SIMILARITY AND NON-SIMILARITY ANALYSIS FOR ENGINEERING BOUNDARY VALUE PROBLEMS

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1. Abstract:

The new era of engineering and technology had given many splendid inventions to the world. Because of the innovative advances in technology, the demand for the qualitative products and enhancement of the performance of products increases day to day. Many industrial products like beauty products, food products, medical products, plastic products, fibre products, etc have an unlimited application of fluid. It is essential to study the principles, characteristics, and parameters of fluid flow which is addressed by the subject Fluid Mechanics. Mathematical models characterize the fluid flow problems which arise in the real world and in the industrial process. This type of problems deal with linear or non-linear partial differential equations or ordinary differential equations.

One of the reasons for nonlinearity in the governing equations is stress-strain expression defined for non-Newtonian type fluid model. To handle the nonlinear models for finding an exact solution is an arduous job. Many researchers attempted to find the solution of the non-linear PDEs analytically. It is proved that similarity methods are one of the most powerful analytic tools currently available in the study of nonlinear partial differential equations.

The present research work is based on the application of group-theoretic methods for finding similarity solution for engineering boundary value problems and another application of the local non-similar method to find the non-similarity solution. Two-dimensional PDE for Sisko fluid, Powell-Eyring and Prandtl Eyring fluids are studied. Also, three-dimensional Nanofluids for Newtonian and non-Newtonian models are studied. After reducing PDE into ODE by applying group-theoretic similarity methods and non-similarity methods further procedure is carried out by numerical techniques using Maple software and MATLAB software and graphs of the velocity profile, temperature profile, etc. are studied for each flow problems.

There are some PDEs that do not possess invariant conditions and hence in such cases we do not get similarity solutions. In such a case, only non-similar solutions or Local non-similar solutions may be available. In literature, it is found that for PDE governing boundary layer flow of non-Newtonian fluids past wedge, a similarity solution exists only for a 90-degree wedge and for the flow other than 90-degree wedge only non-similar solution is available.
2. Brief description on the state of the art of the research topic:

The living organism is surrounded by the fluid and knowingly or unknowingly applying the different fluid characteristics. Natural fluid flow like air flow, water flow in rivers and oceans, blood flow in vain, etc. are following the principle of Fluid Mechanics. The rapid advances in Fluid Mechanics started in the 19th Century. An idea of the boundary layer was given by Prandtl (1904) as a very thin layer in the vicinity of the body. He proved theoretically and practically significant role of friction in this layer and in the remaining region outside this layer friction can be neglected. The real fluid is subcategorized as Newtonian and non-Newtonian fluid according to its properties. Bird et. al. [3], Wilkinsion [26], Skelland [22], had defined a different model for the different behaviour of a non-Newtonian fluid. The relationship between stress and strain for viscous-inelastic fluids is discussed by Patel and Timol [18].

Newtonian and non-Newtonian fluid flow boundary layer problems are modelled mathematically by a system of non-linear partial differential equations. Similarity methods are powerful systematic techniques to solve nonlinear partial differential equations analytically. Similarity techniques have most powerful characteristics to reduce independent variables. Application of the different group theoretic techniques to find similarity solutions are shown in this research work. Mathematician Lie [12] is a pioneer of the technique which gives an invariant solution. This technique known as Group theoretic methods is the application of modern algebra to solve the non-linear problem which gives invariant solutions. In last decades the rapid evolution of the development of a deductive similarity theory based on finite and infinitesimal continuous transformation groups because of the advantage to reduce one and more than one independent variables from differential equations. Bluman and Cole [5], Hansen and Na [17], Brikhoff [4] and Morgan [16], Moran and Gajjoli [13,14], Moran and Masherek [15] had explored the theories on similarity solution. In literature, we surveyed that, Brikhoff [4], Morgan [16], Hansen, Na [17], Ames [2], Sheshadri [21] had contributed for a similarity solution of PDEs in the applied field of science using group theory. Most, recently Timol and his co-worker [1,9,19] had given a significant contribution to analyse different fluid models in different geometry using group-theoretic similarity methods.

Topfer [21] derived the method for converting BVP into IVP and then Klamkin [11] expanded the application for a more general class of BVP. In the book of Sheshadri and Na [21],
discussion about the application of the group-theoretic method to transform BVP into IVP is given in detail.

One of the classical similarity methods is the dimensional analysis method. Buckingham [6] had given a very simple and popular theorem on dimensional analysis known as the Buckingham-pi theorem. This result has a limitation that cannot give suggestion for the form of absolute invariants which is required for analysis. Moran and Marshek had given interesting results which suggests the required form of absolute invariants which depends on the rank of the associated matrix of independent and physical variables. Limitation of similarity solution is that exists only at 90-degree wedge for a non-Newtonian fluid model. So, for the non-similar solution one of the simplest methods is local non-similarity introduced by Sparrow, Quack, and Boerner [23] and implemented on velocity boundary value problems which are non-similar. Sparrow and Yu [24] had extended and applied the method for thermal boundary layer problem.

The concept of flow over stretching surfaces has gained the interest of researchers due to its wide industrial applications. Credit goes to Crane and Skiadas [8,20] for the study of flow pass through stretching sheets.

The word nanofluid is introduced by Choi [7] for suspension of base fluid and nanoparticles of nanometer size. Generally, water, ethylene glycol, oil is taken as base fluid and metals Cu, Ag, Au, Metallic oxides like \( Al_2O_3 \), CuO, Nitrides like AlN, SiN, Carbides like SiC, TiC, semiconductors like Ti02, SiC, and different types of carbon nanotubes like SWCNT, DWCNT, MWCNT are used as nanoparticles. There are many applications of nanofluid in different areas such as in automobiles as coolants, brake fluid and as gear lubrication, also in industrial cooling, in solar devices, in cancer drug, etc, [10]. Because of wide applications of nanofluid many researchers recently focused on it. Choi et al. [7], Xuan et al., [27,28]; Wang et al [25], had attempted on the enhancement of thermal conductivity and augmented heat transfer analysis for low volume of nanoparticles in nanofluid and got unimagined success. So, it is also called new generation heat transfer fluid.

3. Definition of the Problem:

There are various similarity and non-similarity techniques available in the literature to solve non-linear PDEs. Many inventors focused on these techniques to explore theoretical and practical
applications in the area of science and engineering. From that versatile, elegant and easily approachable concept is of group-theoretical procedure in the solution of non-linear PDEs. Newtonian and non-Newtonian fluid flow BVPs are generally described in the form of system of a PDEs. The non-linear relationship of stress and strain for several fluids is defined as different non-Newtonian fluid models like Powell-Eyring, Prandtl-Eyring, Sisko, Power-law, Reiner – Rivlin fluids, Bingham plastics, Ellis Fluids, Reiner-Philippoff fluid, Williamson fluids, Oldroyd fluid, Rivlin–Ericsen fluids, Walters fluid, Maxwell fluids, etc.

Here, we had considered Powell-Eyring, Prandtl-Eyring, Sisko, Power-law fluid models for analysis in two and three dimensions. Our humble endeavour to examine Newtonian and non-Newtonian 3-D nanofluid model is successful. Several group-theoretic techniques like generalised dimensional analysis, one and two parameter deductive group-theoretic methods are implemented to transform governing non-linear boundary value type PDEs into ODEs. Similarity group-theoretic procedure is utilized to transfer BVP into IVP. The concept of local non-similarity is implemented for achieving a non-similar solution. The obtained non-similar equations are treated as the type of similarity solutions. The numerical solution of the transformed ODEs is achieved by different software like Maple and MATLAB. Impact of different physical parameters on fluid flow are depicted graphically for each BVP.

4. Objectives:

- The prime objective of this research is to show effectiveness and significance of similarity methods to analyse BVPs arising in engineering field which are formed in the system of nonlinear PDE and to study effects of different physical parameters on fluid flow for the Newtonian and non-Newtonian fluid model.
- The main goal of this investigation is to demonstrate utilization of potent and simple technique called as a group-theoretic method to convert BVP into IVP.
- The aim behind the purposed study is to suggest a non-similarity solution when similar solutions are not possible or to find a solution other than a 90-degree wedge for a non-Newtonian fluid model.
- Our goal is to obtain a numerical solution of converted ODEs by applying simple Taylor’s series method and by using computer software.
5. Future scope:

- In this study, we had considered steady flow boundary value problems using different similarity methods. In the future, we can apply these methods for time-dependent flow problems.
- In this research work, we had considered two and three-dimensional flow in a cartesian coordinate system for flat plates and over a linearly stretching surfaces, we can expand our work by considering flow over a nonlinearly stretching sheet, over moving boundary for the cylindrical and spherical coordinate system. We used a two-equation model for a local non-similarity method we can extend it for the three-equation model.
- The study is carried out for some of the non-Newtonian fluid models like Powell-Eyring, Prandtl-Eyring, Sisko, Power-law model. We will be carried out a further study for other fluid models by constructing appropriate group and invariant solutions.
- Here, we applied one and two parameter group-theoretic method for solution. we can extend it for multiparameter group method.

6. Original contribution by the thesis:

Our contribution to the field of similarity and non-similarity methods on the application side is as follows:

The thesis consists of seven chapters. The first chapter concerns with the general introduction of the thesis. The second chapter is the foundation of the whole thesis.

The second chapter includes basic definitions and fundamental principles of fluid mechanics, concepts of similarity methods and non-similarity methods and related results.

In the third chapter of the thesis, the Generalized Dimensional Analysis method is applied to find the similarity solution. The steady, two-dimensional boundary layer flow is analysed for non-Newtonian Sisko fluid. The non-linear boundary value problem (BVP) is transformed into an initial value problem (IVP) by applying one parameter linear group method. Simple numerical technique namely Taylor’s series is applied for solution of IVP.

In the fourth chapter, two non-Newtonian fluid models namely Powell-Eyring and Prandtl-Eyring fluid are considered for similarity analysis. Generalized group theoretical method with one
parameter is used to obtain a similarity solution for forced convection flow past a semi-infinite flat plate. Maple ode solver is used for graphical presentations of velocity and temperature profiles for different physical parameters. Also, the comparison for both fluid models is done graphically.

The fifth chapter deals with the three-dimensional Newtonian Nanofluid flow over a flat surface which is continuously stretching linearly in two lateral directions is analysed. Similarity solution is found by reducing two independent variables using two parameter deductive group-theoretic method. The impact on velocity and temperature of nanoparticle volume fraction is examined.

In the sixth chapter, we have extended and investigated three-dimensional MHD nanofluid flow over linearly stretching sheet by considering Brownian motion and thermophoresis effect for Non-Newtonian power-law model. Similarity solution for three-dimensional MHD flow over linearly stretching sheet under the effect of Brownian motion and thermophoresis are found by applying two parameter deductive group-theoretic method. The impact of various physical parameter on flow is examined.

The chapter seventh include the non-similar solution of non-Newtonian fluid for Powell-Eyring model using the local non-similarity method. Non-similar solutions are obtained for different values of power-law index \( m \) for free stream velocity in power-law form \( U = x^m \) and streamwise location \( \xi \). Influence of various parameters on velocity and temperature field are presented graphically using MATLAB bvp4c solver.

7. Methodology of Research and Results:

We had started our work by surveying literature about different similarity and non-similarity methods and fluid flow boundary value problems arising in the engineering field. We conclude from that most of all research work done by assuming similarity variables. We also observed some advantages and limitations of different similarity methods.

We applied generalised dimensional analysis method for analysis of a Sisko fluid model. The method is very simple to apply and it requires only knowledge of the rank of a dimensional matrix associated with independent and physical variables. It is useful for a linear change of scale. Sometimes physical variables are reduced instead of independent variables depending on rank. We
utilized the method of linear group transformations to convert BVP into IVP. The transferred IVP is solved by simple Taylor’s series method.

We had started our similarity analysis for Powell-Eyring and Prandtl-Eyring fluid models using a one-parameter deductive group-theoretic method with the aim to reduce one independent variable from governing equations. The procedure is systematic and no special form of the group is initially assumed. The appropriate group is deduced and auxiliary conditions are also taken into account for the deduction of the group. Limitation of the method is that if we want to reduce two or more than two variables then we have to repeat the same procedure.

The effect of nanoparticles volume fraction on temperature and velocity profile is investigated for three-dimensional nanofluid flow over flat surface stretched continuously in two lateral directions by applying two parameter deductive group-theoretic method. As a result, two independent variables are reduced from governing equations.

We had extended the mathematical model for analysis. The two-parameter deductive group-theoretic method is implemented to investigate three-dimensional MHD nanofluid flow over linearly stretching sheet by considering Brownian motion and thermophoresis effect for non-Newtonian power-law model.

We modified the model for non-similarity. The local non-similarity method with the two-equation model of Sparrow and Yu is applied to achieve local solutions which are independent of the upstream information for non-Newtonian fluid for Powell-Eyring model.

We had started the procedure by considering the general group of transformations and then gained absolute invariants under which system of nonlinear PDEs are transferred into ODEs. All BVPs under consideration governed by a system of nonlinear PDEs are converted into a system of ODEs using different similarity and non-similarity procedures. Systems of transformed ODEs are solved numerically by Taylor’s series method or using software like Maple and MATLAB. All obtained results are expressed in graphical form.

8. Achievements with respect to objectives:

We had studied Newtonian and non-Newtonian fluid flow boundary value problems with governing equations in the form of a system of non-linear PDEs and successfully achieved similarity solutions using various group-theoretic methods. One interesting application of
similarity group-theoretic method to convert BVP into IVP is successfully demonstrated. We found an interesting outcome by employing the local non-similarity method systematically for flow other than a 90-degree wedge. We analysed the impact of different physical parameters on fluid flow by numerical and graphical results which are achieved by software Maple and MATLAB.

9. Conclusion:

- Similarity methods are categorised in two ways one which is very simple known as direct methods and second which are more elegant and satisfies invariant postulate known as group-theoretic methods. Group theoretic methods are subcategorized as the assumed group of transformation and the deductive group of transformations. Deductive group-theoretic methods are those for which auxiliary conditions are also taken into account for invariance analysis.

- We conclude that the similarity solution exists for free-stream velocity is in the form $U = x^m$, where $m = 1/3$ for non-Newtonian models. For other values of $m$, we have to find a non-similar solution. The local non-similarity method of Sparrow and Yu is very simple and systematic. We can treat obtained non-similar equation as treatment of similarity equation.

- The main outcome of the analysis is that nonlinear PDEs are successfully transformed invariantly into ODEs by applying similarity methods. A second observation is that software likes Maple and MATLAB are most powerful to give numerical solution and graphical presentation of BVPs.

- For Sisko fluid model velocity is increases as increasing similarity variable.

- Boundary layer thickness is higher for Powell-Eyring model compare to Prandtl-Eyring model for both velocity and thermal.

- We observed that thermal conductivity enhances as increasing amount of (copper) nanoparticles and opposite trend for velocity for 3-D Newtonian nanofluid model.
• Lewis number, Biot number, Thermophoresis parameter, Brownian motion parameter are affecting concentration and temperature both for 3-D non-Newtonian power-law model.

• Here, we found non-similar solution at shape other than 90-degree wedge and observed that streamwise co-ordinate $\xi$ significantly influences flow.

• Conclusion of each BVPs is given in the concerned chapter of the thesis.

10. List of Paper publications:


Details of the work presented in conference


11. References:


