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A DISCUSSION ON SPECIAL THEORY OF RELATIVITY

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Abstract

A rigid body does not exist in the special theory of relativity; distant simultaneity defined with respect to a given frame of reference without any reference to synchronized clocks; challenges on Einstein's connection of synchronization and contraction; a theory of relativity without light, composition of relative velocities and space of relative velocities.

Keywords: Theory, Relativity, Distant, Synchronization, Contraction.

1. Kinematics of the "Rigid Body"

In 1905, Einstein wrote in his relativity paper, "On the Electrodynamics of Moving Bodies": "The theory to be developed here is based, like all electrodynamics, on the kinematics of the rigid body, since the assertions of any such theory concerns with the relations among rigid bodies (coordinate systems), clocks, and electromagnetic processes". Einstein defined position by "means of rigid measuring rods and using the methods of Euclidean geometry".

John Stachel explains that according to special relativity information cannot travel faster than the speed of light. Thus, there can be no rigid body, which is possible in classical mechanics where forces are transferred at infinite speeds. A rigid body moves in a rigid manner, no matter what forces are imposed on the body. In fact, rigid motions can be defined without any contradiction in special relativity, even though a rigid body does not exist in the special theory of relativity.

Einstein spoke about rigid body because in 1905 he did not realize that this concept of the rigid body is incompatible with the special theory of relativity, and must be replaced by the concept of rigid motions.

At the 81st meeting of the Gesellschaft Deutscher Naturforscher und Ärzte in Salzburg in September 1909, Max Born presented a paper in which he first analyzed this problem and showed the existence of a very limited class of rigid motions in special relativity; although one cannot have rigid bodies in special relativity, one can move the measuring rod rigidly, and Born gave a Lorentz invariant definition for that.

Born explained that, "measuring rods that maintain their length at uniform translation in the co-moving coordinate system, suffer a contraction in the direction of their velocity when viewed from the system at rest. Therefore, the concept of the rigid body drops out [fällt], at least in its form adapted to Newtonian kinematics".

Sommerfeld had commented on Born's talk in a "Diskussion" following the paper. Born noted in a latter paper that, he had discussed with Einstein the subject and they were puzzled that the rigid body at rest can never be brought into uniform motion.

In 1911 in his paper that discusses the problem of rigid bodies in the theory of relativity, "Zur Diskussionüber den starrenKörporen in der Relativitätstheorie", Max Laue responded to Paul Ehrenfest's paradox and to Born's paper. He wrote that the necessity to admit the possibility of shape changes at any body is also explained in a useful way by the example given by Einstein in the conversation (with Born), which was communicated to him in the discussion at the meeting of natural scientists in Königsberg by Somerfield.

A measuring rod is at rest, while another one (forming an angle α with the first) moves with velocity q perpendicular to its direction. The intersection points of both measuring rods travels over the first rod with velocity, $V = q/\sin\alpha$, and if $q = 1$ cm/sec, then V becomes greater than the speed of light c by the choice of sufficiently small values for α. This is a purely geometric conclusion which cannot be changed by any physical theory. Laue asked, whether this fact contradicts Einstein's theorem according to which, physical effects cannot propagate with superluminal speed $V > 1$.

Laue then explained that the measuring rod is again now at rest. At the beginning the second measuring rod is at rest. At one end A of the first rod – the intersection points between the two rods – an event happens. The second rod is immediately set into motion with velocity q (previously defined). Then the other end B would have to go into motion immediately or simultaneously as a result of

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the event that happened at A. Laue said that the transmission from A to B takes place with superluminal speed V. But special relativity says that no signal can be transferred beyond the fundamental velocity. Therefore, the concept of a rigid body contradicts this consequence of special relativity

2. Definition of Distant Simultaneity

In 1905 Einstein had given meaning to the space coordinates by the system of rigid rods at rest in a coordinate system (in a state of inertial motion), and now the question is how to give meaning to the time coordinate. We need to give a definition of simultaneity, because all judgments in which time is involved are always judgments of simultaneous events. Einstein defined measurement conventions using clocks and measuring rods and adopted a definition of distant simultaneity

2.1 Definition of Distant Simultaneity with Reference to Synchronized Clocks

Let us first see how Einstein proceeded with the aid of rods and synchronized clocks to define distant simultaneity. In section Einstein says, "As we know from experience". Einstein implicitly tells his reader: you have been on trains, looked at the hands of clocks: "If for instance, I say, 'That train arrives here at 7 o'clock', I mean something like this: 'This pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events". Einstein substitutes "the position of the small hand of my watch" for "time". This is reasonable when we want to define time for the place where the watch is located. Einstein uses the train and clock imaginative example to exemplify to his reader some experience from his daily life.

However, Stachel says that we must not infer from such examples that this and other similar thought experiments could indicate that Einstein might have been inspired by patents of clocks, trains and clock towers that happened to stand near the patent office he used to work at in Bern.

Next Einstein tells his reader that if he wants to evaluate time of events that are remote from the clock, it could be done, but it is not most convenient. This sounds reasonable as well, but it is a leap of thought.

However, immediately after this intuitive presentation, Einstein says: "We might, of course, content ourselves with evaluating the time of the events determined by an observer stationed together with the clock at the origin of the coordinates, and assigning the corresponding positions of the hands to light signals, given out by every event to be timed, and reaching him through empty space. However, as we know from experience, this coordination has the disadvantage of not being independent of the position of the observer equipped with the clock. We arrive at a much more practical arrangement by the following consideration". Einstein introduces a synchronization procedure.

"If there is a clock at the point A in space, then an observer at A can determine the time of events in the immediate vicinity of A by finding the positions of the hands of the clock, which are simultaneous with these events. If there is at the point B of space another clock – and we should add, 'a clock in all respects resembling the one at A' – it is possible for an observer at B to determine the time values of events in the immediate vicinity of B. But it is not possible, without further assumption, to compare, the time of an event at A with the one at B. We have so far defined only an "A time" and a "B time," but not defined a common "time" for A and B. The latter time can now be defined by establishing by definition distant simultaneity that, the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A. Let a ray of light go from A to B at "A time" t_A, at "B time" t_B is reflected from B towards A, and arrives again at A at "A time" t'_A. By definition the two clocks are synchronous if

$t_B - t_A = t'_A - t_B$.

Or better

$$\frac{t_A - t_A^I}{2}$$

Einstein then assumes two additional hypotheses that are generally valid

- **Transitivity:** if the clock at B runs synchronously with the clock at A, the clock at A runs synchronously with the clock at B.
- **Additivity:** if the clock at A runs synchronously with the clock at B as with the clock at C, then the clocks at B and C also run synchronously relative to each other

Subsequently Einstein says, "With the help of certain (imagined) physical experiments, we have defined what is to be understood by synchronous clocks at rest relative to each other located at different places, and obviously obtained a definition of 'simultaneous' and 'time' ". And also, the definition: "The 'time' of an event is the reading that is given simultaneously with the event by a clock at rest, which is located at the place of the event, this clock for all times is synchronous with a specified clock at rest".
Einstein referred again to experience: "In agreement with experience", we further stipulate the quantity

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$$2AB/(t'A - tA) = c$$

be a universal constant (the velocity of light in empty space). Einstein defined time by means of clocks at rest in the system at rest, and he called it "the time of the system at rest"

3. Relativity of Simultaneity

3.1 On the Relativity of Lengths and Times

In order to evaluate remote events Einstein needed to formulate the two heuristic principles of his theory; these would guide him in his search for the solution to the problem

- The laws by which the states of physical systems change are independent, whether these changes of state are referred to the one or the other of two systems of coordinates in uniform motion of translation.
- Any ray of light moves in the coordinate system "at rest" with the definite velocity c, independent of whether the ray is emitted by a body at rest or in motion. Here

Velocity = light path/time interval

Where "time interval" is understood in the sense of the definition of distant simultaneity given in clock synchronization in one system: the time required by light to travel from A to B = time it requires to travel from B to A.

With the aid of the two principles and the definition of distant simultaneity, in 1905 relativity paper, Einstein presents in the following way the principal result of his kinematics: relativity of simultaneity.

- First there is an observer who moves together with the rod to be measured and with the measuring rod. He measures the length of the rod directly by superposing the measuring rod, in just the same way as if the three were at rest.
- b) "The observer in the rest system ascertains, using synchronous clocks at rest, at what points of the rest system the two ends of the rod to be measured are located at a definite time t. The distance between these two points, measured with the rod used before, but now at rest, is also a length, which may be designated the 'Length of the Rod' ".

Einstein is guided by the principle of relativity and thus calls the length by operation a) – the length of the rod in the moving system. This length is equal to the length l of the rod at rest. Einstein calls the length of the rod by operation b), "the length of the (moving) rod in the system at rest'.

Since, according to Einstein, one could give preference to either of the two observers, as the one was in motion and the other was at rest, this latter statement laid down the following definition: the length of a body in its rest frame is unconstructed.

Imagine again "the observer in the system at rest" of b) who "ascertains, as required by " the kinematical length of the moving rod. The clocks of the observer in the system at rest are synchronous with the clocks of the observers A and B. He sees the two observers A and B exchanging light signals. In the moving system a ray of light is sent from A at time t_A, there it is reflected from B at time t_B, and arrives back at A at time t'_A. The observer in the system at rest uses the definition of distant simultaneity from and finds

$$t_B - t_A = [r_{AB} + v(t_B - t_A)]/c$$
$$t'_A - t_B = [r_{AB} - v(t'_A - t_B)]/c$$

where, rAB denotes the kinematical length of the moving rod, measured in the system at rest. Einstein wrote the equations in the following way

$$tB - tA = rAB/(c - v)$$
$$t'A - tB = rAB/(c + v)$$

This result shows that observers moving with the rod will thus find that their clocks are not synchronous, while observers in the system at rest will declare the clocks to be synchronous. "We thus see that we cannot ascribe any absolute meaning to the concept of simultaneity, but that two events, which are simultaneous as viewed from a system of coordinates, can no longer be considered simultaneous events when observed from a system which is moving relative to that system".

4. Derivation of the Lorentz Transformation

4.1 The Lorentz Transformations Derived by the Principle of Relativity and the Light Postulate

In section §3 Einstein derived the Lorentz transformations (the transformations of the coordinates and times) in the following way: He defined two systems: the first, a rest system K, and a second, system [k] moving with a constant velocity v in the direction of increasing x of the other system K. Einstein defined the two systems of coordinates and time of a specific event one with respect the system K and the other with respect to k. Einstein defined the two systems in the following manner:

"Let there be in the space 'at rest' two systems of coordinates, i.e., two systems, each of three rigid material lines, perpendicular to one another, and originating from a point. Let the X-axes of the two systems coincide, and their Y- and Z-axes respectively be parallel. Each system shall be provided with a rigid measuring-rod and a number of clocks, and let both measuring-rods, and all the clocks of the two systems, be in all respects identical". Now impart to the origin of one of the two systems (k) a (constant) velocity v in the direction of the increasing x of the other system (K) at rest, and let this velocity be communicated to the axes of the coordinates, the relevant measuring-rod, and the clocks. To each time t of the system K at rest there then will correspond a definite position of the axes of the moving system, and from reasons of symmetry we are entitled to assume that the motion of k may be such that the axes of the moving system are at the time t (this "t" always denotes a time of the system at rest) parallel to the axes of the system at rest".

Einstein imagines that space is measured from the system K at rest by means of a measuring-rod at rest, and from the moving system k by means of the measuring-rod moving with it. The coordinates obtained by this way are: x, y, z and ξ, η, ζ, respectively. The time is determined by means of light signals in the manner indicated in section §1 of the relativity paper: there are clocks at many points in system K and observers exchanging light signals between these points at which the clocks are located – the time t is determined for all these points in K where there are clocks. The same method is used in system k, with clocks at rest relative to system k, and time τ is determined for k.32 To any system of values x, y, z, t, which completely define the place and time of the event in the system K at rest, there corresponds a system of values ξ, η, ζ, τ, determining that event relative to the system k. Einstein's task is to find the system of transformations equations – connecting these quantities.

The derivation of the Lorentz transformation, or as Einstein calls them in 1905, the transformations of Coordinates and Time, in section §3 of the relativity paper – the equations connecting the two systems of quantities x, y, z, t and ξ, η, ζ, τ – is very cumbersome. It is based on synchronizing clocks, and expressing "in equations that τ is nothing but the collection of the readings of clocks at rest in system k, which have been synchronized according to the rule given in §1" (distant simultaneity).

The derivation uses the two guiding principles

"[…] and applying the Principle of the Constancy of the Velocity of Light in the rest system", and "expressing in equations that light (as required by the Principle of the Constancy of the Velocity of Light in combination with the Principle of Relativity) is also propagated with velocity c when measured in the moving system".

The final form of Einstein's transformation equations for the coordinates and time is:

$$\tau = \beta\left(t - \frac{vx}{c^2}\right)$$
$$\xi=\beta(x-vt)$$
$$\eta=y$$
$$\zeta=z$$
$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

5. Composition of Relative Velocities

5.1 The Addition Theorem for Velocities

In section §5 of the 1905 relativity paper Einstein obtained the addition theorem for relative velocities. Along the X axis of the system K, a point is moving in the system k with a velocity v according to the equations.

$\xi = \mathbf{w\zeta\tau},\ \eta = w\eta\tau,\ \zeta = 0$, where, w_ξ and $w\eta$ denote constants.

Einstein required the motion of the point relative to the system K. With the help of the system of Lorentz transformations developed in section §3, Einstein obtained for the equations of motion of the point.

$$x = \frac{t\left(\omega_\xi+\vartheta\right)}{\left(1+\vartheta\omega_\xi/c^2\right)},\quad y = \frac{\omega_n{}^{t}\sqrt{1-\vartheta^2/_{c^2}}}{\left(1+\vartheta\omega_\xi/c^2\right)} \quad , \quad z = 0$$

and

$$U^2 = (dx/dt)^2 + (dy/dt)^2,\ w^2 = w^2\xi + w^2\eta$$

Einstein finally obtained U if w has the direction of the X-axis

$$U = (v + w)/(1 + vw/c^2).$$

After presenting the addition theorem of velocities in one direction, not in the general case, he introduced a third system of coordinates k', moving parallel to k. He wrote: "such parallel transformations – necessarily – form a group".

Einstein arrived at this conclusion by obtaining the formula for U if w and v have the same direction; by composition of two transformations from §3: k' that moves parallel to k with velocity ω along the ≡ axis of k, and k moving with velocity v along the X-axis of K, while v is replaced by: $(v + w)/(1 + vw/c^2)$.

Conclusion

After writing formulas for the composition of relative velocities in terms of hyperbolic trigonometric identities and the rapidity,

"Thus, instead of a Euclidean triangle of velocities, we get a Lobatchefckij triangle of rapidities. For small rapidities, however, we may identify rapidity and velocity, and the Lobatchefckij triangle may be treated as a Euclidean one. It is also seen that rapidities in the same straight line are additive".

Therefore, relative velocities and composition of relative velocities correspond to certain properties of hyperbolic triangles in velocity space. Newtonian-Galilean velocity space is then flat, leading to the Galilean law of vector addition of relative velocities.

References

- Albert Einstein (1905) "Zur Elektrodynamikbewegter Körper", Annalen der Physik 17: 891; English translation On the Electrodynamics of Moving Bodies by George Barker Jeffery and Wilfrid Perrett (1923); Another English translation On the Electrodynamics of Moving Bodies by Megh Nad Saha (1920).
- "Science and Common Sense", P. W. Bridgman, The Scientific Monthly, Vol. 79, No. 1 (Jul. 1954), pp. 32–39.
- The Electromagnetic Mass and Momentum of a Spinning Electron, G. Breit, Proceedings of the National Academy of Sciences, Vol. 12, p.451, 1926
- Kinematics of an electron with an axis. Phil. Mag. 3:1-22. L. H. Thomas.]
- Einstein, Autobiographical Notes, 1949.
- Einstein, "Fundamental Ideas and Methods of the Theory of Relativity", 1920
- Einstein, On the Relativity Principle and the Conclusions Drawn from It, 1907; "The Principle of Relativity and Its Consequences in Modern Physics", 1910; "The Theory of Relativity", 1911; Manuscript on the Special Theory of Relativity, 1912; Theory of Relativity, 1913; Einstein, Relativity, the Special and General Theory, 1916; The Principal Ideas of the Theory of Relativity, 1916; What Is The Theory of Relativity?, 1919; The Principle of Relativity (Princeton Lectures), 1921; Physics and Reality, 1936; The Theory of Relativity, 1949.
- Griffiths, David J. (2013). "Electrodynamics and Relativity". Introduction to Electrodynamics (4th ed.). Pearson. Chapter 12.
- Jackson, John D. (1999). "Special Theory of Relativity". Classical Electrodynamics (3rd ed.). John Wiley & Sons, Inc. Chapter 11
- Goldstein, Herbert (1980). "Chapter 7: Special Relativity in Classical Mechanics". Classical Mechanics (2nd ed.). Addison-Wesley Publishing Company
- Lanczos, Cornelius (1970). "Chapter IX: Relativistic Mechanics". The Variational Principles of Mechanics (4th ed.). Dover Publications.
- Tom Roberts & Siegmar Schleif (October 2007). "What is the experimental basis of Special Relativity?". Usenet Physics FAQ. Retrieved 2008-09-
- Albert Einstein (2001). Relativity: The Special and the General Theory (Reprint of 1920translation by Robert W. Lawson ed.). Routledge.
- Richard Phillips Feynman (1998). Six Not-so-easy Pieces: Einstein's relativity, symmetry, and space–time(Reprint of 1995 ed.). Basic Books. p. 68
- Sean Carroll, Lecture Notes on General Relativity, ch. 1, "Special relativity and flat spacetime," http://ned.ipac.caltech.edu/level5/March01/Carroll3/Carroll1.html

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