APRIL 20-24, 2020
LESSON 6
TOPIC: CONFIDENCE INTERVALS FOR THE POPULATION MEAN WHEN σ IS UNKNOWN

ASSUMPTIONS in Computing for the Population Mean When σ is Unknown

When $n \geq 30$, and σ is unknown, the sample standard deviation s can be substituted for σ. However, the following assumptions should be met.

1. The sample is a random sample.
2. Either $n \geq 30$ or the population is normally distributed when $n < 30$.

General Expression for the Confidence Interval When σ is Unknown

$\overline{X} \pm t(\frac{s}{\sqrt{n}})$ and the distribution of values is called t-distribution.

CONDITIONS for using the t-test

1. σ is unknown
2. n < 30

DEGREES OF FREEDOM (df)

Are the number of values that are free to vary after a sample statistic has been computed, and they tell us the specific curve to use when a distribution consists of a family of curves.

FORMULA for COMPUTING the CONFIDENCE INTERVAL USING t-DISTRIBUTION

$$\overline{X} - t(\frac{s}{\sqrt{n}}) < \mu < \overline{X} + t(\frac{s}{\sqrt{n}})$$

The T – table

The t-Table

n	Degrees of Freedom (n – 1)	Confidence Coefficient (amount of α in two tails)		
		0.90	0.95	0.99
2	1	6.314	12.706	63.657
3	2	2.920	4.303	9.925
4	3	2.353	3.182	5.841
5	4	2.132	2.776	4.604
6	5	2.015	2.571	4.032
7	6	1.943	2.447	3.707
8	7	1.895	2.365	3.499
9	8	1.860	2.306	3.355
10	9	1.833	2.262	3.250
11	10	1.812	2.228	3.169
12	11	1.796	2.201	3.106
13	12	1782	2.179	3.055
14	13	1771	2.160	3.012
15	14	1.761	2.145	2.977
16	15	1.753	2.131	2.947
17	16	1.746	2.120	2.921
18	17	1.740	2.110	2.898
19	18	1.734	2.101	2.878
20	19	1.729	2.093	2.861
21	20	1.725	2.086	2.845
22	21	1.721	2.080	2.831
23	22	1.717	2.074	2.819
24	23	1.714	2.069	2.807
25	24	1.711	2.064	2.797
26	25	1.708	2.060	2.787
27	26	1.706	2.056	2.779
28	27	1.703	2.052	2.771
29	28	1.701	2.048	2.763
30	29	1.699	2.045	2.756
31	30	1.697	2.042	2.750
41	40	1.684	2.021	2.714
61	60	1.671	2.000	2.660
∞	∞	1.645	1.960	2.576

Hopkins, K.D. and Glass, G.V. (1978). *Basic Statistics for the Behavioral Sciences*. Englewood Cliffs, New Jersey: Prentice-Hall Inc. and McClave, J.T. (2003). *Statistics*. Upper Saddle River, New Jersey: Prentice-Hall Inc.

CONFIDENCE COEFFICIENTS

Try!

A. Find the confidence coefficients for each of the following:

1. n = 6, 90% confidence
2. n = 7, 90% confidence
3. n = 12, 95% confidence
4. n = 17, 95% confidence
5. n = 24, 99% confidence
6. n = 8, 99% confidence
7. n = 8, 90% confidence

B. Find E given the following:

1. n = 6, s = 2, 90% confidence
2. n = 9, s = 2.8, 90% confidence
3. n = 13, s = 4.5, 95% confidence
4. n = 16, s = 3.1, 95% confidence
5. n = 21, s = 5, 95% confidence

Steps in Computing the Interval Estimate of the Population Mean When σ is Unknown

Step 1: Describe the population parameter of interest.
Step 2: Specify the confidence interval criteria.
a. Check the assumptions. b. Determine the test statistic to be used. In this case, it is the t statistic. c. State the level of confidence.
Step 3: Collect and present the sample evidence.
a. Collect the sample information. b. Find the point estimate.
Step 4: Determine the confidence interval.
a. Determine the confidence coefficients $(t_{\alpha/2})$ from the t-Table. b. Find $\frac{s}{\sqrt{n}}$ or margin of error. c. Find the lower and upper confidence limits. d. Describe/interpret the result.

Problem 1:

Aldrin wants to know if cooperative grouping is an effective strategy in improving the mathematics performance of Grade 7 students. Twenty students were included in the experimental group while another 20 students were included in the control group. The mean achievement score of the students in the experimental group was 82.5 with a standard deviation of 3 while the mean of the students in the control group was 80 with a standard deviation of 6. The two groups come from normally distributed populations. The confidence level adopted was 95%.

1. What is the estimate of the population mean where the experimental group comes from?
2. What is the estimate of the population mean where the control group comes from?
3. Express your confidence as percentage.

Step 1: Describe the population parameter of interest.	**The 1st parameter of interest is the mean μ_1 of the population where the experimental group belongs.** **The 2nd parameter of interest is the mean μ_2 of the population where the control group belong.**
2. Specify the confidence interval criteria.	
A. Check the assumptions.	The samples of size 20 for each group come from normally distributed parent populations and the σ for each group are unknown.

2. Specify the confidence interval criteria.	
b. Determine the test statistic to be used to calculate the interval.	The test statistic is the t, using s_1= _____ and s_2 = _____, respectively.
c. State the level of confidence.	For a 95% confidence, $\alpha = 1 - 0.95 \ = 0 \ \ .05$ From the t-table, with a df = 19 for each group, the confidence coefficients are _____ for each group.

3. Collect and present sample evidence.	
a. Collect the sample information.	The sample information consists of 20 raw scores for each group. From the experimental group: n = 20, so df = 19 $\overline{X} = 82.5$, and s = _____ From the control group: n = 20, so df = 19 $\overline{X} = 80$, and s = _____

4. Determine the confidence interval.	
a. Determine the confidence coefficient.	Since n = 20, then the df = 19. The confidence coefficients in the t-table under 0.95 (for 95%) is _____.
b. Find the maximum error (E).	For the experimental group: $E = t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$ $=$ ____$(\frac{3}{\sqrt{20}})$ $=$ ____(0.67) $=$ ____

4. Determine the confidence interval.	
a. Determine the confidence coefficient.	Since n = 20, then the df = 19. The confidence coefficeints in the t-table under 0.95 (for 95%) is ______.
b. Find the maximum error (E).	For the control group: $E = t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$ $=$ ____$(\frac{6}{\sqrt{20}})$ $=$ ____(1.34) $= 2.8$

4. Determine the confidence interval.	
c. Find the lower and the upper confidence limits.	For the experimental group: $\overline{X} - t\left(\frac{s}{\sqrt{n}}\right) < \mu < \overline{X} + t(\frac{s}{\sqrt{n}})$ So, 82.5 – 1.40 = 81.1 (lower limit) and 82.5 + 1.40 = 83.9 (upper limit) For the control group: So, ______ = 77.2 ______ = 82.8

4. Determine the confidence interval.	
d. Describe/interpret the result.	We can say with 95% confidence that the interval between 81.1 and 83.9 contains the true mean of the experimental population while the interval between 77.2 and 82.8 contains the true mean of the control population based on the given sample data.

Practice Exercises

Direction: Write your complete answer with solution on a one whole yellow paper.

A. Using the t-table, give the confidence coefficients for each of the following:

1. n = 12, 95% confidence
2. n = 15, 95% confidence
3. n = 21, 99% confidence
4. n = 23, 95% confidence
5. n = 25, 99% confidence

B. Assuming that the samples come from the normal distributions, find the margin of error E given the following:

1. n = 10, $\overline{X} = 28$, s = 4.0, 90% confidence
2. n = 16, $\overline{X} = 50$, s = 4.2, 95% confidence
3. n = 20, $\overline{X} = 68.2$, s = 2.5, 90% confidence
4. n = 23, $\overline{X} = 80.6$, s = 3.2, 95% confidence
5. n = 25, $\overline{X} = 92.8$, s = 2.6, 99% confidence

C. Solve.

1. The mean scores of a random sample of 17 students who took a special test is 83.5. If the standard deviation of the scores is 4.1, and the sample comes from an approximately normal population, find the point and the interval estimates of the population mean adopting a confidence level of 95%.

2. The average weight of 25 chocolate bars selected from normally distributed population is 200 g with a standard deviation of 10 g. Find the point and the interval estimates using the 99% confidence level.