LA IMMACULADA CONCEPCION SCHOOL
SENIOR HIGH SCHOOL
GRADE 11– STATISTICS AND PROBABILITY

April 27 – 30, 2020
Lesson 7
TOPIC: POINT ESTIMATE FOR THE POPULATION PROPORTION $\mathcal{P}$

Proportions

Proportions can be obtained from samples or populations.

SYMBOLS USED:
n = number of observations in a simple random sample
$\hat{p}$ = sample proportion (read as "p hat")
P = population proportion

$$\hat{p} = \frac{number\ of\ desired\ outcomes}{numbers\ of\ sample\ elements}$$

Proportion is a fraction expression where the favorable response is in the numerator and the total number of respondents is in the denominator. The basic operation involves division. Thus, the result is a decimal value that can be expressed as percent.
For a sample proportion,

$$\hat{p} = \frac{X}{n}$$
$$\hat{q} = 1 - \hat{p}$$

Where: *X* is the number of sample elements that possess the desired characteristics and
n is the sample size.

Example 1: A New Kind of Snack

A random selection of school children were asked whether they *Like (1)*, Do not like (0), or *Cannot decide (2)* whether they like or not, a new kind of snack served by the school cafeteria. The responses are shown as follows:

1	**0**	**1**	**1**	**2**	**1**	**1**	**2**	**0**	**1**
1	**1**	**0**	**0**	**1**	**1**	**1**	**0**	**1**	**1**
1	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
0	**1**	**0**	**1**	**0**	**1**	**1**	**0**	**0**	**0**
2	**1**	**2**	**2**	**1**	**2**	**1**	**2**	**2**	**0**
2	**1**	**2**	**2**	**0**	**2**	**1**	**2**	**0**	**2**

1. Prepare a three-column table as shown.

Response	Tally	Frequency
Like (code 1)		
Do not like (code 0)		
Cannot decide (code 2)		
Total		

2. Tally the response in the table.
3. Write the tally marks as whole numbers.
4. Complete the entries in the table.
5. Answer the questions that follow using the appropriate formula.

QUESTIONS:

1. What is the proportion of respondents who *like* the new snack?

Solution:

There are 31 who *like* the new snack. The proportion for *like* is:

$\hat{p}_{like} = \frac{31}{60} = 0.516 \approx 0.52 \; or \; 52\%$

2. What is the proportion of respondents who *do not like* the new snack?

Solution:

There are 14 who *do not like* responses. The proportion is:

$\hat{p}_{do\ not\ like} = \frac{14}{60} = 0.23 \; or \; 23\%$

3. What is the proportion of respondents who *cannot decide*?

Solution:

There are 15 *cannot decide* responses and this proportion is:

$\hat{p}_{cannot\ decide} = \frac{15}{60} = 0.25 \; or \; 25\%$

Example 2:

In a job satisfaction survey in a certain mall, 700 employees were asked if they were satisfied with their jobs. There were 518 who responded with a YES. What proportion is this? What proportion responded with a NO?

STEPS	SOLUTION
1. Determine what is asked in the problem.	What proportion responded with a YES? What proportion responded with a NO?
2. Specify the given information.	X = 518 and n = 700
3. Write the representations for computing the desired proportions.	Use $\hat{p}$ to represent the proportion of YES responses. Use $\hat{q}$ to represent the proportion of NO responses.
4. Write a formula for computing the proportions.	$\hat{p} = \frac{X}{n}$ and $\hat{q} = 1 - \hat{p}$

5. Substitute the given values in the computing formula and then solve.	$\hat{p} = \frac{X}{n} = \frac{518}{700} = 0.74$ and $\hat{q} = 1 - \hat{p} = 1 - 0.74 = 0.26$
6. Answer the questions raised in the problem.	So, the proportion of the YES responses is 0.74. This means 74% of the respondents responded YES and the proportion of NO responses is 0.26. That is, 26% responded NO.

The computed values are sample proportions.

- UNDERSTANDING the P DISTRIBUTION

Event of Interest: Getting a *head* on the top side

1. Toss a fair coin.
2. Record the result in the following table. Use 0 for heads and 1 for tails.
3. Count the number of heads occurring; denote this as X. Let n be the total number of tosses.
4. Compute $\hat{p} = \frac{X}{n}$ for the first trial. This is called $\hat{p}_1$.
5. Repeat steps 1 to 4 ten times.

Heads													
Tails													

6. Now you have a distribution called a sampling distribution of values.
7. Compute the mean or average of the $\hat{p}$ values.
8. Recall: What theorem tells us that the relative frequency of the sample mean for any population is approximately normal for sufficiency large samples?
9. Draw a curve showing the sampling distribution of $\hat{p}$. What can you conclude about the mean of the sampling distribution of $\hat{p}$?

CHARACTERISTICS of the SAMPLING DISTRIBUTION of $\hat{p}$

1. The mean of the sampling distribution of $\hat{p}$ is p; that is, $\hat{p}$ is the unbiased estimator of p.
2. The standard deviation of the sampling distribution of $\hat{p}$ is $\sqrt{\frac{pq}{n}}$; that is, $\sigma\mathrm{p} = \sqrt{\frac{pq}{n}}$, where $q = 1 - p$.
3. For large samples, the sampling distribution of $\hat{p}$ is approximately normal. A sample size is considered large if the interval $\hat{p} \pm 3\sigma p$ does not include 0 (like p = 0.001) or 1 (like p = 0.99).

Practice Exercise

A. Find $\hat{p}$ and $\hat{q}$ given X and n.

1. X = 28 and n = 100
2. X = 45 and n = 240
3. X = 120 and n = 1000
4. X = 234 and n = 1500
5. X = 318 and n = 2300

B. Estimate the population proportions for each of the following data:

1. In a plant box consisting of 120 seedlings, 80 seedlings were treated with growth enhancer. Estimate p and q.
2. In a survey of 80 children, 48 like to watch horror films. Find $\hat{p}$ and $\hat{q}$, where $\hat{p}$ is the proportion of children who like to watch horror films.

Solve.

A certain barangay embarked on a QUIT SMOKING program among the constituents. A survey was made among a sample of 200 smoking individuals of the village asking who would join the program. 118 said they would join the program. What are the values of p and q?

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Lesson 8
TOPIC: COMPUTING INTERVAL ESTIMATES OF POPULATION PROPORTIONS

The FORMULA for computing a large-sample confidence interval for a population proportion p is:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

or

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

The following assumptions are made for finding a confidence interval for a population proportion.

1. The sample is a random sample.
2. The conditions for a binomial experiment are satisfied.

COMPUTING INTERVAL ESTIMATES OF p

PROBLEM 1: TRUST THE PRESIDENT?

A survey of 1200 citizens showed that 715 trust the president. Compute a 95% confidence interval for the population of all citizen who trust the president.

a. POINT ESTIMATE	
STEPS	**SOLUTION**
1. Describe the population parameter of interest.	The parameter of interest is the mean proportion p of all citizens who trust the president.
2. Specify the confidence interval criteria.	
a. Check the assumptions.	By the CLT, the sample size of 1200 citizens is normally distributed.
b. Determine the test statistic to be used to calculate the interval.	The test statistic is the p.
c. State the level of confidence.	Confidence level: 95% and $\alpha = 0.05$ Confidence coefficients: _____

3. Collect and present sample evidence.	
a. Collect the sample information.	X = 715 and n = 1200.

b. Find the point estimate of p.	$\hat{p} = \frac{X}{n} = \frac{715}{1200} = 0.595 \approx 0.60$ The point estimate of the population proportion p is _____.

b. 95% Confidence Interval	
4. Compute the interval estimate.	
a. Find $\hat{q}$.	$\hat{q} = 1 - \hat{p} = 1 - 0.60 =$ _____
b. Substitute the value of n, $\hat{q}$ in the formula to find the maximum error of estimate.	$\sqrt{\frac{\hat{p}\hat{q}}{n}} =$ _____
c. Find the limits.	For the lower limit: $\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} =$ _____ $- 1.96x$_____ $=$ _____ $- 1.96\sqrt{0.0002}$ $=$ _____ $- 1.96(0.014)$ $=$ _____ $- 0.027$ $= 0.573$ or 57.3%
c. Find the limits.	For the upper limit: $\hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} =$ _____ $+ 1.96x$_____ $=$ _____ $+ 0.027$ $=$ _____ or 62.7%
d. Describe the results.	Thus, with 95% confidence, we can state that the interval from 57.3% to 62.7% contains the true percentage of all citizens who trust the president.

PROBLEM 2: ENVIRONMENTAL CONCERN

In a survey of 458 random households, people take pride in discussing their methods of environmental protection. 236 manage their trash by separating biodegradables from non-biodegradables are disposed of properly. Use a 95% confidence to estimate the proportion of all households who have good practices of trash management.

a. POINT ESTIMATE	
STEPS	**SOLUTION**
1. Describe the population parameter of interest.	The parameter of interest is the mean proportion p of all households who have

	good practices of trash management.
2. Specify the confidence interval criteria.	
a. Check the assumptions.	By the CLT, the sample size of 236 is normally distributed.
b. Determine the test statistic to be used to calculate the interval.	The test statistic is the p.
c. State the level of confidence.	Confidence level: 95% and $\alpha = 0.05$ Confidence coefficients: _____
3. Collect and present sample evidence.	
a. Collect the sample information.	X = 236 and n = 458
b. Find the point estimate of p.	$\hat{p} = \frac{X}{n} = \frac{236}{450} = 0.515 \approx 0.52$ The point estimate of the population proportion p is _____.
4. Compute the interval estimate.	
a. Find $\hat{q}$.	$\hat{q} = 1 - \hat{p} = 1 - 0.52 =$ _____
b. Find the maximum error E	$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96x$_____
c. Find the limits.	For the lower limit: $\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.52 -$ _____ $= 0.52 - 1.96\sqrt{0.0005}$ $= 0.52 - 1.96(0.023)$ $= 0.52 - 0.046$ $= 0.474$ or 74.4%
c. Find the limits.	For the upper limit: $\hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.52 +$ _____ $= 0.52 + 0.046$ $= 0.566$ or 56.6%
d. Describe the results.	Thus, with 95% confidence, we can state that the interval from 47.4% to 56.6% contains the true percentage of all households who have good practices of trash management.

Practice Exercise

A. Find the proportions $\hat{p}$ and $\hat{q}$ for each of the following.

1. X = 135 n = 378
2. X = 234 n = 512
3. X = 256 n = 624
4. X = 314 n = 850
5. X = 450 n = 1260

B. Compute the population proportion interval estimate given n, $\hat{p}$, and the confidence level.

1. n = 300 $\hat{p} = 0.40$ 95% C.L.
2. n = 500 $\hat{p} = 0.23$ 95% C.L.
3. n = 420 $\hat{p} = 0.61$ 90% C.L.
4. n = 670 $\hat{p} = 0.54$ 99% C.L.
5. n = 710 $\hat{p} = 0.63$ 99% C.L.

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Lesson 9
TOPIC: INTERPRETING INTERVAL ESTIMATE OF A POPULATION PROPORTION

Review the four-step process of computing estimates of population proportions.

INDIVIDUAL ACTIVITY

1. Look at the table of information on enrolment in a college.
2. Answer the questions that follow.

COURSE	NUMBER OF ENROLLEES
Engineering	1 080
Hotel and Restaurant	948
Teacher Education	1 143
Total	**3 171**

3. The proportion of Engineering students is $\frac{1080}{3171} = 0.34$. What is the proportion of Hotel and Restaurant students?
4. What is the proportion of Teacher Education students?
5. Based on the numbers, what can you infer about choice of courses?
6. Can the data provide information to school management to embark on school improvement practices?
7. In what course may the school exert effort to improve if the course has to be offered?

Statistical analysis of numerical data includes an end note called *INTERPRETATION*.

Example 1:

In a graduate teacher college, a survey was conducted to determine the proportion of the students who want to major in Math. If 368 out of 850 students said YES, with 95% confidence, what interpretation can we make regarding the probability that all students in the teacher graduate college want to major in Math?

a. POINT ESTIMATE	
STEPS	**SOLUTION**
1. Describe the population parameter of interest.	The parameter of interest is the mean proportion p of all students in the teacher graduate college who want to major in Mathematics.
2. Specify the confidence interval criteria.	
a. Check the assumptions.	By the CLT, the sample size of 850 is normally distributed.
b. Determine the test statistic to be used to calculate the interval.	The test statistic is the p.
c. State the level of confidence.	Confidence level: 95% and $\alpha = 0.05$ Confidence coefficients: _____
3. Collect and present sample evidence.	
a. Collect the sample information.	X = 368 and n = 850.
b. Find the point estimate of p.	$\hat{p} = \frac{X}{n} = \frac{368}{850} = 0.432 \approx 0.43$ The point estimate of the population proportion p is _____.

b. 95% Confidence Interval	
4. Compute the interval estimate.	
a. Find $\hat{q}$.	$\hat{q} = 1 - \hat{p} = 1 - 0.43 =$ _____
b. Substitute the value of n, $\hat{q}$ in the formula to find the maximum error of estimate.	$\sqrt{\frac{\hat{p}\hat{q}}{n}} =$ _____
c. Find the limits.	For the lower limit: $\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.43 - 1.96x$_____ $= 0.43 - 1.96\sqrt{0.0002}$ $= 0.43 - 1.96(0.014)$ $= 0.43 - 0.034$ $= 0.396$ or 39.6%

c. Find the limits.	For the upper limit: $\hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.43 + 1.96x$_____ $= 0.43 + 0.034$ $= 0.464$ or 46.4%
d. Describe the results.	Thus, with 95% confidence, we can state that the interval from 39.6% to 46.4% contains the true percentage of all graduate students who want to major in Mathematics.

Try!

Example 2:

In a certain food stall, 278 out of 500 randomly selected consumers indicate their preference for a new kind of food combination. Use a 99% confidence interval to estimate the true proportion p who like the new food combination.

A. Estimate the interval for the population proportion from each of the following.

1. X = 500, n = 812, 95% confidence
2. X = 842, n = 1200, 95% confidence
3. X = 610, n = 1050, 95% confidence
4. X = 523, n = 972, 99% confidence
5. X = 415, n = 1678, 99% confidence

B. Solve the problem. Show the complete solutions.

A consumer watch group interviewed a random sample of 1000 consumers and asked them if they are optimistic about the economy or not. There were 612 who said YES. Use a 95% confidence level to estimate the population proportions.