LA IMMACULADA CONCEPCION SCHOOL
SENIOR HIGH SCHOOL
GRADE 11 – STEM: BASIC CALCULUS

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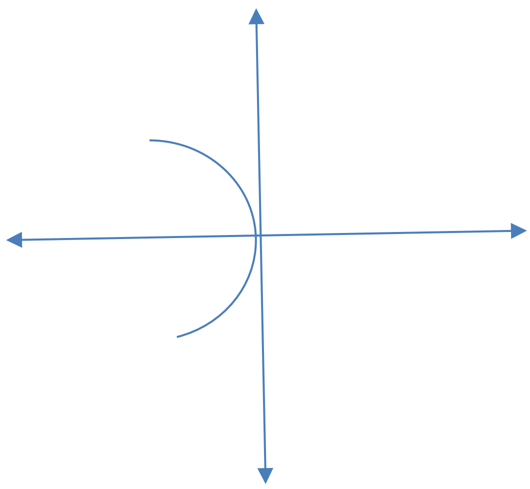
Topic: Tangent Line and Normal Line

A *tangent line* is a le that touches a curve at a point. This point is called *point of tangency*. A line that is perpendicular to a line tangent to a curve at a point of tangency is called *normal line*



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The slope of a normal line is the negative reciprocal of the slope of the tangent line . To find its equationn use the point-slope formula.

Example 1

Find the equation of the tangent line to the parabola $y = x^2 - 5x + 3$ at point (2, -3).

Solution:

a. Find the derivative.
   $y = x^2 - 5x + 3$
   $y' = 2x - 5$

b. Find the slope of the tangent line at $x = 2$
   $Y' = 2x - 5$
   Slope $= 2(2) - 5$
   $m = 4 - 5$
   $m = -1$

c. Find the equation of the tangent line using the Point-Slope Formula
   $y - y_1 = m(x - x_1)$
   $y - (-3) = -1(x - 2)$
   $y + 3 = -x + 2$
   $y = -x + 2 - 3$
   $y = -x - 1$ (the equation of the tangent line)

d. Find the slope of the normal line

   $m_n = -1/m_t$ (slope of the normal line is the negative reciprocal of the slope of the tangent line)
   $m_n = -(1/-1)$
   $m_n = 1$

e. Find the equation of the normal line
   $y - y_1 = m(x - x_1)$
   $y - (-3) = 1(x - 2)$
   $y + 3 = x - 2$

$y = x - 2 - 3$
$y = x - 5$ (equation of the norma line)

Examle 2

Find the equation of the tangent line and the normal line to $y = x^2 - 2x - 1$ at point (3, 2).

Solution:

a. $y = x^2 - 2x - 1$
$dy/dx = 2x - 2$

b. $m = 2x - 2$
$= 2(3) - 2$
$= 6 - 2$
$= 4$

c. $y - y_1 = m (x - x_1)$
$y - 2 = 4 (x - 3)$
$y - 2 = 4x - 12$
$y = 4x - 12 + 2$
$y = 4x - 10$ (equation of the tangent line)

d. $m_n = - 1/m_t$
$m_n = -1/4$

e. $y - y_1 = m (x - x_1)$
$y - 2 = -1/4 (x - 3)$
$y - 2 = -1/4 x + 3/4$
$y = -1/4 x + 3/4 + 2$
$y = -1/4 x + 11/4$ (equation of the normal line)

STUDENT TASK/ACTIVITY

A. Directions: Find the equation of the tangent line and the normal line to the curve at the given point.

1. $Y = x^3 - 3x - 2$ at (-2, 4)
2. $Y = x^3 - 3x + 4$ at (2, 6)
3. $Y = 5x^3 + 6x^2 - 3x$ at (-1, 4)
4. $Y = 2/3 x^2 - 3x + 4$ at (3, 1)
5. $Y = 3x^2 - 2x + 4$ at (1, 5)
6. $Y = x^2 - 4x + 4$ at (3, 1)
7. $Y = x^2 - 2x + 1$ at (2, 1)
8. $Y = 2/3 x^2 - x + 1$ at (2, 5/3)
9. $Y = 1/3 x^2 + 3x + 4$ at (-6, -2)
10. $Y = 1/3 x^3 - 2x + 2$ at (-3, -1)

B. Directions: Determine the equation of the tangent line and the normal line. Then sketch the graph.

1. $F(x) = 2x^2 - 6x + 4$ where x = 2 , y = 1
2. $F(x) = 1/2 x^2 - 2x + 3$ where x = 4 , y = 2
3. $F(x) = 1/2 x^2 - 4x + 4$ where x = 6 , y = -4
4. $F(x) = 2x^2 - 3x - 1$ where x = 5 , y = 3
5. $F(X) = x^2 + 3x - 5$ whwre x- -1 , y = -2

C. Solve the following problems.

1. What is the equation of the line with a slope of 4 that is tangent to $y = x^2 + 2x - 1$?
2. Find the equation of the line with a slope of 6 that is tangent to $y = x^2 + 2x - 6$
3. A line with a slope of –6 is tangent to $y = 2x^2 - 2x - 2$. Find its equation.
4. Determine the equation of the line with a slope of 2 that is tangent to $y = 3x^2 + 3x - 1$.
5. What are the equations of the tangent line and the normal line to the curve $y = x^2 - 4x + 3$ if the slope of the tangent line is 4?

Topic: The Derivative of Exponential Function

Rule # 1: The derivative of e^x is e^x

If $y = e^x$, then $d/dx\ (e^x) = e^x$

The derivative of of the exponential function $f(x) = y = e^x$ is its own function.
This can be combined with the *chain rule.* If u is a function of x, then $d/dx\ (e^u) = e^u\ du/dx$.

Example 1: Differentiate $y = e^{4x+7}$

Solution: Copy e^{4x+7} and then differentiate the exponent 4x + 7

$y = e^{4x+7}$

$dy/dx = e^{4x+7}\ d/dx\ (4x+7)$

$= e^{4x+7}(4)$

$= 4e^{4x+7}$

Example 2: Differentiate $y = e^{3x\text{^}2}$

Solution: Copy $e^{3x\text{^}2}$ and then find the derivative of the exponent $3x^2$

$y = e^{3x\text{^}2}$

$dy/dx = e^{3x\text{^}2}\ d/dx\ (3x^2)$

$= e^{3x\text{^}2}\ (6x)$

$= 6e^{3x\text{^}2}x$ or $6xe^{3x\text{^}2}$

Example 3: Differentiate $y = e^{2\cos(3x)}$

Solution: Copy $e^{2\cos(3x)}$ and then differentiate 2cos(3x)

$y = e^{2\cos(3x)}$

$dy/dx = e^{2\cos(3x)} \cdot 2[-\sin(3x)]\ d/dx\ (3x)$

$= e^{2\cos(3x)} \cdot [-2\sin(3x)](3)$

$= -6e^{2c0s(3x)}\sin(3x)$

Rule # 2: If $y = a^x$, then $d/dx\ (a^x) = a^x \ln a$.

If u is a function of x, then $d/dx\ (a^u) = a^u \ln a\ (du/dx)$

Example 1: Find the derivative of $y = 10^{5x}$

Solution: $y = 10^{5x}$ ($a^u = 10^{5x}$)

$dy/dx = 10^{5x} \ln 10\ d/dx\ (5x)$; a = 10 and u = 5x

$= 10^{5x}(\ln 10)(5)$

$= 5(10^{5x}) \ln 10$

Example 2: Find the derivative of $y = 2^{3x}$

Solution: $y = 2^{3x}$

$dy/dx = 2^{3x} \ln 2\ d/dx\ (3x)$

$= 2^{3x}(\ln 2)(3)$

$= 3 \cdot 2^{3x}\ln 2$

Example 3: Differentiate $y = 5^{3x-1}$

Solution: $y = 5^{3x-1}$

$dy/dx = 5^{3x-1} \ln 5\ d/dx\ (3x - 1)$

$= 5^{3x-1}(\ln 5)(3)$

$= 3 \cdot 5^{3x-1} \ln(5)$

STUDENT TASK

Directions: On a yellow paper, differentiate the following functions. Show your complete soutions.

1. $y = e^{2x-1}$
2. $y = e^{4x^2-5}$
3. $y = e^{3\sin(2x)}$
4. $y = e^{2\tan(5x)}$
5. $y = e^{5\cos(4x)}$
6. $f(x) = 3^{2x+1}$
7. $f(x) = 10^{4\sec(3x)}$
8. $f(x) = 5^{2x^2+3x-1}$
9. $f(x) = 9^{6\cot(6x)}$
10. $f(x) = 8^{3x^2+5x-2}$

Topic: Derivative of Logarithmic Function

Expressions written in exponential form can be converted to logarithmic form and vice versa.

Illustrative Examples:

1. Exponential Form to Logarithmic Form.

a. $5^2 = 25$ can be written as $\log_5 25 = 2$
b. $49^{1/2} = 7$ can be written as $\log_{49} 7 = ½$
c. $3^3 = 27$ can be written as $\log_3 27 = 3$

2. Logarithmic Form to Exponential Form

a. $\log_2 8 = 3$ can be written as $2^3 = 8$
b. $\log_3 81 = 4$ can be written as $3^4 = 81$
c. $\log_{1/2} 32 = -5$ can be written as $(1/2)^{-5} = 32$

Hence $y = \log_b x$ can be written as $b^y = x$ and $y = b^x$ can be written as $\log_b y = x$.

<u>The Derivative of the Natural Logarithmic Function</u>

If $y = \ln x$, then $d/dx (\ln x) = 1/x$

If u is a differentiable function of x, then $d/dx (\ln u) = 1/u \cdot du/dx$

<u>The Derivative of Logarithmic Functions</u>

If $y = \log_b x$, then $d/dx (\log_b x) = 1/x (\ln b)$

If u is a differentiable function of x, then $d/dx (\log_b u) = 1/ (u \ln b) \cdot du/dx$

Examples:

1. Find the derivative of $y = \ln (5x)$
Solution: $y = \ln (5x)$

$dy/dx = 1/5x \cdot d/dx\ (5x)$

$= (1/5x)\ (5)$

$= 1/x$

2. Find the derivative of $y = \ln (5x^2)$

Solution: $y = \ln (5x^2)$

$dy/dx = 1/5x^2 \cdot d/dx\ (5x^2)$

$= (1/5x^2)\ (10x)$

$= 2/x^2$

3. Find the derivative of $y = \ln (x^3 + 4)$

Solution: $y = \ln (x^3 + 4)$

$dy/dx = 1/(x^3 + 4) \cdot d/dx\ (x^3 + 4)$

$= 1/(x^3 + 4) \cdot (3x^2)$

$= 3x^2/(x^3 + 4)$

4. Find the derivative of $y = \ln (4 – 5x)$

Solution: $y = \ln (4 – 5x)$

$dy/dx = 1/(4 – 5x) \cdot d/dx\ (4 – 5x)$

$= 1/(4 – 5x)\ (- 5)$

$= -5/(4-5x)$

5. Find the derivative of $y = \ln (2x^3 + 5)$

Solution: $y = \ln (2x^3 + 5)$

$dy/dx = 1/(2x^3 + 5) \cdot d/dx\ (2x^3 + 5)$

$= 1/ (2x^3 + 5) \cdot (6x^2)$

$= 6x^2/(2x^3 + 5)$

6. Differentiate $y = \log_2 (6x + 5)$

Solution: $y = \log_2 (6x +5)$

$dy/dx = 1/ [(6x +5) \ln 2 \cdot d/dx\ (6x + 5)$

$= 1/ (6x + 5) \ln 2 \cdot (6)$

$= 6/ (6x + 5) \ln 2$

$= 6/ (6x \ln 2) + (5 \ln 2)$

7. Differentiate $y = \log_2 (5x)$

Solution: $y = \log_2 (5x)$

$dy/dx = 1/(5x \ln 2) \cdot d/dx\ (5x)$

$= 1/ (5x \ln 2) \cdot (5)$

$= 5/ (5x \ln 2)$

$= 1/ x\ln 2$

8. Differentiate $y = \log_2 (x^2 - 1)$

Solution: $y = \log_2 (x^2 - 1)$

$dy/dx = 1/ ((x^2 -1) (\ln 2) \cdot d/dx (x^2 - 1)$

$= 1/ (x^2 - 1) \ln 2 \cdot (2x)$

$= 2x/ (x^2 - 1) (\ln 2)$

$= 2x/ [(x^2 \ln 2) - \ln 2]$

9. Differentiate $y = \ln[\sin (x)]$

Solution: $y = \ln [\sin (x)]$

$dy/dx = 1/\sin(x) \cdot d/dx \sin(x)$

$= 1/\sin (x) \cdot \cos (x)$

$= \cos (x)/ \sin(x)$

$= \cot(x)$

10. Differentiate $y = \ln [\cos (3x)$

Solution: $y = \ln [\cos (3x)]$

$dy/dx = 1/\cos(3x) \cdot d/dx \cos (3x)$

$= 1/\cos (3x) \cdot [-\sin (3x)] (3)$

$= -3 \sin (3x)/\cos (3x)$

STUDENT TASK:

A. Directions: Find the derivative of each logarithmic functions.

1. $y = \ln (12x)$
2. $y = \ln (6 - 3x^2)$
3. $y = \ln (5x^3)$
4. $y = \ln (3x^2 - 7)$
5. $y = \ln (x^2 +2x - 3)$
6. $F(x) = \log_2 (9x)$
7. $F(x) = \log_2 (x + 15)$
8. $F(x) = \log_2 (x^2 - 3)$
9. $F(x) = \log_3 (3x)$
10. $F(x) = \log_3 (x^2 - 20)$

B. Directions: Differentiate each given function.

1. $y = \ln \cos (4x)$
2. $y = \ln [\sin (3x) \cos(x)]$
3. $y = \ln [\sin(x) \cos(x)]$
4. $y = \log \tan (3x -5)$
5. $y = \log \cot (1 - 4x^2)$