Lagrangian and Eulerian lateral diffusivities in zonal jets

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Abstract

Meridional diffusivities from Lagrangian particle dispersion and Eulerian diffusivities from a flux-gradient relationship are estimated in an idealized primitive equation channel model featuring eddy-driven zonal jets.

The Eulerian estimate shows an increase with depth and clear minima of meridional diffusivities within the zonal jets, indicating mixing barriers. The Lagrangian estimates agree with the Eulerian method on the vertical variation and also show indications of meridional mimima, although meridional variations are poorly resolved. We found early maxima in the particle spreading rates which should not be related to diffusivities since they are caused by the meandering zonal jets. The meanders also produce rotational eddy fluxes, which can obscure the Eulerian diffusivity estimates.

Zonal particle dispersion rates do not converge within the chosen lag interval, because of shear dispersion by the mean flow, i.e. it is not possible to estimate Lagrangian zonal
diffusivities representative for regions of similar size of the zonal jet spacing. Removing
the zonal mean flow, zonal and meridional dispersion rates converge and show much higher
zonal than meridional diffusivities. Further, the pronounced vertical increase and indica-
tions of meridional minima in the Lagrangian meridional diffusivities disappear, pointing
towards the importance of shear dispersion by the mean flow for the suppression of merid-
ional mixing by zonal jets.

1 Introduction

Zonal jets might play an important role in the ocean circulation and their effects on tracer
transport should be included in ocean climate models. Zonal jets are a characteristic
feature of many observed flows in the ocean (Treguier et al., 2003; Maximenko et al., 2005)
and atmosphere (Dritschel and McIntyre, 2008) and are believed to be both cause for, and
effect of inhomogeneous transport in the sense that they partition the flow into regions of
strong mixing separated by transport barriers.

Large-scale ocean (climate) models do not resolve mesoscale fluctuations and therefore
cannot reproduce eddy-driven mean flow in general, and zonal jets in particular. In order
to include these effects, proper parameterizations for the impact of zonal jets on turbulent
mixing are needed. This study is a continuation of previous studies (Eden, 2009, 2010) in
which the zonal and meridional transport properties of an eddying channel model have been
analyzed in an Eulerian framework and in which an eddy momentum flux parameterization
for the zonally averaged flow was successfully tested, in the sense that it reproduced the
zonal jets. In the present study, we extend the previous work by considering turbulent
mixing in a Lagrangian framework, focussing on zonal and meridional mixing appropriate
to passive tracers.

Lagrangian analysis is a useful complement to Eulerian analysis, since the former de-
scribes the purely advective stirring of a passive tracer in the flow field, without being
influenced by additional processes acting on a passive tracer, like numerical diffusion or
explicit relaxation. Such processes may have a large effect on Eulerian mixing estimates
for passive tracers. Further, the Lagrangian analysis may provide a more direct linkage
to observational estimates of mixing in the ocean where drifters and floats are often used
for mixing estimates. The channel model used in this study provides an idealized frame-
work to illuminate the relationship between Eulerian and Lagrangian analysis of turbulent
mixing: It provides quasi-stationary velocity- and tracer fields featuring spatially inhomoge-
neous fluctuation statistics and minimal drift, and therefore yields reliable Eulerian and
Lagrangian statistics with strong spatial dependence. Note that such statistics are often
difficult to obtain from observations or realistic ocean model simulation.

In section 2 we detail the numerical model and the methods which we use to estimate
diffusivities from particle dispersion, while in section 3 the possibility of a localization of
the non-local Lagrangian particle statistics is explored. Section 4 discusses the results in
terms of the meridional diffusivities inferred from the particle dispersion, while section 5
focusses on the zonal dispersion. The last section summarizes and discusses the results of
this study.

2 Methods

2.1 Lagrangian method

The Lagrangian method we employ in this study is based on the assumption that the
evolution of the ensemble-mean field of a conservative passive tracer field is fully determined
by Lagrangian single-particle statistics (Batchelor, 1949, 1952). Davis (1987) applied this
theory to Lagrangian drifters in the ocean, and Davis (1991) outlined how diffusivities,
appropriate to a flux-gradient parametrization of unresolved tracer transport, can be found
Figure 1: a) Snapshot of velocity (arrows) and temperature at \( z = -500 \) m after the spinup phase of the integration. Zonal and meridional axis are in km, the north-south temperature gradient is about 20 K and the maximal velocities are about 1 m/s. b) The same snapshot of temperature and eight arbitrarily chosen trajectories during 400 days each at the same depth level.

Particle trajectories are obtained by integrating the velocity field of a primitive equation model configured to simulate flow in an idealized zonally periodic channel. The model is the same as the one used by Eden (2009) (specifically the experiment with Rossby radius of 96 km and Eady growth rate 4.8 d). The model equations are formulated in Cartesian coordinates and the domain is a square, zonally periodic channel of 4000 km width and 2000 m depth with flat bottom. Fig. 1 shows a snapshot of velocity and temperature (salinity is held constant) and some sample trajectories at 500 m. The horizontal and vertical resolution of the model are 16 km and 50 m, respectively. Boundary conditions are free slip and zero buoyancy fluxes at top, bottom and lateral boundaries. The zonal mean flow is established by relaxing the buoyancy at the northern and southern boundaries towards prescribed vertical profiles, resulting in a northward pointing (baroclinically...
unstable) horizontal temperature gradient throughout the domain.

After a four year long spinup phase of the integration, 10000 particles are distributed randomly within the domain and then integrated for about 4 years (6 years for experiment NOMEAN, which is described below) using the instantaneous velocity field of the model. Particle positions are stored in intervals of 12 hours. Every four-year trajectory is cut into overlapping pieces (so called pseudo-trajectories) of 200 days (Fig. 2). The overlap is 100 days. Treating these pseudo-trajectories as individual trajectories, we obtain about 120000 individual 200-day trajectories. The trajectory of each float can be written as \( X(t, X_0) \), where \( t \) is a time-lag coordinate ranging from day -100 to day 100, and \( X_0 \) is the position of the trajectory at \( t = 0 \). The meridional plane is partitioned into 20x10 bins, 20 in y-direction and 10 in z-direction, corresponding to grid boxes of an equally spaced grid (Fig. 2 b). The turbulent velocity statistics and particle statistics are expected to be constant in zonal direction; the bin size therefore extends over the whole domain in zonal direction. Each pseudo-trajectory is mapped to the bin which encloses its position \( X_0 \) at \( t = 0 \), marked with a red dot in the schematic Fig. 2a. In this way, every bin is associated with a set of pseudo-trajectories, and the Lagrangian statistics computed from this set can be mapped to the corresponding bin (see Fig. 2). After a pseudo-float \( X(t, X_0) \) has been mapped to a grid box, we redefine the pseudo-trajectory as \( X(t) := X(t, X_0) - X_0 \). All pseudo-trajectories in a bin then satisfy \( X(0) = 0 \), and it is convenient to imagine an individual coordinate system for each bin, with the origin located at the grid box center, and to assume that all pseudo-trajectories associated with the bin have been released at the grid box center. Note that we assume that all statistics are homogeneous in the individual bin. The time-axis in this coordinate frame ranges from \( t = -100 \) days to \( t = 100 \) days. Where necessary, we distinguish the timeseries \( X(t) \) from the original one, \( X(t, X_0) \), by referring to the former as \textit{normalized} particle trajectory. In a sense, this procedure can be thought of constructing an ensemble of single-particle point-release experiments from
Figure 2: a) A schematic outlining the binning of pseudo-trajectories. b) A transect through the channel, showing the grid of the bin boxes. The meridional plane is partitioned into 20 bins in y-direction and 10 bins in z-direction. Each gridbox has a width of about 200 km and a vertical extent of 200 m. The y-axis points northward.

trajectories that where actually collected by random-encounter sampling in a finite volume (bin): We construct one set of pseudo-trajectories for each bin (using all pseudo-particles mapped to the bin during the multi-year experiment), and analyze this set imagining that each individual element was released exactly at the bin center, with the time-label of release being \( t = 0 \). This is similar to the approach of Swenson and Niiler (1996).

We use two different single-particle dispersion rate estimates \( D_1 \) and \( D_2 \) that have also been used in previous studies and are extensively described in Taylor (1922) and Davis (1991). A brief introduction to Taylor’s discussion about the analogy between turbulent dispersion and diffusion can be found in the appendix. The lateral (isopycnal) diffusivity is expressed as a 2x2 flux-gradient tensor \( K_{ij} \), where \( i, j = 1, 2 \), and may depend on position (bin). Note that since we assume the turbulence to be homogeneous in zonal direction, \( K_{ij} \) is a function of \( y \) and \( z \) only. In this article we will consider only the diagonal components \( K_{11} \) and \( K_{22} \), and for simplicity we use the notation \( K_x := K_{11} \) and \( K_y := K_{22} \). In the following we only discuss \( K_y \), but it is straightforward to obtain the definition of \( K_x \) by substituting the variable \( y \) with \( x \). Since the diffusivity is an asymptotic value of the particle dispersion rate, it is time-independent by definition. However, it is convenient to use the notation \( K(t) \) to denote the complete time series of dispersion rate and to define the
diffusivity accordingly as \( \lim_{t \to \infty} K(t) \). Note that for statistically homogeneous turbulence, the dispersion rate is an odd function, such that \( K(t) = -K(-t) \) holds (Davis, 1991).

The mean position (or mean displacement) of the particle cloud in meridional direction is defined as \( \overline{Y}(t) \), where the overbar is an average over all trajectories in a bin. The dispersion rate \( K_y \) is defined as

\[
K_y(t) := \frac{d}{dt} \overline{Y'^2}(t),
\]

where \( Y'(t) := Y(t) - \overline{Y}(t) \) is the deviation of a particle’s meridional position \( Y(t) \) from the mean meridional position at time \( t \). We use the term dispersion to refer to the quantity \( \overline{Y'^2}(t) \), i.e. the mean square displacement from the mean position of the particle cloud.

The two dispersion rate estimates are:

- \( D1 \):

\[
K_y(t) \approx \frac{\overline{Y'^2}(t + \Delta t) - \overline{Y'^2}(t)}{2\Delta t},
\]

The time increment \( \Delta t \) is 12 hours.

- \( D2 \):

\[
K_y(t) \approx \overline{U'(0)X'(t)},
\]

where \( U'(0) = U(0) - \overline{U}(0) \) is the deviation of a particle’s velocity at \( t = 0 \) from the mean particle velocity \( \overline{U} \) obtained from ensemble volume averaging over a grid box. This formulation is analogous to the definition of turbulent diffusivity in mixing length theory and formally equivalent to definition (2) for ergodic flow. Its quantitative estimate, however, differs from (2) in the flow studied here. Davis (1991) shows that \( D2 \) is the preferred method for point-release experiments in shear flows (his formula 2.13). However, Oh et al. (2000) suggest that there is no clear difference between the methods when random-encounter sampling in finite volumes is used.
Although we do not attempt to evaluate the performance of the different estimates, we will point out differences in the results. The methods were chosen because they both represent suitable tools for the analysis of single particle statistics and a comparison indicates which results of our analysis are robust enough to be produced by all estimates, and which are not. Detailed information on estimates of dispensor rates with single particle statistics can be found in Davis (1991). Davis (1991) specifically focuses on \( D_2 \), which is therefore chosen as the default method if only results of one method are shown.

2.2 Eulerian method

The Eulerian estimate is based on meridional eddy fluxes of buoyancy and potential vorticity and a flux-gradient relationship:

\[
\tilde{K}_b := -\frac{v'b'}{b_y},
\]

\[
\tilde{K}_q := -\left[ f\left(\frac{v'b'/N^2}{z} - (u'v')_y\right) / \left[-u_y + f\left(\frac{b}{N^2}\right)_z\right]_y \right],
\]

where \( u, v \) and \( b \) denote the Eulerian horizontal velocity and buoyancy, respectively, and primes denote instantaneous perturbations from zonal averages. \( N^2 \) is the stability frequency with respect to a horizontally constant background buoyancy. The overbar stands for the operation of zonal averaging and subsequent time-averaging over the multi-year experiment. Note that \( \tilde{K}_b \) corresponds to thickness diffusivity appropriate to the Gent and McWilliams (1990) parameterisation and that \( \tilde{K}_q \) resembles a lateral diffusivity attached to the zonally averaged potential vorticity \(-u_y + f(\overline{b}/N^2)_z\) in quasi-geostrophic approximation.

3 Displacement statistics and binning

Lagrangian particles released at a point change position in time and therefore sample velocity at different locations. The set of pseudo-trajectories associated with a bin will be
Figure 3: a) to c): Estimated meridional location probability density as a function of $Y/L$ at lag $25\,\text{d}$ (a), lag $50\,\text{d}$ (b) and lag $100\,\text{d}$ (c). Shown is the probability that a float has been advected a meridional distance $Y/L$ away from the origin. $Y$ denotes the normalized meridional position and $L$ is the width of a bin in meridional direction. Red lines indicate the standard deviation of $Y/L$ (drawn in positive and negative meridional direction). d) Empirical variance of $Y/L$ (solid) as a function of time lag in d. Only positive lags are shown. Also shown is the slope of variance at lag $t = 60\,\text{d}$ (dashed) and the variance at lag $t = 60\,\text{d}$ is marked with a dot.

Advected out of the bin after a certain time and sample velocity statistics of neighbouring bins. The method of estimating diffusivities with Lagrangian data requires knowledge of how long after release the velocity statistics sampled by a drifter cloud are representative for the region in which it was released. Relevant statistics are the mean particle displacement, i.e. the displacement of the center of mass of the particle cloud, and the dispersion of the cloud from its center of mass. The displacement is caused by a mean velocity, which might advect most particles out of the bin before $K$ has converged to its asymptotic limit. In such a case, the spatial resolution of the bin grid is too high to yield information on diffusivities. A similar consideration applies to the dispersion of the particle cloud; to attribute a diffusivity to a bin, the dispersion must reach the diffusive regime while the majority of particles are still in the bin or at least close to it. The turbulent transport might disperse all particles from the bin of origin into surrounding neighbouring bins before the diffusive regime is reached. For the flow in this study, the normalized mean particle displacement of all pseudo-trajectories in meridional direction $\bar{Y}$ is on the order of kilometers after 100 days, and therefore small compared to the bin width of about 200 km
(note that the zonal mean velocity cannot transport particles out of a bin, since the bins extend across the whole length of the periodic channel). The mean displacement in the individual bins is similar to the global mean displacement in most cases, with the exception of bins located at the boundaries below mid-depth. In any case, we choose to ignore any statistics for lags \( t \) at which \( \sqrt{\overline{Y^2}(t)} > 100 \text{ km} \) (there exists such a \( t \) in 23 bins). In total there are 200 lag days in each of the 200 bins, and the overall percentage of undefined float days is 3.8% in the base experiment. For experiment NOMEAN (discussed below), undefined values in 61 bins amount to 8.3% of the total number of float days.

To quantify the combined effect of meridional displacement and dispersion, Fig. 3 a) to c) shows the estimated probability that a float has been advected a meridional distance \( Y/L_b \) away from the origin (i.e. the center of the bin), where \( Y \) is the normalized \( y \)-position and \( L_b \) is the width of a bin in meridional direction. The plotted data consists of all pseudo-trajectories in the domain and shows probability densities at three different lags. The expected meridional position is about equal to the bin width at a lag of 50 d. At lag \( t = 100 \text{ d} \) the expected meridional position has slightly increased to about 1.3 times the bin width, and it therefore seems reasonable to consider lags up to 100 d for the chosen bin size. Note that an increase of bin size would cause the floats to reside longer within a bin, but would also result in a lower spatial resolution for diffusivity; considering that the width of a zonal jet in the model is about 500 km, it is desirable to keep the bin width smaller.

The evolution of the variance of \( Y/L_b \) is shown in Fig. 3 d). The data suggests that the spreading (i.e. the increase of variance) can be approximated by a linear function after a lag of about 50 d. We find that at lag 60 d, about 41% of the normalized particles are still in the bin in which they have been released, and about 86% are in the three bins consisting of the bin of release and the neighbouring bins to the north and south. The
Figure 4: a) to c): $K_y$ in $10^4 \text{m}^2\text{s}^{-1}$ as a function of time lag in d in three representative bins with their centers located at $y = 1944 \text{km}$ and at $z = -100 \text{m}$ (a), $z = -900 \text{m}$ (b) and $z = -1700 \text{m}$ (c). Error bars were obtained with a bootstrapping method: The width of the error bars is the standard deviation of $K_y$ obtained from 50 resamples of the pseudo-trajectories contained in a bin. Timeseries were obtained with method $D1$. d) to f): Same for $D2$. g) and h): Percentage of (lag) days with significantly negative (d) or positive (e) values of $K_y$ as a function of depth in m for the three different methods as indicated in d). Only values at positive time lags are considered and values for bins at the same depth levels are averaged. The lag duration with significantly positive values for each bin is determined, summed over bins with the same depth and divided by the total amount of lag days at the depth. Time lags for which $K_y$ is undefined are not considered and the bins located $z = -1900 \text{m}$ are not shown. Note the different horizontal axis in d) and e).

spatial pattern of the standard deviation of $Y/L_b$ at lag $t = 60 \text{d}$ for each individual bin shows values between 0.6 and 1.4 (not shown). We will discuss the spatial dependency in greater detail below. Fig. 3 also shows that the dispersion is approximately ballistic (i.e. proportional to $t^2$) at lags smaller than about 10 d, after which it decelerates considerably before reaching the diffusive regime (discussed below).

4 Meridional particle dispersion

Ideally, the set of pseudo-trajectories from which single-particle Lagrangian statistics are computed for a bin consists of particle paths which are separated temporally long enough, so that the perturbation velocity field by which a drifter is advected through the bin and out of the bin becomes decorrelated until the arrival of the next drifter in the bin. Since the statistics are computed from particles which arrive in random intervals in the bin and
reside there with an unknown number of other particles simultaneously, the set of pseudo-trajectories within a bin might not be statistically independent and the results might be biased. To estimate the robustness of the statistics to variations in the data set, we apply a simple bootstrapping method and construct 50 resamples of the data set associated with the individual bin, each of which is obtained by random sampling (with replacement) of the original data set. The resampled data sets contain the same number of pseudo-trajectories as the original data set. Timeseries for $K_y$ are computed for each of the 50 resampled data sets, and error bars are obtained by taking the standard deviation of the 50 values at each time lag. Fig. 4 a) to c) shows $K_y$ obtained from $D_1$ in three bins located at different depths at the same meridional position in the center of the channel. The three arbitrarily picked results exemplify the growth of error obtained in most bins. Method $D_2$, shown in Fig. 4 d) to f) for the same bins, yields error bars of similar width. Note that the growth of error with time is expected, since the dispersion rate is a lag correlation of timeseries with finite length. Except for the deepest bin, there exist lags at which dispersion rates are not significantly different from zero. A comparison between the two methods shows that method $D_2$ produces smoother estimates of $K_y(t)$, a result that seems to contradict the fact that the width of bootstrapping error bars is similar for both methods. Fig. 4 a) suggests that there are also bins with significant negative (positive) dispersion rates for positive (negative) time lags. Since those values are related to oscillations around low values for $K_y$, we assume here that negative (positive) values of $K_y$ at positive (negative) lags are caused by undersampling of the flow.

To illustrate the overall amount of negative or insignificantly positive values of $K_y$ at positive lags, Fig. 4 g) and h) show the vertical distribution of (lag) days in which $K_y$ attains values which are significantly different from zero. Fig. 4 g) panel shows the percentage of days for positive lags with significant negative values. Both methods yield significant negative values for about 5-10% of the total lag in the bins centered above 300 m
Figure 5: Timeseries of $K_y$ for method $D2$ for all bins. Shown is the mean value of dispersion rates at positive and negative lags $(K_y(t) - K_y(-t))/2$. The axis on the right and the numbers on the bottom of the figure indicate the position of the bin center in the meridional plane, whereas the axes in the top left corner refer to an individual bin.

To get an overview of the spatial variation of $K_y$ in the meridional plane, Fig. 5 shows timeseries of $K_y$ for all bins, obtained with method $D2$. Note that only positive values in the range of $0$ to $2 \cdot 10^4 \text{ m}^2/\text{s}$ are shown. Note also that values are undefined for lags at which the mean normalized meridional position exceeds the half-width of the bin. Above 1300 m depth, the values of $K_y$ attain a global maximum with the first 20 days, after which they decrease to some fraction of the maximal value. The highest global maxima are generally attained close to the surface, whereas the highest values at later lags are
Figure 6: $K_y$ in $10^4 \text{m}^2/\text{s}$ for all pseudo-trajectories above 600 m depth (solid), between $-600 \text{ m}$ and $-1200 \text{ m}$ (dashed) and between $-1200 \text{ m}$ and $-1800 \text{ m}$ (dashed dotted) as a function of positive lag in d (values for negative lags are virtually identical but of opposite sign). Error bars are shown for the lag at which $K_y$ is maximal and for subsequent lags and was calculated by computing $K_y$ for each bin separately in the respective depth range and obtaining the standard deviation considering all but the northernmost and southernmost bins.

The values obtained with $D1$ behave in a similar way (not shown).

To exemplify the vertical variation of $K_y$, Fig. 6 shows the averaged dispersion rates in three depth layers with thickness of 600 m. The timeseries show a pronounced global maximum within the first 15 days and are relatively flat in the second half of the 100 day lag period. As already seen from Fig. 5, Fig. 6 suggest an inverse relationship between the maximum of a dispersion rate and its asymptotic value. Method $D1$ yields a similar picture (not shown), although the timeseries obtained from $D1$ oscillate weakly at later lags. The changes in time of $K_y(t)$ will be discussed in the concluding section and we proceed to discuss the asymptotic values of $K_y$.

Fig. 7 shows $K_y$ estimated with $D1$ and $D2$ in the meridional plane. Shown is the average of $K_y$ during the periods $t = 60$ to 100 d and $t = -100$ to $-60$ d, i.e. both values at positive and negative lags are combined in the estimate. As already seen in Fig.6, there is a strong increase of $K_y$ with depth in both estimates and at all latitudes from about 1000 $\text{m}^2/\text{s}$ at the surface to about 5000 $\text{m}^2/\text{s}$ at depth. Furthermore, there is a pronounced
Figure 7: a) and b): Mean value of $K_y$ in $\log^{10}(\text{m}^2/\text{s})$ during the intervals from $t = -100$ d to $t = -60$ d and $t = 60$ d to $t = 100$ d, estimated with $D1$ (a) and $D2$ (b). Bins are painted in white if $K_y$ is undefined at one or more lags or the average value of $K_y$ is negative. The thick black contour line describes the depth of zero zonal mean Lagrangian particle velocity. Thin lines are positive (solid) and negative (dashed) zonal mean Lagrangian particle velocity in intervals of 5 cm/s. c) and d): Error estimate based on a bootstrapping method. Shown is the mean of standard deviation of $K_y$ in m$^2$/s in the intervals of averaging, $t = -100$ d to $t = -60$ d and $t = 60$ d to $t = 100$ d. Note the different color scales in the last row.

Fig. 7 also shows the zonal mean particle velocity in the bins, indicating the position of the zonal jets. The result of $D2$ indicates that values are reduced within the eastward zonal jet cores, i.e. indicating mixing barriers within the jets. Note that this meridional variation persists for $D2$ when the respective bootstrapping error is added or subtracted (not shown). Method $D1$ does not yield minima of diffusivity in the 2 northernmost jets, and the estimate for the jet at position $y \approx 1750$ km features a minimum only at the surface. We quantify the correspondence of the minima in diffusivity with the zonal jet by calculating mean values of $K_y$ separately for regions where the Lagrangian zonal velocity is positive (the jets) and negative (outside the jets) in bins above a depth of 1000 m: For $D1$ we obtain a mean of $1.1 \cdot 10^3 (2.2 \cdot 10^3)$ m$^2$/s inside (outside) the jets, while for $D2$ we obtain $1.1 \cdot 10^3 (1.3 \cdot 10^3)$ m$^2$/s, i.e. both estimates show on average decreased
Figure 8: Eulerian diffusivity in $\log_{10} m^2/s$ estimated from the meridional eddy buoyancy fluxes (a) and meridional eddy potential vorticity fluxes (b) from a flux-gradient relationship. Contour lines show the Eulerian mean flow with a thick line for $\bar{u} = 0$ m/s and contour interval of 10 cm/s; values above (below) the thick line are positive (negative).

values of $K_y$ within the zonal jets.

The error estimate for methods $D1$ and $D2$ show a different meridional structure in the sense that method $D1$ seems to be less noisy in the jet cores than in between them, whereas $D2$ has slightly increased errors in the jet cores. The error for method $D2$ ranges between 1000-2000 m$^2$/s in the upper 600 m. Note that these error bars represent errors for estimates in individual bins, which happen to be of the same order of magnitude as the standard deviation of $K_y$ in different bins above 600 m, as can be seen comparing with the error bars in Fig. 6. Both methods produce noisy results in the bins close to the bottom. An error estimate based on the difference between values at positive lags and corresponding negative lags shows no clear depth dependence (not shown), but values similar to the ones shown Fig. 7.

Fig. 8 shows an estimate of the Eulerian diffusivities $\tilde{K}_b$ and $\tilde{K}_q$ in the same integration of the channel model, similar to the results in Eden (2009). Although the mean fields of buoyancy and potential vorticity differ a lot (not shown) the flux-gradient relationship is similar. Note that the meridional derivative of the mean potential vorticity changes sign at mid-depth (white areas of Fig. 8) for which the estimated diffusivity has a singularity.
For more details about the estimates the reader is referred to Eden (2009). Both Eulerian diffusivities show a similar depth dependency as $K_y$, i.e. a strong increase with depth, although the Eulerian estimate is mostly larger (lower) than the Lagrangian one above (below) middepth (not shown). This difference is of about the same magnitude as the error bounds obtained from the bootstrapping method.

Fig. 8 also shows the mean Eulerian zonal velocity, featuring four eastward zonal jets, coinciding with the positions of the zonal jets in the mean particle velocities. Magnitudes of the jets vary between 20 and 35 cm/s in the Lagrangian zonal velocity, while the Eulerian mean zonal velocity peaks at 30 to 50 cm/s, although the variations in the magnitude of the individual jets appear similar, i.e. the two northern ones are weaker than the southern ones in both zonal velocities. Further, the jets in the Eulerian mean reach the bottom, while the Lagrangian estimate features a flow reversal around 1500 m depth. In any case, we see that in both Eulerian estimates, diffusivities are clearly reduced indicating mixing barriers within the eastward jets, and the region of reduced diffusivities extends down to the bottom of the channel.

5 Zonal particle dispersion

The timeseries of $K_x$ are in the range of 0 to $2 \times 10^5$ m$^2$/s (not shown) and generally increase monotonically in time for all lags, i.e. they do not approach an asymptotic limit within the period in which the particles stay close to their respective bins. We interpret this behaviour as an effect of dispersion caused by the sheared zonal mean flow as discussed by Oh et al. (2000), i.e. the dispersion of the particles is not due to the eddying flow, but due to the shear in the mean flow. We conclude that shear dispersion is the dominating process causing zonal dispersion, masking the smaller effect of zonal dispersion by mesoscale activity.
We repeat the analysis with a set of trajectories obtained from a velocity field from which the instantaneous zonal mean flow was subtracted, since it is this part which generates the shear dispersion and which biases the estimate of $K_x$. In this experiment (NOMEAN), the non-diffusive particle dispersion caused by the sheared mean flow should thus be eliminated to a large extent. In fact, the timeseries for $K_x$ flatten after several days for experiment NOMEAN, with asymptotic values in the range of $0$ to $4 \times 10^4 \text{m}^2/\text{s}$ (not shown). Fig. 9 shows a set of 200-day pseudo-trajectories from a representative bin at the surface, integrated with the full velocity (Fig. 9 a) and in experiment NOMEAN (Fig. 9 b). The particle cloud in the experiment with the full velocity is clearly more elongated in the zonal direction, due to the meridional shear of the zonal mean current, while this is not the case in NOMEAN. It therefore appears that the non-diffusive shear dispersion is effectively removed in NOMEAN. Note also that the zonal mean zonal particle velocity in experiment NOMEAN ranges from $-3 \text{cm/s}$ to $2 \text{cm/s}$, as opposed to $-0.2 \text{m/s}$ to $0.4 \text{m/s}$.
Figure 10: a) and b) Asymptotic values of the zonal diffusivity $K_x$ in experiment NOMEAN in $\log_{10} \text{m}^2/\text{s}$ estimated with $D1$ (a) and $D2$ (b). Bins are painted in white if $K_x$ is undefined at all lags in the averaging intervals from $t = -100 \text{d}$ to $t = -60 \text{d}$ and $t = 60 \text{d}$ to $t = 100 \text{d}$. Also shown is the zonal mean zonal Lagrangian particle velocity of the base experiment. The thick black contour line describes depths of zero velocity; thin lines are positive (solid) and negative (dashed) zonal mean Lagrangian particle velocity in intervals of 5 cm/s. d) to f) same but for $K_y$ in NOMEAN, except that additional bins are painted in white if the mean value of $K_y$ in the averaging interval is negative. Contour lines show the zonal mean zonal Lagrangian particle velocity of NOMEAN, in intervals of 2 mm/s. Note the different color scale compared to Fig. 7 and 8.

in the base experiment (Fig. 10, contour lines).

Fig. 10 a) and b) show the asymptotic value of $K_x$ in experiment NOMEAN as a function of latitude and depth. The two methods agree in their result. However, the meridional and vertical structure is different from $K_y$ in the base experiment (Fig. 7), i.e. there is a decrease with depth of $K_x$ in NOMEAN instead of an increase and there are maxima of $K_x$ in NOMEAN within the (removed) eastward zonal jets. It is already evident from Fig. 9 that the meridional dispersion also appears to be different in NOMEAN: while the trajectories in Fig. 9 a) are contained within the two neighbouring southward bins with respect to the bin of release, the trajectories in Fig. 9 b) extend almost to the
Figure 11: Empirical variance of $Y/L_b$ as a function of positive time lag for all pseudo-trajectories above 600 m depth (black), between −600 m and −1200 m (red) and between −1200 m and −1800 m (green). Also shown is the respective slope of variance at lag $t = 60$ d (dashed) and the variance at lag $t = 60$ d is marked with a dot.

southernmost bin of the channel. Fig. 10 c) and d) show the asymptotic value of $K_y$ for experiment NOMEAN. Here, the two methods agree in showing much smaller values for $K_y$ than for $K_x$ in NOMEAN and a less pronounced vertical structure as for $K_y$ in the base experiment, such that surface (deep) values of $K_y$ in NOMEAN are larger (smaller) than the corresponding values in the base experiment. A decrease with depth, however, as seen in $K_x$ for NOMEAN, cannot be noticed for $K_y$ in NOMEAN. Further, the two methods again do not agree in their meridional structure: while $D1$ tends to feature still minima of $K_y$ within the eastward jets, $D2$ suggest maxima of $K_y$ within the jets.

6 Summary and Discussion

6.1 Differences between Lagrangian and Eulerian diffusivity estimates

Eulerian and Lagrangian estimates agree on the vertical variation of diffusivity, although Eulerian estimates averaged on the Lagrangian bin grid are generally larger (lower) than Lagrangian estimates above (below) middepth. The difference in amplitude is of comparable magnitude as the error bounds obtained from the bootstrapping method and might be therefore left unconsidered. However, Fig 11. shows a possible reason for the difference
in amplitude of both estimates: The time series of the mean square displacement averaged over all particles shows for the upper layers a pronounced transition from a non-linear regime with larger increase to the asymptotic spreading rates with smaller increase after about 10 to 20 days lag, while for the lower layer the mean square displacement appears to converge to the asymptotic spreading regime much earlier. This effect leads in the shallower layers to an underestimation of the particle dispersion, when estimating the dispersion rate in the asymptotic limit.

A comment on length scales is appropriate here; The Eulerian method captures the partition of regions with strong mixing between the jets and mixing barriers in the jet cores much clearer than the Lagrangian method, i.e. it captures variations of mixing on small length scales. Since the global asymptotic meridional dispersion (relevant for scales larger than the jet spacing) is defined by the permeability of the barriers (the jets) and not by strong mixing between the jets, we expect that it is constant in meridional direction and a lower bound for any local estimate of transient diffusive dispersion. The reason why our Lagrangian estimates yield a blurrier picture of meridional variations and generally lower values of diffusivity in shallow layers may be that they represent larger spatial scales than the Eulerian method.

6.2 Meridional diffusivities

Two different Lagrangian estimates of a spatially variable diffusivity are discussed in this study, based on particle dispersion rates in an idealized channel model with a highly turbulent flow featuring eddy-driven zonal jets in the zonal mean. The flow can be thought as representative of zonal jets in the interior of ocean basins. The spatial inhomogeneity of the flow due to the zonal jets dictates an upper bound for the bin size for drifter trajectories, while the advection by the mean flow and the dispersion by turbulent fluctuations require
that the bin size is not to be chosen too small. Note that this dilemma is typical for diffusivity estimates from particle dispersion in the ocean, in particular for those from sparse in situ float or surface drifter observations. Using ideal particle trajectories in this specific model configuration, we have shown that a meridional bin size of 200 km is appropriate for both conditions.

The timeseries for meridional dispersion rates \( (K_y(t)) \) attain a global maximum of 1-2 \( \times 10^4 \) m\(^2\)/s during the first 20 days after deployment. The maxima of \( K_y(t) \) are decreasing with depth. For larger lags, \( K_y(t) \) is decreasing and reaches an asymptotic limit of 10-11000 m\(^2\)/s during the interval from day 60 to 100 for most of the bins. In contrast to the maxima, the asymptotic values of \( K_y(t) \) are increasing with depth. These results are similar for both methods. In several previous studies using sparse in situ float and surface drifter data, the first maximum of the dispersion rate was interpreted as its asymptotic limit (e.g., Freeland et al. (1975); Krauss and Boening (1987) and Lumpkin et al. (2002)), since values beyond the first zero crossing were judged to be too noisy, due to the limited amount of data. Note that the maximum of the dispersion rate is identical to its asymptotic limit only if the velocity autocorrelation function of the Lagrangian particles does not become negative, which in turn implies that the dispersion rate increases monotonically. However, Lagrangian particles in the ocean and in numerical ocean models do feature pronounced negative lobes, caused by mesoscale eddies, wave activity and meanders (Veneziani et al. (2004), Berloff et al. (2002)). Therefore, the first maximum of the dispersion rate is not a good estimator for its asymptotic limit in the ocean.

We have compared the Lagrangian diffusivity estimates to an Eulerian estimate of meridional diffusivity using a turbulent flux/mean gradient relationship for buoyancy and potential vorticity from the same model integration. Both estimates agree with respect to the vertical structure of the diffusivity, i.e. they feature a decrease with depth. They
differ, on the other hand, in their magnitudes; the Eulerian estimate is larger for both diffusivities related to buoyancy and potential vorticity. Both the Eulerian estimate and the Lagrangian estimate $D1$ tend to feature minima of meridional diffusivities within the eastward, surface intensified zonal jets, indicating mixing barriers associated with these zonal jets (Dritschel and McIntyre, 2008). However, the Lagrangian estimate $D2$ shows no clear tendency for minima in the jets, and further investigation is needed to understand this difference between the Lagrangian estimates. Moreover, we can only speculate about the asymmetry of $K_y$ in meridional direction (Fig. 7); Due to the varying $f$ (we do use a full primitive equation model) environmental parameters vary, such that jet width, depth scale and eddy energy all vary with latitude.

6.3 Particle dispersion and rotational eddy fluxes

The vertical dependence of the maximum value of $K_y(t)$ has been noted by other authors (Griesel et al., 2010), who also found that the magnitude of the maximum of $K_y$ might not be representative to its asymptotic value. For the flow investigated in this study, our results show in fact an inverse relationship between the vertical dependency of the maximum and the asymptotic value of $K_y(t)$. Griesel et al. (2010) argue that the first peak of the dispersion rate is dependent on the details of the mesoscale-dynamics which might influence, but do not determine its asymptotic behavior. They suggest that meanders and, to a lesser extent, coherent eddies can dominate the short time lag dependence of the Lagrangian meridional dispersion, and furthermore suggest a link between the initial maximum of Lagrangian dispersion rates and non-divergent rotational fluxes. Griesel et al. (2010) used a 0.1° ocean model of the Antarctic Circumpolar Current to study Lagrangian diffusivities and obtain similarly shaped cross-stream dispersion rates (their Fig. 7) as in this study, in the sense that the global maximum value is attained in the first 20 days, and that this maximum decreases with depth. However, their results do not suggest a
significant depth dependence of asymptotic values.

Fig. 12 a) (from Eden (2010)) shows the horizontal eddy buoyancy flux in the eastern part of the southernmost zonal jet at a depth of 500 m. According to the definition of rotational fluxes after Eden et al. (2007), the zonally directed fluxes at the flanks of the jet are non-divergent and therefore do not impact the mean buoyancy. Only the meridional (down-gradient) part of the flux is divergent, which is, however, much smaller than the non-divergent part and thus hard to see in the figure. These non-divergent fluxes are produced to a large extent by the meandering zonal jet: the meanders yield in the time mean a turbulent buoyancy flux directed to the west (east) at the northward (southward) flank of the jet, i.e. circulating around regions of enhanced variance (related in turn to the meandering zonal jet) and have thus no divergence and no impact on the mean buoyancy. A similar mechanism will effect the particles; they will initially disperse rapidly, followed by a phase in which the meandering jets act to contain the spreading, leading to coherent (spiraling) and well correlated trajectories. Only when the trajectories become uncorrelated, they can reach the diffusive regime, for which $K_y$ reaches in turn its asymptotic value.

Fig. 12 b) illustrates the possible relation between Lagrangian meridional displacement
perturbation \( Y' \), the Eulerian zonal velocity perturbation \( u' \) and the Eulerian buoyancy perturbation \( b' \). The meridional displacement is caused by the meandering of the jet, the zonal velocity perturbation is caused by the high (low) speeds within (outside) the meandering jet core, and the buoyancy perturbation results from buoyancy signal of the jet. Note that the horizontal buoyancy gradient points southward, i.e. a southward (northward) meandering causes a negative (positive) buoyancy perturbation at the southern (northern) flank of the mean velocity profile.

Note also that this goes hand in hand with the notion that regions of enhanced eddy kinetic energy (EKE) are not generally regions of intense mixing; a coherent meandering jet induces large fluctuations in the velocity field, but acts at the same time as a transport barrier. From this perspective, the attenuation of the meandering with depth (not shown) implies a decrease of EKE with depth, which corresponds to the observed decrease in the maxima of the dispersion rate in our model. However, these maxima represent non-divergent rotational fluxes which do not figure in a transport equation. The divergent fluxes, which are relevant for parameterizations of unresolved transport in the mean budget, are manifested in the asymptotic values of dispersion rate, which show an opposite vertical dependence (increasing with depth).

6.4 Mixing barriers and anisotropic mixing in zonal jets

In contrast to the meridional dispersion rates, the zonal dispersion rates in the base experiments do not reach an asymptotic limit. Therefore, we conclude that dispersion rates cannot be used to estimate zonal diffusivities in this model configuration. A similar behavior of zonal particle dispersion was found e.g. in realistic simulations by Kamenkovich et al. (2009) and questions the applicability of this method to estimate meaningful zonal diffusivities in the ocean in general.
In order to resolve this problem, we have removed the zonal mean flow from the velocity field in experiment NO_MEAN. This artificial flow field yields particle dispersions without the non-diffusive shear dispersion. In that experiment, dispersion rates in zonal and meridional direction do converge and reach asymptotic values which can be related to zonal and meridional diffusivities. However, a vertical increase of diffusivity cannot be seen anymore, which means that shear dispersion is an essential component of mixing barriers; Although a sheared zonal mean flow acts to disperse particles in zonal direction only, its removal from the full velocity field leads to a breakdown of the meridional mixing barrier in the remaining velocity field. This confirms the results of Ferrari and Nikurashin (2010), who infer eddy-diffusivities from surface altimetric observations in the Southern Ocean using the method of Nakamura (1996). They find that estimates of cross-jet diffusivities in the Antarctic Circumpolar Current increase dramatically when the mean flow is subtracted from the velocity field. On the other hand, Bauer et al. (1998) study an ensemble of Lagrangian surface drifters in the region 160°W- 100°W, 4°N-12°N in the tropical Pacific, which includes part of the westward North Equatorial Current and the eastward North Equatorial Countercurrent, resulting in strong meridional gradients of the mean flow. Their estimate for asymptotic meridional spreading rates is rather insensitive to the removal of the mean flow, which may be attributed to different dynamics of the flow under consideration.

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Taylor (1922) assumed that the time-series of the velocity of a particle is an ergodic process, which implies that the particle is advected in a field of homogenous and stationary turbulence. He showed that the rate of mean square displacement of an ensemble of particles, which are released at a point, asymptotes to a time-independent constant. Considering only the component in x-direction and assuming that there is no mean flow,

$$\lim_{t \to \infty} \frac{d}{dt} \left( \overline{X^2} \right) = \lim_{t \to \infty} \left( \frac{2u_l(t) \int_0^t u_l(\tau) d\tau}{u_l^2} \right) = 2 \overline{u_l^2} \int_0^\infty r(\tau) d\tau = 2u_l^2 T_L$$

(6)

where

$$T_L := \int_0^\infty r(\tau) d\tau, \quad r(\tau) := \frac{\overline{u_l(t)u_l(t+\tau)}}{u_l^2}, \quad \tau := t - t', \quad X(t) = \int_0^t u_l(t') dt' \tag{7}$$

the overbar is an ensemble average and $X(0)$ is the point of release. $u_l$ is the particle velocity caused by the turbulent velocity fluctuations and $T_L$ is called the Lagrangian time-scale. The velocity autocorrelation $r$ and the statistic $\overline{u_l^2}$ are independent of $t$ because the particle velocity is an ergodic process, and therefore the last equation states that the mean-square displacement grows linearly in time. This is analogous to diffusive transport, which also leads to a linear increase of spatial variance. In a fluid which is characterized by quasi-stationary, homogeneous fluctuation statistics and in absence of time-mean flow or a wave-field leading to Stokes-drift, the concentration evolution of a tracer is governed by

$$\frac{\partial C}{\partial t} = \kappa \frac{\partial^2 C}{\partial x^2}, \quad \text{where} \quad \kappa := \overline{u_l^2} T_L \tag{8}$$

provided that the tracer field varies smoothly at length-scales which are large in comparison to the respective Lagrangian scale. Let $t_b$ be chosen such that

$$T_L \approx \int_0^{t_b} r(\tau) d\tau \tag{9}$$
From Eq. (6) follows
\[
\kappa \approx \frac{1}{2} \frac{d}{dt} \left( X^2(t_b) \right) = u_l(t_b) \int_0^{t_b} u_l(\tau) d\tau
\]  
(10)

A time-dependent finite difference approximation corresponding to the second term in Eq. (10) is
\[
\kappa(t) \approx \frac{X^2(t + \Delta t) - X^2(t)}{2\Delta t}
\]  
(11)

which is identical to the estimate \( D1 \). Considering that the time label for the time of release in the ensemble-average experiment can be shifted from 0 to \(-t_b\), the third term in Eq. (10) can be written as
\[
\kappa(t_b) \approx -u_l(0)X(-t_b).
\]  
(12)

In homogeneous turbulence, \( \kappa \) is an odd function of time, and the expression on the right hand side in Eq. (12) is analogous to \( D2 \).

References


