ n \rightarrow \infty}} \frac{24}{n^3} \cdot \frac{n(n-1)(2n-1)}{6} + \frac{24}{n^2} \cdot \frac{n(n-1)}{2} + 6 + he^2 \cdot \frac{e^{2nh} - 1}{e^{2h} - 1}$$

2

$$= \lim_{\substack{h \rightarrow 0 \\ n \rightarrow \infty}} \frac{24 \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right)}{6} + \frac{24 \left(1 - \frac{1}{n}\right)}{2} + 6 + e^2$$

1

$$= 26 + \frac{1}{2}(e^6 - e^2)$$

1

Alternately $f(x) = (3x^2 + e^{2x})$

$$f(x)dx = \lim_{h \rightarrow 0} \{ [3(1)^2 + e^2] + [3(1+h)^2 + e^{2(1+h)}] + 3[3(1+2h)^2 + e^{2(1+2h)}] + \dots + [3(1+(n-1)h)^2] + e^{2(1+(n-1)h)} \} \quad 2$$

$$= \lim_{h \rightarrow 0} h \left[\frac{3n + 6hn(n-1)}{2} + \frac{3h^2 n(n-1)(2n-1)}{6} + e^2 \{1 + e^{2h} + e^{4h} + \dots + e^{2(n-1)h}\} \right] \quad 2$$

$$= \lim_{h \rightarrow 0} h \left[3nh + 3nh(nh-h) + \frac{nh(nh-h)(2nh-h)}{2} + \frac{e^2 h e^{2nh} - 1}{e^{2h} - 1} \right] \quad 1$$

$$= 6 + 12 + 8 + \frac{e^2(e^4 - 1)}{2} \quad 1$$