E-Companion - "A Computational Analysis of Bundle Trading Markets Design for Distributed Resource Allocation"

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This e-companion contains four sets of supporting materials for the main paper. §EC.1 provides algorithmic treatments to handle key market implementation issues. §EC.2 examines effects of active market intermediation on market performance and the dealer's wealth under the controlled market experiment. §EC.3 studies market liquidity and heterogeneous market participation in a randomized market environment. §EC.4 includes proofs of Lemmas and Corollaries.

EC.1 Algorithmic Treatment to Market Implementation

There are several implementation challenges when adapting the original BTM algorithm to the asynchronous market trading environment under the dealer's active market intervention and agent learning. The first is premature market closure. The second is cycling.

EC.1.1 Premature Market Closure

In the extended BTM mechanism, three new factors that are related to agents and the dealer's behavior may lead to premature market closure: the dealer's active inventory policy, asynchronous communication and agent learning.

Since the dealer does not engage in production, her inventory holding aims to temporarily balance buying and selling pressures in the market. At some point, the dealer has to switch to the naïve inventory policy to possibly release all of her inventory to agents for their efficient use. To time the switch, we define the moving average price norm (MAPN) in round R as $\text{MAPN}_R = \frac{1}{5}\sum_{t=R-4}^{R} \|p_t - p_{t-1}\|$, $R \geq 5$. When MAPN is smaller than a pre-specified threshold level (which we set as 1) the dealer switches to the NA inventory policy. The rationale for this is that usually market prices vary significantly in initial rounds of market trading but tend to stabilize as the market converges. The aggregate price variation decreases non-monotonically over time. We impose the condition of 5 rounds to ensure that the dealer won't switch her inventory policy too early after observing relatively stable market prices in early rounds due to the lack of active trades.

In the asynchronous market trading environment agents may not be able to participate in each round of market trading due to communication cost or other considerations. In order to prevent premature market closure, the dealer has periodic contact with an inactive agent to make sure they won't be inactive forever. Specifically, define the periodic contact cycle length as X_j so that the dealer will communicate with Agent j if she finds the agent is inactive for X_j rounds.

The reason that agent learning may lead to premature market closure is because of forecast error. If an agent's forecasted market prices do not agree with the equilibrium market clearing prices, the agent may choose different bundles (in contrast to "correct" bundles under the true equilibrium prices) or simply not be interested in submitting any new bundles based on his distorted market information. The latter may trigger the algorithm stopping condition therefore directly affecting full preference elicitation.

We distinguish two types of learning strategies. The first type is myopic learning, under which an agent just uses the most recently observed market price as a proxy for the future market price. Any other more sophisticated learning models, such as those that adopt historical time series data to estimate future market movement, belong to the second type of strategy which we call forecast learning.

The following Lemmas ensure an effective algorithmic treatment to guarantee market convergence to an optimal solution under these two types of learning strategies. Proofs of all Lemmas are in Section EC.4.

Lemma 4. (Convergence under Myopic Learning) *Under a myopic learning strategy, if an agent's communication frequency is P_j, for $j = 1, ..., k$, and the dealer contacts the agent with the current market prices if the agent is inactive for $X_j \geq 1/P_j$ rounds, then the market converges to an equilibrium allocation if there are no new orders in $X \geq 2max(X_1, ..., X_k)$ rounds.*

Lemma 5. (Convergence under Forecast Learning) *Under a forecast learning strategy, suppose an agent's communication frequency is P_j, for $j = 1, ..., k$, and the dealer contacts the agent with the current market prices if the agent is inactive for $X_j \geq 1/P_j$ rounds. When there are no new orders in $X \geq 2max(X_1, ..., X_k)$ rounds, all agents are called and they use the current market prices in* *their bundle selection. If there are no new orders upon the market call, then the market converges to an equilibrium allocation.*

EC.1.2 Cycling

Cycling is a situation where the same agent bundles are repeatedly submitted and settlements among several agents, possibly involving the dealer, return to earlier points of market allocation. We can show that the following treatment can effectively deal with potential cycling.

Lemma 6. (Anti-Cycling) *In a non-value-added trading round, if only one agent has a positive trade (either a match from the dealer's inventory or matches among its own bundles), then void the transaction; if more than one agent trades, and the dealer's inventory repeats a previous level in the oscillation list, then freezing the dealer's inventory provision until the next new order by non-excluded agents prevents cycling.*

We assume that the dealer does not honor a non-value-added trade if she is the only counter-party in the transaction or if the agent trades with himself, but does honor a non-value-added trade if other agents are involved as the counter-parties. With the anti-cycling treatment, the market monotonically improves the system allocation toward an optimal solution after each profitable trade.

EC.2 Effects of Active Market Intermediation

To gain further insight about how the dealer's inventory policies would lead to different market outcomes, we conducted paired t-tests under different levels of agent asynchronous communication and learning models. We selectively present results under the learning model L = MA. Table EC.1 shows the preferred inventory policies based on two criteria: market performance and the dealer's profit.

There are several interesting observations regarding market performance. First, no significant market performance differences were observed when the market communication is highly asymmetric and asynchronous (L=MA, P=0.2, 0.5). This is probably due to the difficulty of matching bundles when agents cannot coordinate timing of their decision-making to facilitate trades among themselves. In addition, complexity of the trading environment such as large bundle size and large number of agents contributes to the difficulty of market matching. This is evident by the statistically indifferent policy implication under the 50 agents 8 shared resource markets. The complicated trading process makes the market convergence paths highly unpredictable. So no inventory policy

Table EC.1: Preferred Inventory Policies Based on Market Performance and Dealer Wealth: O = 0

P	Market Performance				Dealer's Profit			
	m=2		m=8		m=2		m=8	
	k=10	k=50	k=10	k=50	k=10	k=50	k=10	k=50
P=0.2	–	–	–	–	–	NA≻FISS*	NA≻FISS*** NA≻FISP*** SP≻FISP**	NA≻(FI)SS*** NA≻FISP*** FISS≻SS*** SP≻FISP***
P=0.5	–	–	–	–	NA≻SS* FISS≻SS*	NA≻(FI)SS**	NA≻(FI)SS*** NA≻FISP*** FISS≻SS*** SP≻FISP**	NA≻(FI)SS*** NA≻FISP*** FISS≻SS*** SP≻FISP***
P=0.8	NA≻(FI)SS*	–	NA≻SP* FISS≻SS*	–	NA≻SS*	–	NA≻(FI)SS*** NA≻FISP*** FISS≻SS*** SP≻FISP**	NA≻(FI)SS*** NA≻FISP*** FISS≻SS*** SP≻FISP***
P=1	NA≻All* FISS≻SS**	NA≻SS***	NA≻SS** NA≻(FI)SP* FISS≻SS***	–	NA≻SS*** FISS≻SS***	NA≻SS* SP≻FISP*	NA≻(FI)SS*** NA≻FISP*** FISS≻SS*** SP≻FISP*	NA≻(FI)SS*** NA≻(FI)SP*** FISS≻SS*** SP≻FISP***

Notes: a) "–" indicates no policy difference; b) "≻" indicates policy preference; c) We mark the lowest significance level when NA outperforms all other inventory policies (NA ≻ All).

seems to systematically outperform the others.

Second, when the market is fully synchronized (P=1), we observe the naïve inventory policy either outperformed or yielded comparable performance than other active inventory policies (statistically significant at 0.05 level). It seems to suggest that there is no need for the dealer to perform active market intervention because the synchronized market has an inherent market liquidity effect in bundle execution. Furthermore, under the same policy, the dealer's informational advantage helps improve market performance. We see that the corresponding FISS or FISP policies outperformed SS or SP policies in many cases.

The wealth effect under different policies crucially depends on market size and bundle complexity. For example, in a small market trading simple bundles (e.g., 10 agents and 2 shared resources), different inventory policies do not seem to significantly affect the dealer's wealth except for the dominated SS policy. However, if it is in a large market or a market trading complex bundles, the naïve inventory policy outperformed both SS and SP policies regardless of the dealer's informational advantage. Although the dealer's informational advantage helps improve market performance for both SS and SP policies, it has positive wealth implication for the SS policy only.

EC.3 Extensions of the Computational Market Experiment

In the main text, we mainly focus on the controlled experiment design to systematically study the impacts of key market design factors on market performance as well as agents and the dealer's wealth. In this extension, we perform additional randomized market experiments to analyze more realistic market scenarios and heterogeneous market participation.

EC.3.1 Market Liquidity under Different Inventory Policies and Agent Strategies

In finance, liquidity is defined as the ease of trading. It is the ability to trade large sizes quickly. Therefore, it is expected that liquid market prices are less volatile than illiquid ones. Apparently, liquidity differs across assets and markets. While exchanges mainly rely on endogenous submission of limit orders for liquidity provision, most major stock markets designate market makers with affirmative obligations to supply liquidity. In the following, we present several measurements related price variation and trading volume. The data in Table EC.2 are based on the market experiment described in Section 7.1.

Recall that we use the moving average price norm MAPN to measure price variation during the trading process. We record the maximum MAPN value, maxMAPN, for each market experiment. We report the minimum maxMAPN value (minimum price volatility, or MPV) under each treatment in Panel (a). Since bundle sizes vary in our experimented markets, we define a unified measurement: the average trading volume (ATV), which is the summed trading volume for all shared resources divided by the number of shared resources traded in the market. Panel (b) shows the mean number of ATV for the dealer. Panel (c) reports the total market ATV, where both the dealer and all agents' trading volumes are counted.

We observe that both the dealer's and the market total trading volume increase as either the number of market participants or the number of shared resources increases. However, the effect of market size is larger than the effect of bundle complexity. This is evident from the fact that, regardless of the bidding strategy and the bundle size, markets with 50 agents had much higher average ATV than markets with 10 agents.

To see whether price volatility and trading volume would differ under different inventory policies, we perform paired t-test. Results are presented in Table EC.3.

Table EC.2: Summary Statistics: Market Price Variation and Trading Volume

Market		V	Panel (a) MPV			Panel (b) Dealer ATV			Panel (c) Market ATV		
k	m		TRUE	RAN	FIX	TRUE	RAN	FIX	TRUE	RAN	FIX
10	2	NA	0.45	0.43	0.58	24.9	29.20	32.07	408.2	403.6	424.2
		SS	0.41	0.27	0.41	113.9	84.66	100.7	524.6	406.8	446.3
		SP	0.40	0.43	0.49	38.58	45.42	45.57	488.3	355.3	463.6
		FISS	0.45	0.36	0.44	103.9	70.06	81.33	507.7	327.5	451.4
		FISP	0.38	0.65	0.50	75.02	67.21	59.92	379.0	453.5	455.2
	8	NA	1.21	1.63	1.52	139.0	139.0	140.4	451.3	543.1	513.8
		SS	1.17	1.48	1.76	244.3	284.0	335.6	597.2	714.8	905.9
		SP	1.29	1.35	1.78	155.5	156.5	149.2	486.7	544.3	519.5
		FISS	0.89	1.30	0.86	198.0	177.5	190.8	587.8	504.4	609.6
		FISP	1.44	0.91	1.45	178.1	146.8	167.9	547.4	443.7	526.1
50	2	NA	0.52	0.62	0.66	191.3	56.24	89.74	6101.7	28560.4	3644.2
		SS	0.54	0.63	0.57	509.9	648.4	770.9	3214.1	29085.5	3960.2
		SP	0.61	0.60	0.58	168.3	176.9	182.2	30500	3869.0	3471.8
		FISS	0.63	0.59	0.52	649.2	535.4	659.0	35936	3452.6	4798.0
		FISP	0.55	0.65	0.66	625.6	844.5	606.2	4567.7	4626.3	25448
	8	NA	2.26	1.45	1.54	354.9	345.9	378.1	3237.7	3094.0	2844.5
		SS	1.28	1.78	1.41	680.1	619.6	768.0	3969.4	3333.4	3785.6
		SP	1.66	1.69	1.28	450.5	440.5	450.9	3019.9	3615.8	3489.1
		FISS	1.17	1.18	1.14	907.6	547.8	468.7	5248.7	3027.3	2814.9
		FISP	1.60	1.33	1.59	457.9	546.8	449.7	2970.7	5842.4	3001.2

Table EC.3: T-Test: Market Price Variation and Trading Volume

Market		V	Panel (a) MPV			Panel (b) Dealer ATV			Panel (c) Market ATV		
k	m		TRUE	RAN	FIX	TRUE	RAN	FIX	TRUE	RAN	FIX
10	2	NA-SS	−0.53	−1.71	−0.60	−5.93***	−6.68***	−6.46***	−1.12	−0.05	−0.33
		NA-SP	1.01	−0.10	−0.40	−1.96	−1.67	−1.01	−1.02	0.81	−0.38
		SS-FISS	0.81	0.44	−0.90	0.43	1.31	1.28	0.15	1.38	−0.07
		SP-FISP	−0.80	−1.38	−0.85	−2.0	−1.41	−0.78	1.53	−1.44	0.08
	8	NA-SS	1.00	−0.69	−0.33	−4.19***	−2.68*	−4.12***	−1.88	−1.03	−2.67**
		NA-SP	0.99	−0.42	−0.97	−1.10	−1.11	−0.60	−0.60	−0.01	−0.07
		SS-FISS	−1.38	1.44	1.91	1.57	1.80	2.72	0.10	1.31	1.67
		SP-FISP	−0.32	1.66	1.00	−1.18	0.68	−1.08	−0.89	1.50	−0.09
50	2	NA-SS	1.26	−0.96	0.65	−3.22**	−6.00***	−4.27***	1.71	−0.01	−0.35
		NA-SP	0.28	0.92	0.38	0.29	−1.90	−2.0	−0.95	0.93	0.20
		SS-FISS	−0.91	0.92	−1.18	−0.90	0.81	0.51	−1.04	1.07	−0.54
		SP-FISP	0.85	−0.58	−0.75	−2.98**	−1.90	−2.29*	1.01	−0.55	−0.97
	8	NA-SS	−0.18	−1.64	1.05	−5.68***	−5.42***	−3.57***	−0.69	−0.38	−1.60
		NA-SP	0.05	−1.76	1.15	−2.72**	−2.43*	−1.35	0.55	−0.49	−1.26
		SS-FISS	0.69	1.47	1.07	−0.74	1.09	3.07**	−0.66	0.61	1.71
		SP-FISP	0.77	1.19	−0.68	−0.19	−1.30	0.02	0.13	−0.74	0.88

From Panel (a) we see that no statistically significant difference was found in market price variation under different agent bidding strategies. In all market scenarios, Panel (b) shows that the dealer's average trading volume under the safety stock policy is significantly higher than the naïve inventory policy at the 0.05 level. However, the total average trading volume difference between NA and SS was not statistically significant except for $k10m8$ market under the fixed percentage bidding strategy. It is evident that the dealer indeed actively traded in the market under the SS policy, where more trades occur between agents and the dealer rather than agents themselves. Moreover, in large markets with complex bundles being traded, the dealer's informational advantage will make her trade more conservatively (statistically significant under the fixed percentage strategy).

Under the SP policy, the dealer's average trading volume is significantly higher than the naïve policy only in large markets trading complex bundles. In large markets with simple bundles being traded, the dealer's informational advantage will make her trade more aggressively (statistically significant under truthful bidding and fixed percentage strategy). There was no observed difference in the average trading volume when the number of market participants is small regardless of whether the dealer has informational advantage.

The fact that the dealer has actively traded in complex market environments without significantly improving market performance implies the difficulty to directly apply conventional wisdom in financial markets to the BTM market environment. The dealer's active intermediation does not necessarily lead to quicker price discovery. Moreover, although financial market theory suggests that high trading volume corresponds to low price volatility, this does not seem to be supported by our trading data. We caution the mechanism designer to be careful when implementing the BTM framework. Traditional financial market insights may not be directly applied and transferred to the BTM trading environment.

EC.3.2 Randomization of Agent Structure and Bidding Strategies

In this experiment, we take into account the heterogeneity of agent internal structure (the number of independent resources and the number of activities). We further allow agents to randomly select their communication frequencies, learning models, and bidding strategies from the respective treatment sets. Table EC.4 presents the mean number of market iterations and the mean WealthRatio under the five inventory policies and four market scenarios. Data are based on the market experiment described in Section 7.2.

Table EC.4: Summary Statistics under Randomized Experiment

	V	Panel (a) Mean Number of Iterations				Panel (b) Mean WealthRatio			
		m=2		m=8		m=2		m=8	
		k=10	k=50	k=10	k=50	k=10	k=50	k=10	k=50
Mean									
	NA	56.54	63.07	155.44	147.81	0.97	0.99	0.81	0.93
	SS	54.53	61.36	153.17	173.12	1.09	1.13	1.24	1.11
	SP	55.65	62.23	153.18	147.08	0.96	0.99	0.93	1.01
	FISS	58.13	64.66	163.27	154.30	0.97	1.02	0.90	1.01
	FISP	55.15	60.14	211.47	178.87	1.05	1.16	0.90	1.04
t-Stat									
	NA-SS	0.69	0.50	0.18	−0.81	−4.72***	−6.82***	−8.73***	−21.48***
	NS-SP	0.28	0.22	0.18	0.04	0.23	−0.55	−5.13***	−7.03***
	SS-SP	−0.38	−0.23	0.01	0.77	4.48***	5.86***	5.86***	7.35***
	NA-FISS	−0.48	−0.42	−0.51	−0.32	−0.02	−5.19***	−6.80***	−13.77***
	NA-FISP	0.43	0.85	−0.91	−0.65	−3.79***	−6.24***	−5.84***	−10.39***
	SS-FISS	−1.14	−0.90	−0.76	0.53	4.51***	4.91***	6.85***	10.57***
	SP-FISP	0.15	0.56	−0.95	−0.64	−3.56***	−5.64***	1.07	−1.92
	FISS-FISP	0.87	1.24	−0.78	0.48	−3.57***	−4.83***	−0.10	−2.48*

We see that markets with 2 shared resources converge under 100 rounds for all five inventory policies, while markets with 8 shared resources take over 100 rounds to converge. This shows that bundle size or complexity has a major impact on market convergence. However, we see that markets trading 8 shared resources on average converge in 155.44 rounds with 10 agents, but 147.81 rounds with 50 agents under the naïve inventory policy. In fact, except for the safety stock inventory policy, all other inventory policies yielded quicker market convergence in larger markets when trading complex bundles. This suggests that our BTM framework is scalable to larger sized auctions with larger sized bundles. However, as shown in the paired t-tests, the market performance differences under different inventory policies are not statistically significant.

Although the dealer's choice of the inventory policy does not significantly impact the market performance, it has significant effect on the dealer's profit. For example, the negative t-values in the WealthRatio tests NA-SS imply that the naïve inventory policy yields higher profit than the safety stock policy. The profit difference is significant at the 0.001 level under all tested market scenarios. In fact, both the NA and SP policies outperform the SS policy. The NA policy further outperforms the SP policy in complex trading environments involving large number of resources or large number of market participants. However, in markets that trade simple bundles, there was no observed profit difference between the naïve and the speculative price policies.

The value of information has different wealth effect under different policies. If the dealer adopts the safety stock policy, then the informational advantage could significantly improve the dealer's wealth. In contrast, if the dealer adopts the speculative price policy, then the informational advantage could lead to a statistically significant wealth decrease in markets trading simple bundles, though the wealth effect was not significant in markets trading complex bundles. Although the SP policy yields higher profit than the SS policy, the FISP policy yields lower profit than the FISS policy. Therefore, the dealer would prefer the speculative price policy when she does not have informational advantage, but would prefer the safety stock policy when she has better market price information than agents.

EC.4 Proofs

Proof of Lemma 1

Proof. For limited orders, we see that $U_j(w_j) = v_j(w_j) - p^{'}w_j = z_j(c_j) - z_j(c_j + w_j) - p^{'}w_j = z_j(c_j) - [d_j^{'}\bar{x}_j + p^{'}w_j]$. Since $z_j(c_j)$ is a constant, minimizing the objective function in problem (3) is equivalent to identifying the highest utility bundle.

For unlimited orders, $U_j(u_j) = v_j(u_j) - p^{'}u_j = -d_j^{'}\hat{x}_j - p^{'}u_j = -[d_j^{'}\hat{x}_j + p^{'}u_j]$. Minimizing the objective function in problem (3) is equivalent to identifying the highest utility bundle. $\square$

Proof of Lemma 2

Proof. Theorem 1 in Guo et al. (2007) shows that $p^{'}w^* \leq v_j(w_j^*)$. Since $v_j(w_j^*) = z_j(c_j) - z_j(c_j + w_j^*)$, we have $p^{'}w_j^* \leq z_j(c_j) - z_j(c_j + w_j^*)$. Accordingly, we have $e_j(c_j) - p^{'}w_j^* - z_j(c_j + w_j^*) \geq e_j(c_j) - z_j(c_j)$. Since $e_j(c_j) - p^{'}w_j^* - z_j(c_j + w_j^*) = e_j(c_j + w_j^*) - z_j(c_j + w_j^*) = W_j(c_j + w_j^*)$, and $e_j(c_j) - z_j(c_j) = W_j(c_j)$, we have $W_j(c_j) \leq W_j(c_j + w_j^*)$. So Agent j's wealth after trading bundle w_j^* is no less than it before trading. That is, the trading is wealth non-decreasing. $\square$

Proof of Lemma 3

Proof. If the identity $Min \sum_{j=1}^{k} z_j(c_j^0 + \bar{w}_j) = Z(c)$ holds, it is straightforward that the market mechanism efficiently allocates resources because it achieves the allocation under complete information. This completes the sufficiency.

Now we prove the necessity. In auction, an efficient allocation maximizes total value over all agents. By the definition of bundle valuation, $\sum_{j=1}^{k} v_j(\bar{w}_j) = \sum_{j=1}^{k} z_j(c_j^0) - \sum_{j=1}^{k} z_j(c_j^0 + \bar{w}_j)$. Since

$\sum_{j=1}^{k} z_j(c_j^0)$ is fixed, we have $Max \sum_{j=1}^{k} v_j(\bar{w}_j) = \sum_{j=1}^{k} z_j(c_j^0) - Min \sum_{j=1}^{k} z_j(c_j^0 + \bar{w}_j)$. Allocative efficiency of the auction mechanism implies that total operating cost of all agents is minimized. We know from the Central Problem that the minimal operating cost of all agents is $Z(c)$. So efficient allocation must guarantee that $Min \sum_{j=1}^{k} z_j(c_j^0 + \bar{w}_j) = Z(c)$. □

Under agent learning with asynchronous communication, the key for algorithm convergence to an optimal solution is that no agents are interested in trading new bundles at the current market prices which support an equilibrium allocation. Proofs of Lemmas 4 and 5 use this line of reasoning.

Proof of Lemma 4

Proof. Under the myopic learning strategy, an agent uses his most recently observed market price to solve his bundle determination problem. Note that there is a two-way communication in the asynchronous market environment. On average, an agent will contact the dealer in $1/P_j$ rounds. If not, the dealer has periodic asynchronous contact with an inactive agent to make sure by any chance they won't be inactive forever. Let the periodic contact $X_j \geq 1/P_j$. Then agent j will get the current market price in at most $2X_j$ rounds. In the worst case scenario, at most $2\max(X_1, ..., X_k)$ rounds all k agents obtain the current market price information. Theorems 3 and 4 in Guo et al. (2007) ensure that the market converges to an equilibrium allocation in finite number of trades. □

Proof of Lemma 5

Proof. Under the forecast learning strategy, an agent uses his forecasted market price to solve his bundle determination problem. In the EWMA price forecast model, $\tilde{p}_{jR} = \alpha\pi_{jR} + (1-\alpha)\tilde{p}_{j,R-1}$, where $0 < \alpha < 1$ is the smoothing constant. Note that $\tilde{p}_{j0} = p_0$. Repeatedly substituting prior round forecasted prices into the forecast equation, we have an equivalent expression $\tilde{p}_{jR} = \alpha \sum_{i=0}^{R-1}(1-\alpha)^i \pi_{j,R-i} + (1-\alpha)^R p_0$. Since the realized prices vary in prior rounds, we see that $\tilde{p}_{jR} \neq \pi_{jR}$. This requires an extra step of a market call to inform agents that it might be the last round trading. Since forecast learning does not have future value for decision making in the last round trading, all agents use current market prices to determine the most preferred bundles. The remaining proof follows a similar reasoning as in Lemma 5. □

Proof of Lemma 6

Proof. From Lemma A in Guo et al. (2007) we know that there is a one-to-one correspondence of bundles and the extreme points or extreme rays in the Central Problem defined in (1). The following equalities hold: $w_j^* = C_j x_j^* - c_j$ for limited bundles and $u_j^* = C_j x_j^*$ for unlimited bundle, respectively. Therefore, from the Simplex method we know that if a round results in $w_j^* \neq 0$ or $u_j^* \neq 0$, the Central Problem moves to a new basic feasible solution. Then it is termed a value-added trading.

There are two types of cycling that prevents the Central Problem from moving to a new basic feasible solution. Algorithmically, it is identified as a series of non-value-added trading that returns to an earlier market allocation. The first is agent self-trading. It is possible that an agent submits limit sell or buy orders in different rounds and his own orders get matched. This is handled in Step 2 where only one agent is added in the positive trade set $Trade$. The algorithm goes back to Step 1 and no settlement is carried out. So the agent's outstanding order book I_j and H_j are not cleared and no such orders can be resubmitted.

The second type of cycling involves the repeated exchange of resources between agents and the dealer. This can happen when there are slack resources in the market. Agents repeatedly submit the same sets of orders in a sequence. Slack resources are switched among agents and the dealer at zero cost. Two treatments prevent such cycling. If the trade involves only one agent and the dealer, then the transaction is voided so the agent's outstanding order book I_j and H_j are not cleared and no such orders can be resubmitted (see Step 2 in the adapted algorithm). Since the Central Problem is non-degenerate by assumption, eventually new bundles will come in which iterates the Central Problem solution to a new basic feasible solution that breaks the cycle.

If the trade involves more than one agent and the dealer, the trade is honored. Agents are added to the positive trade set $Trade$ and their outstanding order books I_j and H_j, for $j \in Trade$ are cleared. Therefore, it is possible that the same sequence of orders is repeatedly submitted and gets matched. In order to detect such oscillation, we track the dealer's inventory levels. Once the dealer's inventory oscillation is detected, freezing the dealer's inventory can break the cycling because it either prevents a full match of the orders that only result in inventory changes of slack resources in agent problem or forces an agent to self-trade. The former may lead to clearance of the outstanding order book so that a different set of orders can be submitted. The latter can be handled as discussed in the first case. As long as the market has not converged to the system optimal allocation, there must exist profitable trades from non-excluded agents (recall that the excluded agents only perform non-profitable trades). This will trigger the dealer's inventory release to allow her to participate in future market trades. This also guarantees that the market continues without risking a premature closure.

Note that it is impossible that different agents switch resources among each other without positive effect on their wealth. If an agent has a positive trade with another agent, the wealth-improving trading bundles must have non-zero prices. Therefore, matching among different agents will have a non-zero wealth effect. This won't result in cycling. □

Proof of Corollary 1

Proof. Lemma 4 ensures that the continuous market operation converges to an optimal allocation in a finite number of trades under the myopic learning strategy. As shown in Lemma 5, $\tilde{p}_{jR} \neq \pi_{jR}$. From linear programming sensitivity analysis, we know the solution to the agent bundle determination problem (3) under $\tilde{p}_{jR}$ and π_{jR} might be different if the two prices sufficiently differ from each other. There is positive probability that there will be new bundles under π_{jR}, but such bundles cannot be discovered under $\tilde{p}_{jR}$. Therefore, it is necessary to use a market call. When the dealer calls the market to signal that it might be the last round of trading, agents find there is no need to forecast market prices anymore. They use the current market prices in their bundle determination problem. If there are new orders, the market continues as usual. Otherwise, the stopping criteria satisfy Theorem 3 in Guo et al. (2007). Lemma 6 treatment ensures that the hybrid market design can guarantee the market convergence to the optimal allocation and equilibrium market prices. □

Proof of Corollary 2

Proof. Guo et al. (2007) have shown that agent strategic bundle pricing merely slows down market convergence without affecting the algorithm finite termination and optimality properties. Under the extended framework and adapted algorithm, now it is sufficient to show that 1) the strategic bundle selection, 2) agent learning with asynchronous communication, and 3) the dealer's inventory policies do not affect the finite termination and optimality property.

As to 1), the predicted market prices based on different forecasting models will only affect the sequence of bundle elicitation. By Lemma A in Guo et al. (2007), each bundle selected from (3) corresponds to an extreme point or extreme ray solution to the Agent Problem (2). Theorem 2 in Guo et al. (2007) further shows the equivalence between solving the market-matching problem (4) and the Central Problem (1). Therefore, the sequence of bundle elicitation only determines which feasible solutions to the Central Problem should be evaluated in each round of market iteration. Because the set of feasible solutions is finite and no feasible solutions can be visited twice by the Simplex method, it is straightforward that the finite convergence and optimality properties are preserved under the assumption that the Central Problem is non-degenerate.

Regarding 2), Lemmas 4 and 5 guarantee market convergence and optimality under agent learning with asynchronous communication.

In terms of 3), recall that the dealer eventually switches to the naïve inventory policy if she initially adopts a different inventory policy. So the final market convergence is affected by the market-matching problem defined in (4). Its finite termination and optimality are guaranteed by Theorems 3&4 in Guo et al. (2007). $\square$