LA IMMACULADA CONCEPCION SCHOOL
SENIOR HIGH SCHOOL
GRADE 11 – STEM: BASIC CALCULUS

April 20 - 24, 2020

Topic: Proving the Derivatives of Trigonometric Functions

Derivative of Sine Function

PROOF OF THE DERIVATIVES OF SIN(X)

IN PROVING THE DERIVATIVES OF SIN X WE WILL USE THE LIMIT DEFINITION:

$$\frac{d}{dx} sinx = cosx$$

$$\frac{d}{dx} sinx$$

$$\lim_{\Delta x \to 0} \frac{\sin(x + \Delta x) - sinx}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{cosx\ sin\Delta x + sinx\ cos\Delta x - sinx}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{cosx\ sin\Delta x}{\Delta x} + \frac{sinx\ cos\Delta x - sinx}{\Delta x}$$

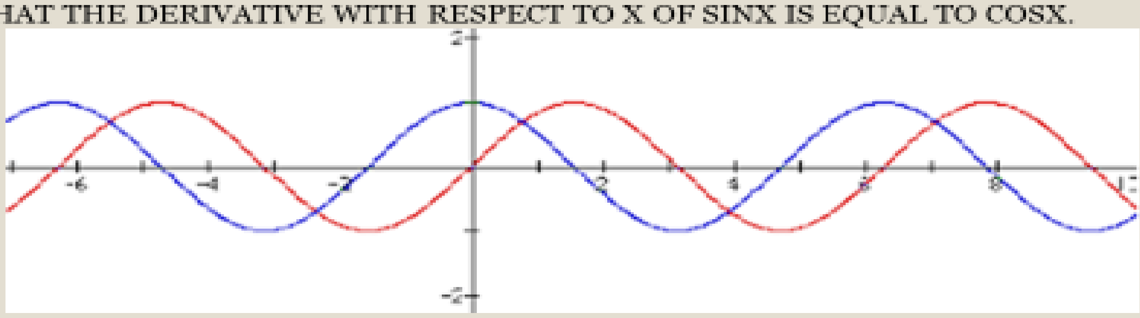
$$\lim_{\Delta x \to 0} cosx(\frac{sin\Delta x}{\Delta x}) + \lim_{\Delta x \to 0} sinx(\frac{cos\Delta x - 1}{\Delta x})$$

$$cosx \lim_{\Delta x \to 0} (\frac{sin\Delta x}{\Delta x}) - sinx \lim_{\Delta x \to 0} (\frac{1 - cos\Delta x}{\Delta x}) = cosx(1) - \text{sinx}(0) = \boldsymbol{cosx}$$

Derivative of Cosine Function

PROOF OF THE DERIVATIVES OF COS(X)

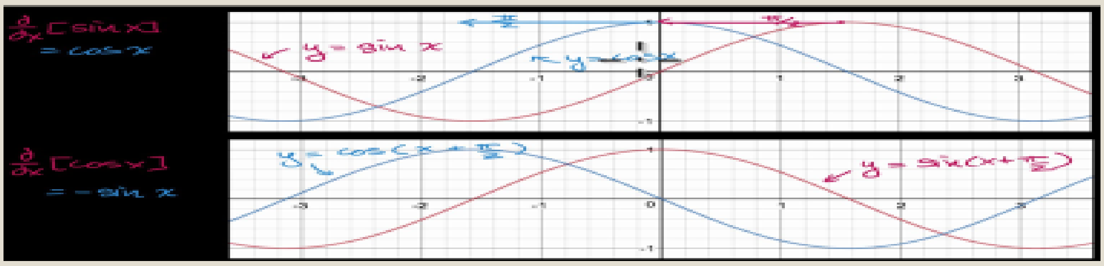
IN THIS PROVING WE'RE GOING TO BASED IT ON A PREVIOUS PROOF. WE MADE THAT THE DERIVATIVE WITH RESPECT TO X OF SINX IS EQUAL TO COSX.



For better understanding we will use the graph of y=sin(x) and y=cos(x) to prove that cos(x)= -sin(x).

PROOF OF THE DERIVATIVES OF COS(X)

The red line is y=sin(x) while the blue line is y=cos(x).



- We're going to do is shifted the graphs to the left by pi over 2.
- As we can see the y=sin(x + pi/2) is at the same spot as the y=cos(x) when move the graph by pi/2 which means y=sin(x + pi/2) is equal to the equation y=cos(x).

PROOF OF THE DERIVATIVES OF COS(X)

$$\frac{d}{dx} sin(x + \frac{\pi}{2}) = \cos(x + \frac{\pi}{2})$$

- As we can see in the graph derivatives of sin(x+pi/2) is also the same as cos(x) and cos(x+pi/2) is also the same as −sin(x).
- derivatives of sin(x+pi/2) is equal to cos(x+pi/2) which means the derivatives of cos(x) is −sin(x).

$$cosx = -sinx$$

PROOF OF THE DERIVATIVES OF COS(X)

As an alternative, this is the solution in how we get the derivatives of cosx.

(2) Determine the Derivative of cos(x):

$$\frac{d}{dx}(\cos x) = \lim_{\Delta x \to 0}\left[\frac{\cos(x+\Delta x)-\cos(x)}{\Delta x}\right]$$

$$= \lim_{\Delta x \to 0}\left(\frac{\cos(x)\cos(\Delta x)-\sin(\Delta x)\sin(x)-\cos(x)}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}\left(\frac{\cos(x)\cdot 1-\sin(\Delta x)\sin(x)-\sin(x)}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}\left(\frac{-\sin(\Delta x)\sin(x)}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}(-\sin x) = -\sin x$$

Derivative of Tangent Function

PROOF OF THE DERIVATIVES OF TAN(x)

In proving the derivatives of tan(x) we will use the trigonometric identity:
$\sin x + \cos^2 x = 1.$

$$d(\tan x)/dx = f'(\tan x)$$

$$= f'\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \sin' x - \sin x \cos' x}{\cos^2 x}$$

$$= \frac{\cos x \cos x + \sin x \sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Derivative of Cotangent Function

PROOF OF THE DERIVATIVES OF COT(X)

In proving the derivatives of cot(x) we will use the trigonometric identity.

$$f(x) = \cot x$$

$$= \frac{1}{\tan x}$$

$$= \frac{\cos x}{\sin x}$$

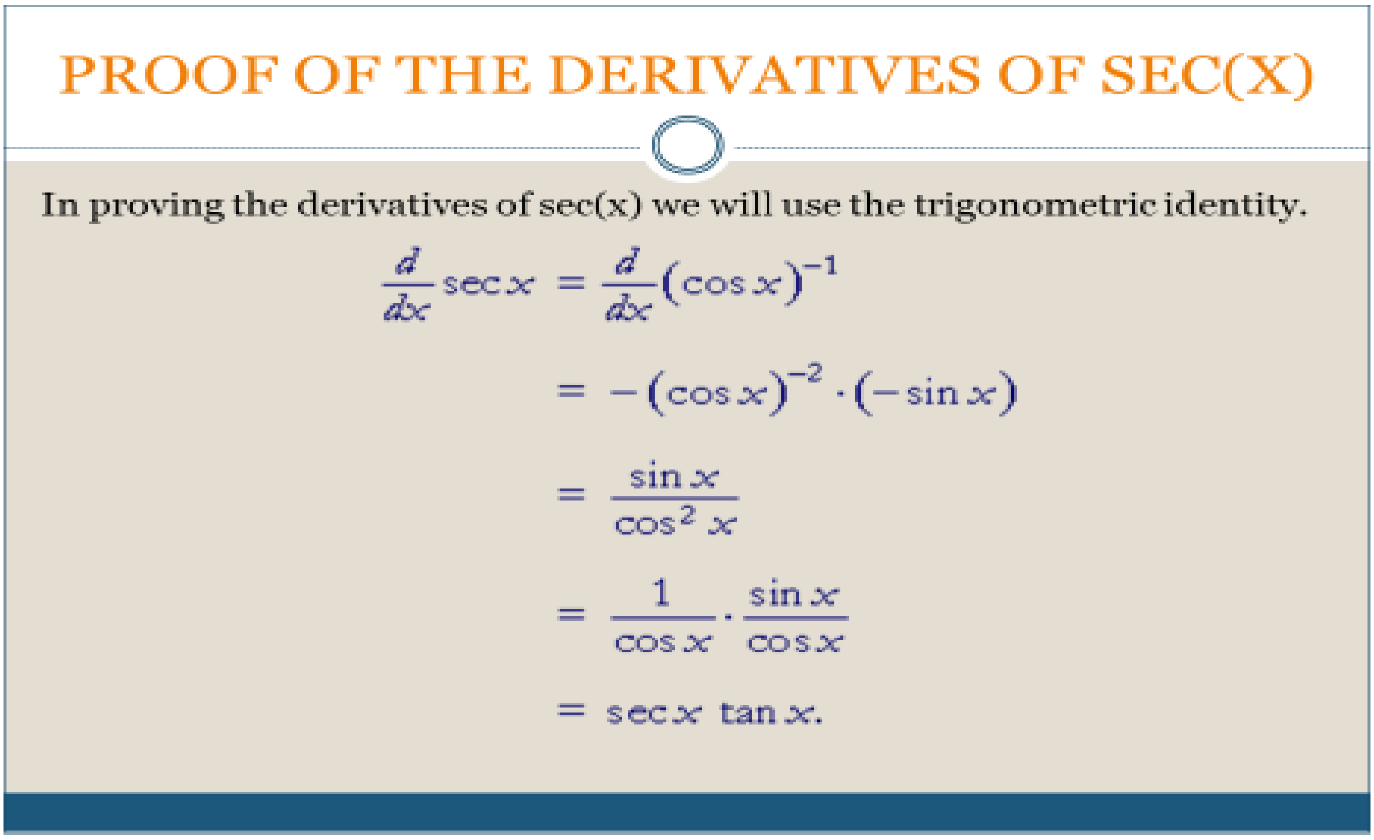
$$f'(x) = \frac{\sin x . -\sin x - \cos x . \cos x}{\sin^2 x} \quad \text{using the Quotient Rule}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

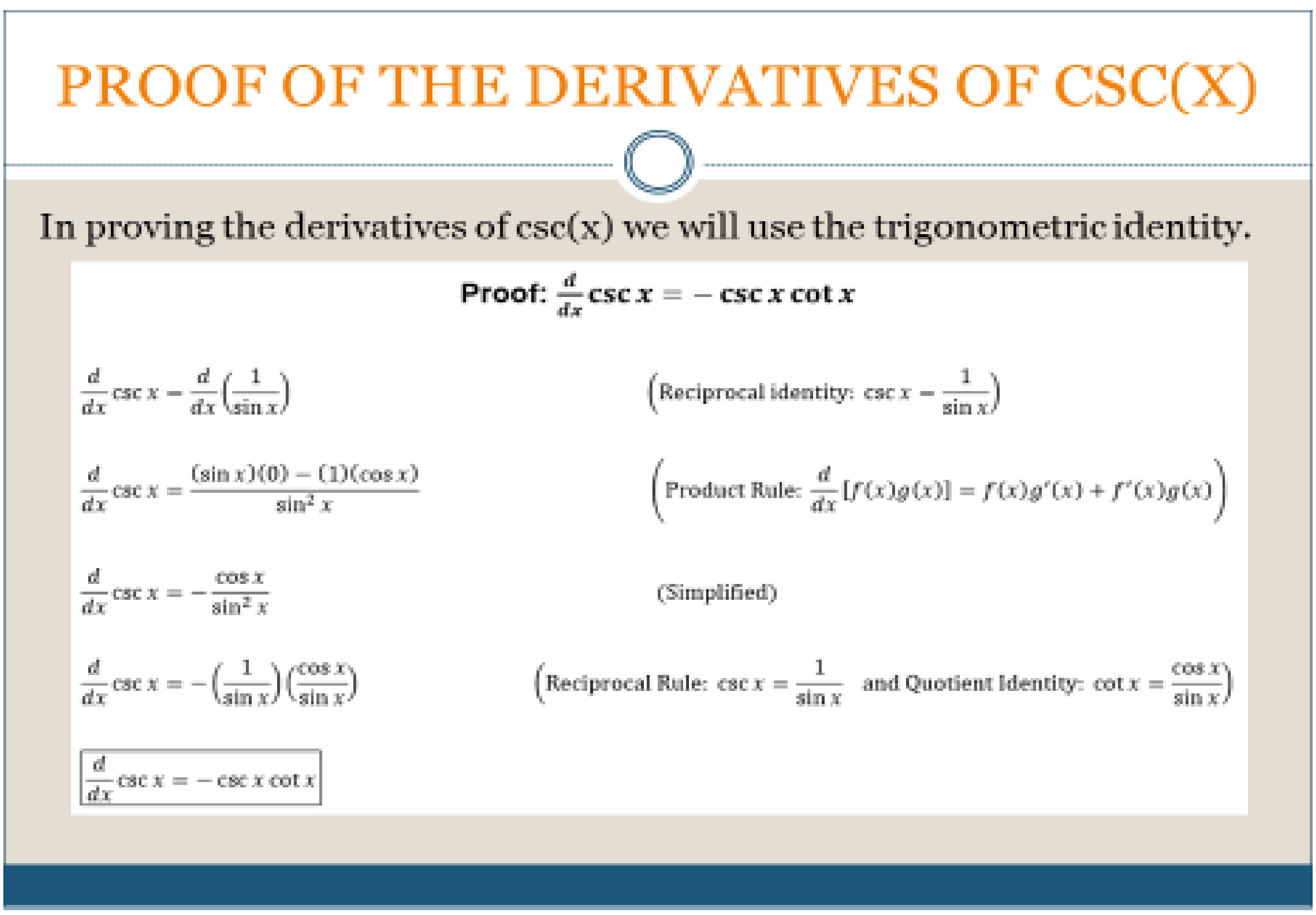
$$= \frac{-1}{\sin^2 x}$$

$$= -\text{cosec}^2 x$$

Derivative of Secant Function



Derivative of Cosecant Function



STUDENT TASK

A. Directions: Find the derivatives of the following functions.

1. y = sin (8x)
2. y = sin ($4x^2$)
3. y = xsin (3x)
4. y = cos ($x^2 + 3$)
5. y = x cos (2x)
6. Y = sin (5x – 2)
7. Y = sin ($2 - 3x^2$)
8. Y = cos (7x
9. Y = tan (3x)
10. Y = tan ($2x^3 +5$)

B. Directions: Differentiate the following.

1. F(x) = 5 sec (2x) + 20
2. F(x) = 3 sec (x^2) - 45
3. F(x) = csc ($5x^3$) + 32
4. F(x) = cot ($2 - 4x^2$)
5. F(x) = cot (secx)