
New Results in Various Aspects of Graph Theory

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Bosmia Mohitkumar Ishvarlal

Enrollment No.: 139997673003

under supervision of

Dr Kailas K Kanani

Head, Department of Mathematics,
Government Engineering College, Rajkot



GUJARAT TECHNOLOGICAL UNIVERSITY

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1 Abstract

Graph theory and its applications have grown exponentially in twentieth century. The development of computer science and optimization techniques have accelerated the research activities in the subject. At present, this branch has the status of one of the fastest growing field of research with multifaceted applications ranging from electrical engineering to management science and computer science to social science.

Graph theory has close correlation with other branches of mathematics such as matrix theory, statistics, algebra, geometry and topology. Algebraic graph theory, domination in graphs, algorithmic graph theory, energy of graphs and labeling of graphs are potential fields of research in graph theory.

The labeling of discrete structures is one of the emerging areas of research due to its diversified applications. Most of the graph labeling problems trace their origin with graceful labeling which was introduced by Rosa[8] in 1967.

This research work deals with various types of graph labeling. This work is divided into five chapters. Chapter 1 is of introductory nature which provides an overview of the remaining chapters.

Chapter 2 contains all basic definitions and concepts which are necessary to understand this research work.

Chapter 3 is focused on cordial labeling as well as its variants namely product cordial, total product cordial, edge product cordial and total edge product cordial labelings for the line graph of bistar $B_{n,n}$.

In chapter 4, the divisor cordial labeling for the graphs obtained by graph operations corona product, splitting, degree splitting, barycentric subdivision, shadow, switching of a vertex, join, restricted square and m-splitting have been discussed.

Chapter 5 is focused on square divisor cordial labeling and cube divisor cordial labeling as variants of divisor cordial labeling.

2 Brief description on the state of the art of the research topic

In keeping with the title of the research topic we have introduced a new graph labeling technique cube divisor cordial labeling. We have also analyzed the technique to find that there are certain graphs that admit cube divisor cordial labeling and certain graphs do not. Also, we have derived several results for various graph labeling techniques like cordial labeling, product cordial labeling, total product cordial labeling, edge product cordial labeling, total edge product labeling, divisor cordial labeling and square divisor cordial labeling.

3 Definition of the Problem

We found that some graph admit cordial labeling and its variants like product cordial labeling, total product cordial labeling, edge product cordial labeling, total edge product labeling, divisor cordial labeling, square divisor cordial labeling, cube divisor cordial labeling. So in this research this situation is investigated and analyzed. We have even explored the reason for which some graphs do not admit certain graph labeling techniques.

4 Objectives and Scope of work

The objectives of this research work are:

- To survey different graph labeling techniques.
- To define new graph labeling techniques.
- To find new families of graphs which admit cordial labeling
- To find new families of graphs which do not admit cordial labeling.
- To find which graphs admit or do not admit variants of cordial labeling

5 Original contribution by the thesis

The following results are derived in this thesis.

- $L(B_{n,n})$ is cordial if and only if $n = t^2$ or $n = (t + 1)^2 - 1$ for $t \in \mathbb{N}$.
- $L(B_{n,n})$ is a product cordial graph.
- $L(B_{n,n})$ is a total product cordial graph.
- $L(B_{n,n})$ is an edge product cordial graph.
- $B_{n,n}$ is a total edge product cordial graph.
- $L(B_{n,n})$ is a total edge product cordial graph.
- $K_{1,n} \odot K_1$ is a divisor cordial graph.
- $K_{2,n} \odot K_1$ is a divisor cordial graph.
- $K_{3,n} \odot K_1$ is a divisor cordial graph.
- $W_n \odot K_1$ is a divisor cordial graph.
- $H_n \odot K_1$ is a divisor cordial graph.
- $Fl_n \odot K_1$ is a divisor cordial graph.
- $f_n \odot K_1$ is a divisor cordial graph.
- $Df_n \odot K_1$ is a divisor cordial graph.
- $S(K_{1,n}) \odot K_1$ is a divisor cordial graph.
- The bistar $B_{m,n}$ is a divisor cordial graph.
- $S'(B_{m,n})$ is a divisor cordial graph.
- $DS(B_{m,n})$ is a divisor cordial graph.

- $D_2(B_{m,n})$ is a divisor cordial graph.
- Restricted $B_{m,n}^2$ is a divisor cordial graph.
- The barycentric subdivision $S(B_{m,n})$ of the bistar $B_{m,n}$ is a divisor cordial graph.
- $B_{m,n} \odot K_1$ is a divisor cordial graph.
- The graph G_v obtained by switching of a vertex in the crown $C_n \odot K_1$ is a divisor cordial graph.
- The graph G_v obtained by switching of a vertex in the armed crown AC_n is a divisor cordial graph.
- The graph G_v obtained by switching of a vertex in the helm H_n is a divisor cordial graph.
- The graph G_v obtained by switching of a vertex in the bistar $B_{m,n}$ is a divisor cordial graph.
- $AC_n + K_1$ is a divisor cordial graph.
- $\left(\bigcup_{i=1}^n C_{m_i}\right) + K_1$ is a divisor cordial graph.
- $\left(P_m \cup \bigcup_{i=1}^n C_{m_i}\right) + K_1$ is a divisor cordial graph.
- $\left(K_{1,m} \cup \bigcup_{i=1}^n C_{m_i}\right) + K_1$ is a divisor cordial graph.
- The barycentric subdivision $S(K_{2,n})$ of $K_{2,n}$ is a divisor cordial graph.
- The barycentric subdivision $S(K_{3,n})$ of $K_{3,n}$ is a divisor cordial graph.

- The m -splitting graph $Spl_m(P_n)$ of path P_n is divisor cordial for $n \leq 11$.
- The m -splitting graph $Spl_m(C_n)$ of cycle C_n is divisor cordial for $n \leq 11$.
- The graph G_v obtained by switching of a vertex in the bistar $B_{m,n}$ is square divisor cordial.
- The graph G_v obtained by switching of a vertex in the comb graph $P_n \odot K_1$ is square divisor cordial.
- The graph G_v obtained by switching of a vertex in the crown $C_n \odot K_1$ is square divisor cordial.
- The graph G_v obtained by switching of a vertex in the armed crown AC_n is square divisor cordial.
- The graph G_v obtained by switching of a vertex except apex vertex in the helm H_n is square divisor cordial.
- The graph G_v obtained by switching of a vertex except apex vertex in the gear graph G_n is square divisor cordial.
- Given a positive integer n , there is a cube divisor cordial graph G which has n vertices.
- If G is a cube divisor cordial graph of even size, then $G - e$ is also cube divisor cordial for all $e \in E(G)$.

- If G is a cube divisor cordial graph of odd size, then $G - e$ is also cube divisor cordial for some $e \in E(G)$.
- The path P_n is a cube divisor cordial graph if and only if $n = 1, 2, 3, 4, 5, 6, 8$.
- The cycle C_n is a cube divisor cordial graph if and only if $n = 3, 4, 5$.
- The star graph $K_{1,n}$ is a cube divisor cordial graph if and only if $n = 1, 2, 3$.
- The complete bipartite graph $K_{2,n}$ is a cube divisor cordial graph.
- The complete bipartite graph $K_{3,n}$ is cube divisor cordial if and only if $n = 1, 2$.
- The complete graph K_n is cube divisor cordial if and only if $n = 1, 2, 3, 4$.
- The wheel $W_n = K_1 + C_n$ is a cube divisor cordial graph for each n .
- The flower graph Fl_n is a cube divisor cordial graph for each n .
- The fan graph $f_n = K_1 + P_n$ is a cube divisor cordial graph for each n .
- The bistar $B_{n,n}$ is a cube divisor cordial graph.
- Restricted $B_{n,n}^2$ is a cube divisor cordial graph.
- The barycentric subdivision $S(K_{1,n})$ of the star $K_{1,n}$ is a cube divisor cordial graph.

- The graph G_v obtained by switching of a vertex in cycle C_n is a cube divisor cordial graph.
- $DS(B_{n,n})$ is a cube divisor cordial graph.
- $DS(P_n)$ is a cube divisor cordial graph if and only if $n \neq 5$.
- The graph G_v obtained by switching of a vertex in $P_n \odot K_1$ is cube divisor cordial.
- The graph G_v obtained by switching of a vertex in $B_{m,n}$ is cube divisor cordial.
- The graph G_v obtained by switching of a vertex in $C_n \odot K_1$ is cube divisor cordial.
- The graph G_v obtained by switching of a vertex in AC_n is cube divisor cordial.

6 Methodology of Research and Results/Comparisons

Different techniques and methodologies are applied according to the need of research work. Many softwares, internet tools and electronic journals are used for concept building, confirmation and representation of this work.

Various graph labeling techniques, namely graceful labeling, harmonious labeling, cordial labeling, product cordial labeling, total product cordial labeling, edge product cordial labeling, total edge product cordial labeling, divisor cordial labeling and square divisor cordial labeling are surveyed

at the outset of this work to develop the basic understanding. The existing labeling techniques are analyzed and the findings are implemented to derive the final results.

7 Achievements with respect to objectives

- Gained knowledge of different graph labeling techniques.
- Defined a new graph labeling technique namely cube divisor cordial labeling.
- New families of graphs which admit cordial labeling are found.
- New families of graphs which do not admit cordial labeling are found.
- Variants of cordial labeling are analyzed and new results are derived.

8 Conclusion

The findings of this research are follows:

- Discussed cordial labeling, product cordial labeling, total product cordial labeling, edge product cordial labeling and total edge product cordial labeling for line graph of bistar $B_{n,n}$.
- Twenty eight results of divisor cordial labeling.
- Six results of square divisor cordial labeling.

- Introduced new graph labeling technique namely cube divisor cordial labeling.
- Twenty two results of cube divisor cordial labeling.

9 List of Publications

List of Publications derived from the Thesis

- On cube divisor cordial graphs, International Journal of Mathematics and Computer Applications Research, 5(4), 2015, 117-128.
(<http://www.tjprc.org/journals/journal-of-mathematics>)
ISSN : 2249-6955 (Print), 2249-8060 (Online)
- Further Results on Cube Divisor Cordial Labeling, Elixir Discrete Mathematics, 88, 2015, 36597-36601.
([http://www.elixirpublishers.com/articles/1451025456_88%20\(2015\)%2036597-36601.pdf](http://www.elixirpublishers.com/articles/1451025456_88%20(2015)%2036597-36601.pdf))
ISSN : 2229-712X
- Some Standard Cube Divisor Cordial Graphs, International Journal of Mathematics and Soft Computing, 6(1), 2016, 163-172.
(<https://www.ijmsc.com/index.php/ijmsc/article/view/389/ijmsc-6-1-14>)
ISSN : 2249-3328 (Print), 2319-5215 (Online)
- Divisor Cordial Labeling in the Context of Graph Operations on Bistar, Global Journal of Pure and Applied Mathematics, 12(3), 2016,

2605-2618.

(https://www.ripublication.com/gjpam16/gjpamv12n3_50.pdf)

ISSN : 0973-1768

- Divisor Cordial Labeling in the Context of Corona Product, Proceedings of 9th National Level Science Symposium 2016 on Recent Trends in Science and Technology, 14 February 2016, 178-182.
ISBN : 9788192952123
- Square Divisor Cordial Labeling in the Context of Vertex Switching, International Journal of Mathematics And its Applications, 6(1-D), 2018, 687-697.
(<http://ijmaa.in/v6n1-d/687-697.pdf>)
ISSN : 2347-1557
- Various Graph Labeling Techniques for the Line Graph of Bistar, *International Journal of Technical Innovation in Modern Engineering & Science*, 4(9), 2018, 851-858.
(http://ijtimes.com/papers/finished_papers/IJTIMESV04I09150921191146.pdf)
ISSN : 2455-2585
- Cube Divisor Cordial Labeling in the Context of Switching of a Vertex, *Mathematics Today*, 34, 2018, 111-124.
(http://mathematicstoday.org/currentissue/V34_Dec_2018_10.pdf)
ISSN : 0976-3228 (Print), 2455-9601 (Online)

Details of the Work Presented in Conference

- The paper entitled as “Cube Divisor Cordial Labeling of Some Standard Graphs” was presented in *The Annual Conference of ADMA & Graph Theory Day-XI* at B. S. Abdur Rahman University campus, Chennai, during June 10-12, 2015.
- The paper entitled as “Divisor Cordial Labeling in the Context of Corona Product” was presented in *9th National Level Science Symposium 2016 on Recent Trends in Science and Technology* at Christ College, Rajkot, on February 14, 2016.
- The paper entitled as “Divisor Cordial Labeling in the Context of Vertex Switching” was presented in *International Conference on Discrete Mathematics-2016 and Graph Theory Day-XII* at Siddaganga Institute of Technology, Tumakuru, Karnatak, during June 9-11, 2016 .
- The paper entitled as “Cube Divisor Cordial Labeling in the Context of Vertex Switching” was presented in *National Conference on Computer Engineering, Information & Communication Technology* at Government Engineering College, Gandhinagar, during September 15-16, 2016.
- The paper entitled as “New Families of Cube Divisor Cordial Graphs” was presented in *National Conference on Algebra, Analysis and Graph Theory* at Saurashtra University, Rajkot, during February 9-11, 2017.
- The paper entitled as “Divisor Cordial Labeling of Bistar Related Graphs” was presented in *Annual Conference of ADMA and Graph The-*

ory Day-XIII at SSN College of Engineering, Chennai, during June 8-10, 2017 .

- The paper entitled as “Divisor Cordial Labeling in the Context of Graph Operations” was presented in *National Conference on Applied Mathematical Sciences* at Gujarat University, Ahmedabad, during April 14-15, 2018.
- The paper entitled as “Various Graph Labeling Techniques for the Line Graph of Bistar” was presented in 14th *Annual ADMA Conference & Graph Theory Day* at Dhirubhai Ambani Institute of Information and Communication Technology, Gandhinagar, during June 6-10, 2018.

Details of the Work accepted for Publication

- Divisor Cordial Labeling in the context of Join and Barycentric Subdivison, *TWMS Journal of Applied and Engineering Mathematics*.

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