Study of 1-D Flow in Porous Media

A Synopsis submitted to Gujarat Technological University

for the Award of

Doctor of Philosophy

in

Science - Maths

by

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June, 2018

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1 Abstract

The thesis has been devoted to study of 1-D flow in porous media and the development of mathematical model of fluid flow through porous media. The fluid flow through porous media is one of the most important concept in many areas of applied science and engineering such as filtration, hydrogeology, petroleum engineering, geomechanics, water resource engineering, soil mechanics, environment engineering, chemical engineering, construction engineering, civil engineering, geophysics, biophysics etc.

In recent years, extensive research works have been carried out to study the fluid flow through porous media. This field has gained extensive attention due to its broad range of applications in science and industry. In particular, the modeling of fluid flow through porous media is a central problem within the field of various applications in such areas. The scope of the present study lies in an increasing importance of the hydrodynamics of single phase flow and multiphase flow through porous media. Thus, an investigation of such research area leads directly or indirectly to study in another research area far away from the first in terms of physical context, yet related through the fundamental principles of fluid dynamics of multiphase flow. Due to vast scope of multiphase flow through porous media, the specific problems are almost unlimited and therefore it is reasonable to select such types of problems for discussion here. Accordingly a selection of more interesting problems of current interest have been made for mathematical treatment in the thesis. The investigated problems of the present study are concerned with the flow of immiscible fluids.

This work has been devoted to the study of some problems with fluid

flow through porous media. The development and testing of mathematical models describe the fluid flow through porous media. The solution of the problems have been studied numerically and graphically. The thesis consists of eight chapters. The first chapter deals with the introductory nature and the general introduction of the subject matter of the thesis. The second chapter includes a brief discussion on certain relevant topics, like basic equations governing the flow of an incompressible fluid through porous medium and related literature review. The problems related to the present work are studied in Chapters 3 to 8 with the homotopy analysis solution [17]. An approximate analytical solution of fingering phenomenon arising in two phase flow through homogeneous porous medium is studied in Chapter 3. The mathematical model for countercurrent imbibition phenomenon in the inclined oil formatted homogeneous porous medium with homotopy analysis solution is discussed in Chapter 4. Chapter 5 discusses the mathematical models of cocurrent imbibition phenomenon in inclined homogeneous and heterogeneous porous media. Chapter 6 is divided in two parts, the first discusses the homotopy series solution for fingero-imbibition phenomenon in the heterogeneous porous medium during secondary oil recovery process, second part discusses fingero-imbibition phenomenon in heterogeneous porous medium with magnetic field effect and its solution is compared with first part. The homotopy series solution of Boussinesq's equation for infiltration phenomenon in unsaturated porous medium is studied in Chapter 7. On the basis of linear and nonlinear conductivity and diffusivity functions, the groundwater recharge by spreading of water in vertical direction is studied in Chapter 8.

2 Brief description on the state of the art of the research topic

Scheidegger [34] has introduced the physics of flow through porous media. The study of the physics of fluid flow through porous media has become basic for many scientific and engineering applications like as groundwater hydrology, ceramic engineering, petroleum engineering, water resource engineering, soil mechanics, etc. All these branches of science and engineering have contributed a vast amount of literature on this topic.

The fluid flow through porous media can be classified into single phase flow and multiphase flow. Many researchers have discussed to the single phase flow and multiple phase flow with different view points. Different type of problems of fluid flow through porous media in the various fields have been discussed by Muskat [23]. The theory of dynamics of fluids in porous media, as applicable to many disciplines of science and engineering are given in the book "Dynamics of fluids in porous media" by Bear [1]. It helps to researchers where flow through porous media plays a fundamental role in fields such as soil mechanics, soil physics, groundwater hydrology, petroleum engineering, drainage and irrigation engineering, chemical engineering and sanitary engineering. By reading this literature, I have learnt which has made easier to understand and analyze my research topic. The phenomenon of fingering (instability) has been of great importance in many engineering fields such as agriculture engineering, soil mechanics, groundwater hydrology and petroleum engineering [2, 3, 7, 32, 35, 39]. The phenomenon of fingering in homogeneous porous media without capillary pressure was examined from

a statistical viewpoint by Scheidegger and Johnson [35]. The stabilization of fingers in a specific oil-water displacement process with capillary pressure has been statistically discussed for a heterogeneous porous medium by Verma [42].

Spontaneous imbibition is the process in which the wetting phase is drawn into a porous medium by means of capillary pressures and the curved interfaces between the wetting and non-wetting phase without any external force. There are two types of spontaneous imbibition: cocurrent and countercurrent. In the cocurrent imbibition both wetting and non-wetting phases move in the same direction while during countercurrent imbibition both move in the opposite directions. The imbibition phenomenon has been investigated by many researchers such as Blair [4], Brownscombe and Dyes [8], Graham and Richardson [13], Mehta and Verma [20], Scheidegger [34], Tavassoli et al. [38]. Imbibition in water-wet porous media is commonly considered as countercurrent imbibition [4, 5, 11, 12, 19, 31, 46]. When a porous medium is partially filled with wetting phase, oil recovery is dominated by cocurrent imbibition phenomenon, not countercurrent imbibition [5, 19, 31, 46]. The simultaneous occurrence of both phenomena fingering and imbibition is known as fingero-imbibition phenomenon which is investigated by researchers with different viewpoints [11, 14, 21, 22, 25, 26, 27, 28, 36, 43].

The groundwater flow has great importance in various fields of science and engineering like as fluid mechanics, hydrology, environment engineering, water resource engineering, soil science [1, 6, 10, 11, 29, 30, 40, 34]. Infiltration is the process in which precipitation or water on ground surface enters into the subsurface soils and moves into rocks through cracks and pore

spaces. Philip [29] described the development of infiltration equation and its numerical solution. Srivastava and Yeh [37] described one dimensional, vertical infiltration toward the water table in homogeneous and layered soils by analytical solutions. Witelski [44] extended the applicability of the Boltzmann similarity solution by introducing a time-shift constant to describe the long time behavior for absorption into slightly wet soil layers. Wojnar [45] have discussed Boussinesq equation for flow in an aquifer with time dependent porosity. Chavan and Panchal [9] have discussed the solution of porous medium equation arising in fluid flow through porous media by homotopy perturbation method using Elzaki transform.

One dimensional groundwater recharge problem is related to hydrology, environment engineering, soil mechanics, water resource engineering etc. The flow of water in unsaturated soil has been considered with some specific assumptions. The problem of groundwater flow has been discussed by many researchers with different aspects, like as Klute [16] reduced diffusion equation to an ordinary differential equation and applied a forward integration and iteration method, Verma [41] obtained solution of a one dimensional groundwater recharge for constant diffusivity and linear conductivity by Laplace transform, Prasad et al. [33] developed numerical model to simulate moisture flow through unsaturated zones using the finite element method, Joshi et al. [15] obtained solution of one dimensional vertical groundwater recharge by group theoretical approach, Nasseri et al. [24] studied solution of advection-diffusion equation on the basis of the simplified Brooks-Corey model for soil conductivity and diffusivity.

3 Definition of the Problem

In the secondary oil recovery process, physical phenomena like as fingering phenomenon, countercurrent imbibition phenomenon, cocurrent imbibition phenomenon, fingero-imbibition phenomenon occur in the fluid flow through porous medium. The mathematical models of 1-D flow in porous medium were studied with some specific assumptions in the present work. The aim of the present study was to investigate the behavior of the saturation of injected water in different physical phenomena which are arising in the fluid flow through homogeneous or heterogeneous porous medium. To study the infiltration phenomenon through unsaturated porous medium and the solution of the Boussinesq's equation for infiltration phenomenon. The objective of the work is to investigate the behavior of the moisture content of soil in the groundwater flow when excess water on the ground surface is spreading in the vertical direction through unsaturated porous medium.

4 Objective and Scope of work

The main goal of the present work is to study the 1-D flow in porous media. The primary objective of our study is to discuss the one dimensional mathematical problem arising in the fluid flow through porous media. The secondary objective is to study the solution of one dimensional nonlinear partial differential equation by homotopy analysis method. The solution represents:

- the saturation of the injected water which helps us to predict the amount of water required to inject for recovering oil,

- the moisture content which helps us to predict the amount of water spread in the unsaturated soil.

This type of mathematical model is useful for predicting oil recovery from petroleum reservoir and for predicting moisture content increase in unsaturated soil. The scope of the current work is to study of problems of 2-D flow in porous media.

5 Original contribution by the thesis

The original contribution made by the study is created/modified mathematical models of

- countercurrent imbibition phenomenon in the inclined oil formatted homogeneous porous medium,
- cocurrent imbibition phenomenon in the inclined homogeneous porous medium,
- cocurrent imbibition phenomenon in the inclined heterogeneous porous medium
- fingero-imbibition phenomenon in the heterogeneous porous medium with magnetic field effect.

The solution of various problems of fluid flow through porous media are discussed using homotopy analysis method with appropriate boundary conditions.

Comparison study of the solution of fingero-imbibition phenomenon in the heterogeneous porous medium with magnetic field effect and without magnetic field effect are given.

6 Methodology of Research and Results/Comparisons

We have studied various literatures related to fluid flow through porous media and done a comparative analysis to find out research gap and problem statement. The literature survey helped us to define an objective of the research.

We have used Mathematica BVPh package for nonlinear boundary value problems [18]. It is a combination of the homotopy analysis method and the computer algebra system Mathematica, and provides us a convenient analytic tool to solve many nonlinear differential equations.

We have studied various problems of fluid flow through porous medium and according to research gap we create/modify the mathematical model for different physical phenomena arising in two phase flow through porous medium during secondary oil recovery process.

An approximate analytical solution of one dimensional nonlinear partial differential equation arising in the oil-water displacement process (instability) in a homogeneous porous medium is investigated. The homotopy analysis method has been used to find solution of this equation with appropriate boundary conditions. The solution interpreted numerically and graphically using Mathematica BVPh package for homotopy analysis method [18] and it is compared with the exact solution.

The mathematical model is developed for the problem of countercurrent imbibition phenomenon occurring in the inclined oil formatted homogeneous porous medium. The homotopy analysis method has been applied to solve mathematical equation with suitable boundary conditions. The c_0-curves are obtained using Mathematica BVPh package.

One dimensional nonlinear partial differential equation for fingero-imbibition phenomenon arising in two phase flow through heterogeneous porous medium during secondary oil recovery process is discussed. The homotopy analysis method is used to solve the equation with appropriate boundary conditions. Using Mathematica software, the graphical and numerical interpretations of solution are given. Comparison of the solution of fingero-imbibition phenomenon in heterogeneous porous medium with magnetic field effect and without magnetic field effect are given.

The mathematical model is developed for cocurrent imbibition phenomenon in the inclined oil formatted homogeneous porous medium and heterogeneous porous medium. An approximate analytical solution of the governing equation is derived by homotopy analysis method. The graphical and numerical solutions are discussed.

The infiltration phenomenon through unsaturated porous medium has been discussed. The mathematical formulation yields a Boussinesq's equation. The homotopy series solution of the Boussinesq's equation for infiltration phenomenon is obtained. The numerical interpretation of solution and the graphical interpretation of solution are also discussed by Mathematica software.

The problem of one dimensional groundwater recharge in the vertical di-

rection is discussed. The groundwater is recharged by spreading of water in vertical direction and the moisture content of soil increases. On the basis of linear and nonlinear conductivity and diffusivity functions, three cases are considered for Brooks-Corey model. The governing nonlinear partial differential equations has been solved by homotopy analysis method. The proper value of convergence control parameter for convergent solution has been chosen from c_0-curve. The numerical and graphical solutions are obtained.

7 Achievements with respect to objectives

The mathematical models of the problems in fluid flow through porous media are created/modified for

- countercurrent imbibition phenomenon in the inclined oil formatted homogeneous porous medium,
- cocurrent imbibition phenomenon in the inclined homogeneous porous medium,
- cocurrent imbibition phenomenon in the inclined in heterogeneous porous medium
- fingero-imbibition phenomenon in the heterogenous porous medium with magnetic field effect.

We have discussed the problems of fluid flow through porous media for

- oil-water displacement process (instability) in a homogeneous porous medium,

- countercurrent imbibition phenomenon occurring in the inclined oil formatted homogeneous porous medium,
- cocurrent imbibition phenomenon in the inclined oil formatted homogeneous porous medium,
- cocurrent imbibition phenomenon in the inclined oil formatted heterogeneous porous medium,
- fingero-imbibition phenomenon in the heterogeneous porous medium,
- fingero-imbibition phenomenon occurring in the fluid flow through heterogeneous porous medium with magnetic field effect,
- the infiltration phenomenon through unsaturated porous medium,
- groundwater recharge in the vertical direction.

The solutions of mathematical problems are obtained by homotopy analysis method with appropriate boundary conditions.

8 Conclusion

We have studied the mathematical models of different physical phenomena like as fingering phenomenon, countercurrent imbibition phenomenon, cocurrent imbibition phenomenon, fingero-imbibition phenomenon which are arising during secondary oil recovery process. We have discussed the infiltration phenomenon through unsaturated porous medium and an approximate analytical solution of the Boussinesq's equation for infiltration phenomenon.

Also we have investigated the mathematical model of groundwater recharge in vertical downward direction through unsaturated porous medium. All the problems are consider in one dimensional fluid flow and then the governing equations come in the form of one dimensional nonlinear partial differential equations. Homotopy analysis method is adopted to find solution of nonlinear partial differential equations with appropriate boundary conditions. The solutions are interpreted graphically as well as numerically using Mathematica BVPh package for homotopy analysis method.

9 List of Publications

List of Publications Arising From the Thesis

- An approximate analytical solution of nonlinear differential equation arising in fluid flow through homogeneous porous media, *International Journal of Innovative Research in Science, Engineering and Technology*, 4(8), (2015), 7655-7662.

- Homotopy analysis solution of countercurrent imbibition phenomenon in inclined homogeneous porous medium, *Global Journal of Pure and Applied Mathematics*, 12(1), (2016), 1035-1052.

- Homotopy analysis method for fingero-imbibition phenomenon in heterogeneous porous medium, *Nonlinear Science Letters A: Mathematics, Physics and Mechanics*, 8(1), (2017), 90-100.

- Homotopy analysis method for nonlinear partial differential equation arising in fluid flow through porous medium, *International Journal of*

Computer & Mathematical Sciences, 6(5) (2017), 14-18.

- Mathematical modelling and analysis of cocurrent imbibition phenomenon in inclined heterogeneous porous medium, *International Journal of Computational and Applied Mathematics*, 12(3) (2017), 639-652.

- An approximate analytical solution of the Burger's equation for longitudinal dispersion phenomenon arising in fluid flow through porous medium, *International Journal on Recent and Innovation Trends in Computing and Communication*, 5(5) (2017), 1103-1107.

- Mathematical modelling of fingero-imbibition phenomenon in heterogeneous porous medium with magnetic field effect, *PRAJNA - Journal of Pure and Applied Sciences*, 14-15 (2017), 15-22.

- An approximate analytical solution of Boussinesq's equation for infiltration phenomenon in unsaturated porous medium, *International Journal of Mathematics And its Applications*, 6(1-C) (2018), 463-470.

Details of the Work Accepted

- An approximate analytical solution of one-dimensional groundwater recharge by spreading, *TWMS Journal of Applied and Engineering Mathematics*.

Details of the Work Presented in Conference

- The paper entitled as "A mathematical model of cocurrent imbibition phenomenon in inclined homogeneous porous medium" presented in *ICRISET2017* at B. V. M. Engineering College, V. V. Nagar, Anand

(Gujarat) on 17^{th} February, 2017 and published in Kalpa Publications in Computing, ICRISET2017, Selected Papers in Computing, 2, (2017), 51-61.

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