

*EXPERIMENTAL INVESTIGATION AND ANALYSIS OF
FRICTION TORQUE IN TAPERED ROLLER BEARINGS*

A Synopsis

submitted to

Gujarat Technological University

for the Award of

Doctor of Philosophy

in

Mechanical Engineering

by

Khant Amishaben Babubhai
[219999919002]

under supervision of

Dr. Jaydeep K. Dadhaniya



GUJARAT TECHNOLOGICAL UNIVERSITY
AHMEDABAD

February – 2026

INDEX

Sr. No.	Topic	Page No.
1.	Title Page	1
2.	Index	2
3.	Abstract	3
4.	State of the Art of the Research Topic	4
5.	Problem definition	5
6.	Objective of work	5
7.	Scope of work	5
8.	Original contribution by the thesis	6
9.	Methodology of Research, Results and Comparisons	6
10.	Achievements with Respect to Objectives	19
11.	Conclusion	19
12.	Limitation and Future scope of the work	20
13.	List of Papers Published	20
14.	References	21
15.	Copies of Paper Published	24

Abstract

Tapered roller bearings (TRBs) are widely used in automotive and industrial applications for supporting combined radial and axial loads. In TRBs at high axial load, Friction torque increase rapidly which also raise the bearing temperature (BT) in tapered roller bearings (TRBs) that leads to major power loss and heat generation within bearing and ultimately lower bearing performance and reduced bearing reliability. The present study aims to experimentally evaluate and optimize friction torque (FT) by considering both operational and geometrical parameters. In TRBs, at high axial load, inner raceway rib convexity plays an important role by providing resistance to friction and support the bearing. An experimental investigation was carried out on a friction torque test rig to study the combined influence of inner raceway rib convexity, axial load, and bearing rotational speed on FT and BT. A Full Factorial Design of Experiments (DOE) was initially employed to develop predictive regression models and identify significant factors and interactions. The developed models demonstrated high predictive accuracy, with coefficients of determination exceeding 98% for both responses. Model adequacy was verified through Analysis of Variance (ANOVA) and regression diagnostics using Minitab 19. To further enhance prediction capability and optimization accuracy, Response Surface Methodology (RSM) based on a Box–Behnken Design (BBD) was implemented. The RSM models effectively captured nonlinear effects and interaction behavior, enabling precise estimation of frictional and thermal responses. Minimum friction torque and bearing temperature were predicted at a rib convexity of approximately 3–3.21 μm , an axial load of 2000 kgf, and a bearing speed of 400 rpm, corresponding to $\text{FT} \approx 7.96 \text{ Nm}$ and $\text{BT} \approx 54.57 \text{ }^{\circ}\text{C}$. For simultaneous multi-response optimization, Desirability Function Analysis (DFA) and Grey Relational Analysis (GRA) were applied and compared with Minitab-based optimization results. The comparative analysis confirmed the superiority of the RSM-BBD optimization, yielding the highest composite desirability value and ranking first among all methods. Confirmation experiments were conducted at optimal settings, and the experimental results closely matched the predicted values with deviations within $\pm 5\%$, validating the robustness of the proposed approach. Overall, the study demonstrates that the integration of Full Factorial DOE, RSM, DFA, and GRA provides a reliable and effective framework for minimizing friction torque and thermal losses in tapered roller bearings, highlighting the critical role of inner race rib convexity in improving bearing performance under severe operating conditions.

1. State of the Art of the Research Topic

Tapered roller bearings (TRBs) are widely used in aerospace, automotive, and manufacturing applications due to their high precision, low friction, and ability to operate reliably under high load and speed conditions [1], [2], [3], [4]. Friction torque (FT) is a critical performance parameter in TRBs, as it directly affects energy consumption and bearing temperature rise, thereby influencing overall bearing efficiency and durability [5]. Accurate prediction of friction torque is essential for estimating frictional power loss and heat generation in bearings [6]. Friction torque in TRBs mainly arises from rolling contact between rollers and raceways and sliding friction at the roller large-end and inner raceway rib interface, while cage–roller friction and lubricant viscous drag are comparatively minor and often neglected [7], [8], [9]. Numerous analytical and experimental studies have been conducted to model and validate friction torque under varying operating conditions. Witte developed torque prediction equations for TRBs under pure thrust, pure radial, and combined loading conditions [10], [11]. Zhou et al. proposed an analytical torque model and demonstrated good agreement with experimental results under different lubricants, loads, and speeds [8]. Bercea et al. reported that power loss in the inner ring exceeds that in the outer ring using an analytical torque loss model [12]. Matsuyama introduced simplified expressions for predicting viscous friction torque at roller–raceway contacts and validated them experimentally, later extending the model to fully flooded lubrication conditions over a wide operating range [13], [14]. Comparative studies between ball bearings and TRBs were presented by Houpert to support bearing selection for specific applications [15], while Black et al. proposed rapid experimental methods for measuring running friction torque [16]. Experimental investigations by Tiago et al. and Cousseau et al. enabled accurate monitoring of friction torque and bearing temperature under high axial loads and rotational speeds [17]. Several studies emphasized the critical role of inner raceway rib–roller contact in carrying axial loads and contributing significantly to friction torque and power loss [11], [17], [18], [19], [20]. Contact-based friction prediction methods were shown to provide accurate results across roller bearing applications [18], and analytical models validated through dedicated test rigs further confirmed these findings [20]. Advanced models incorporating elastohydrodynamic lubrication (EHL), surface roughness, raceway convexity, and defect effects were also developed to improve friction torque prediction accuracy [7], [21], [22], [23]. Recent numerical and parametric studies employed multibody simulations, evolutionary optimization, and design of experiments to assess the influence of rib geometry, roller geometry, and lubrication conditions on friction torque and bearing temperature [22], [24], [25], [26], [27]. Experimental evidence indicates that maximum temperature rise typically

occurs at the roller end–inner raceway rib interface, especially under high axial loads and speeds [28]. Despite extensive research on friction torque modeling and validation, the combined effects of high pure axial load, shaft speed, and inner raceway rib convexity have not been comprehensively addressed. These parameters are particularly critical for TRBs used in heavy-duty vehicle wheel hub assemblies, where high axial loads occur during vehicle turning, leading to increased friction torque and bearing temperature [29]. Therefore, precise evaluation of bearing load, shaft speed, and rib convexity is essential to reduce friction torque, manage thermal behavior, and enhance the service life of TRBs under such demanding operating conditions.

2. Problem Definition

Tapered roller bearings are widely used in automotive and industrial applications due to their ability to support combined radial and axial loads. However, during operation, at high bearing load, high frictional torque generated in TRBs leads to high heat generation resulting high bearing temperature, reduced performance, and may cause premature bearing failure. The magnitude of this friction torque is strongly influenced by geometric and operating parameters. Addressing this problem is essential for improving bearing performance, enhancing reliability, and supporting the development of optimized bearing operating conditions for automotive and industrial applications.

3. Objectives of the Work

The objectives of the present research are:

1. To experimentally evaluate the effect of operating parameters- axial load and rotational speed on frictional torque of tapered roller bearings with varying rib convexity.
2. To proposed a predictive and optimization model for frictional torque in tapered roller bearings within an acceptable bearing temperature range by systematically evaluating the effects of bearing axial load, rotational speed, and rib convexity.
3. To validate the proposed friction torque model through conformation of experiments.

4. Scope of the Work

The scope of the present study is limited to:

- Tapered roller bearings operating under controlled pure axial load conditions.
- A defined range of axial loads, rotational speeds, and rib convexity geometries.
- Measurement and analysis of friction torque and bearing temperature.

- Provide framework for predicting and optimizing frictional performance in TRBs using full factorial DOE (Design of Experiments), RSM (Response Surface Methodology), and DFA (Desirability function analysis), GRA (Grey Relational Analysis).

5. Original Contributions of the Thesis

The original contributions of the present research are summarized as follows:

- Development of an experimental methodology to analyse friction torque in tapered roller bearings under axial loading conditions.
- Experimental investigation of the influence of inner race rib convexity on friction torque and bearing temperature.
- Development of statistically significant predictive models for friction torque and bearing temperature using Full Factorial Design and Response Surface Methodology using MINITAB 19.
- Identification of optimal combinations of axial load, rotational speed, and rib convexity to minimize friction torque while maintaining acceptable bearing temperature.
- Experimental validation of optimized results through confirmation experiments, demonstrating the reliability of the proposed models.

6. Methodology of Research, Results and Comparisons

6.1. Experimental Procedure

The experimental setup as shown in Figure 1 consisted of a motor-driven shaft, a grease-lubricated bearing hub, a coupling with a torque sensor, a hydraulic axial loading unit, and a SCADA system for real-time measurement of friction torque, temperature, speed, and load. Three identical TRBs with geometry as shown in Figure 2 and specifications as Table 1 are selected with inner race rib convexities of 0 μm , 3 μm , and 6 μm , measured using a TALYSURF surface roughness tester, were tested. Preliminary experiments identified safe operating limits, showing that axial loads above 6000 kgf and speeds beyond 600 rpm caused bearing temperatures to exceed 120 °C which is higher than the permissible temperature range for sealed bearings according to the bearing standards and resulted in surface discoloration. Hertzian contact stress at the roller-rib interface was evaluated assuming point contact for TRBs [11], [20] using MESYS AG software, based on established contact stress formulations [30]. The analysis confirmed that rib convexity up to 6 μm maintained contact stress below 2400 MPa, while higher convexity exceeded permissible limits defined by bearing standards

[31]. Accordingly, rib convexity (A), axial load (B), and operating speed (C) were selected as control factors, each at three levels, as presented in Table 2.

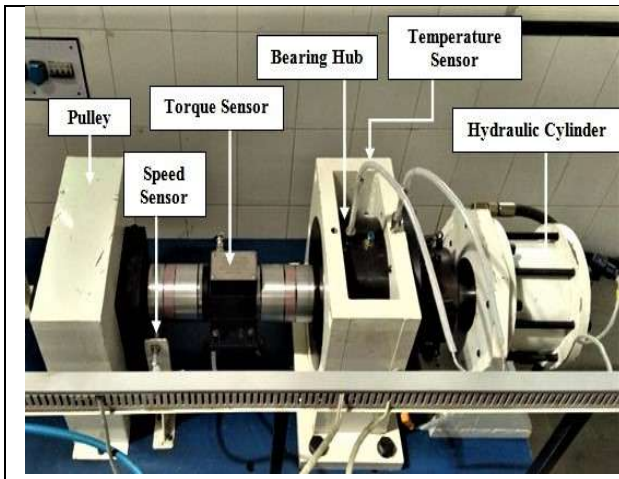


Figure 1 Experimental setup

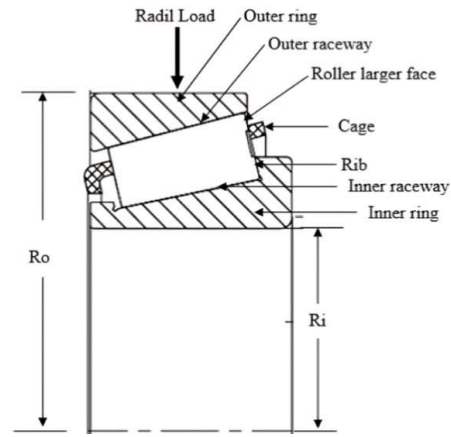


Figure 2 Tapered Roller Bearing

Table 1 Specification of Test TRBs

Nominal Outer diameter, mm	150
Nominal inner diameter, mm	90
Nominal inner ring width, mm	45
Nominal outer ring width, mm	35
Number of rollers	23
Material	SAE52100

Table 2 Factors and their selected levels

Factors	Code	Level			Unit
		-1	0	1	
Rib Convexity	A	0	3	6	μ
Axial Load	B	2000	4000	6000	kgf
Bearing Speed	C	400	500	600	rpm

6.2. Design of Experiments

Mechanical experiments always involve lots of physical effort and different kinds of costs like material costs, experimental design running costs, etc. It is also time consuming if not planned properly. Design of Experiments (DOE) is the set of methods that analyze the possibilities of experimental work with the minimum number of experiments [32], [33], saving lots of human efforts

and overall cost. Full Factorial Design (FFD), Fractional Factorial Design, Taguchi Design, and Response Surface Methodology (RSM) are popular types of DOE often preferred by researchers.

In this study, FFD and RSM were used to develop predictive models and identify significant factors, while Desirability Function Analysis and Grey Relational Analysis were applied for simultaneous optimization of multiple responses. Both statistical and multi-response optimization techniques were employed to ensure reliable and validated optimization results. The comparison between Minitab-based optimization and Desirability Function Analysis (DFA) and Grey Relational Analysis (GRA) outcomes provides cross-validation and enhances confidence in the optimized parameter settings.

6.3. Full Factorial Design (FFD) Analysis

FFD is the most widely used method that allows all the possible interaction effects between all the factors and offers higher precision in estimating factor effects [33]. In order to create a statistical model for the Friction Torque (FT) in Nm and Bearing Temperature (BT) in °C, L27 FFD with all possible interactions has been considered for the experimentation. Each experiment is replicated three times, and the average of all three readings is considered to be the response of the experiments. Minitab19 analytical software is used to analyze the interaction effects.

The experimental results listed in Table 3 along with regression model fits with percentage errors for FT and BT. The main effect plots of FT and BT shown in Figures 3 and 4, generated using MINITAB 19, reveal the interaction effects of operating and geometric parameters on the responses. Figure 3 indicates that friction torque (FT) increases with higher axial load and speed and decreases with increasing rib convexity. FT is maximum at 0 μ rib convexity, 6000 kgf axial load, and 600 rpm, and minimum at 6 μ rib convexity, 2000 kgf axial load, and 400 rpm. However, Figure 4 shows that bearing temperature (BT) can increase at certain convexity levels due to higher contact stress between the rib and roller tip as convexity decreases. BT also increases with increasing load and speed.

The interaction plots for FT and BT shown in Figures 5 and 6, based on Table 3, illustrate the combined effects of independent variables. In Figure 5, the non-parallel lines in A×B and A×C plots indicate a strong interaction effect of rib convexity on FT with axial load and speed, while the parallel lines in the B×C plot show a weaker interaction. Figure 6 presents similar interaction trends for BT. Overall, rib convexity has a dominant influence on both FT and BT; increasing convexity reduces FT (Figure 5), whereas BT increases at higher convexity levels (Figure 6).

Table 3 L27 Full Factorial DOE with Experimental Results FT and BT

Sr. No	Factors			Experiment		Regression		Percentage Error	
	A	B	C	FT	BT	FT FITS	BT FITS	FT	BT
1	1	1	1	12	65	11.8611	66.2315	1.16	1.89
2	1	1	2	14	72	13.7500	72.0926	1.79	0.13
3	1	1	3	15	79	14.6389	78.7315	2.41	0.34
4	1	2	1	13	74	13.2222	73.4259	1.71	0.78
5	1	2	2	15	80	15.4444	80.0370	2.96	0.05
6	1	2	3	16	87	16.6667	87.4259	4.17	0.49
7	1	3	1	15	79	14.5833	79.0648	2.78	0.08
8	1	3	2	17	86	17.1389	86.4259	0.82	0.50
9	1	3	3	19	96	18.6944	94.5648	1.61	1.49
10	2	1	1	8	55	8.0000	53.7870	0.00	2.21
11	2	1	2	10	61	10.2222	59.4815	2.22	2.49
12	2	1	3	11	65	11.4444	65.9537	4.04	1.47
13	2	2	1	9	61	9.1111	60.1481	1.23	1.40
14	2	2	2	12	67	11.6667	66.5926	2.78	0.61
15	2	2	3	14	73	13.2222	73.8148	5.56	1.12
16	2	3	1	10	64	10.2222	64.9537	2.22	1.49
17	2	3	2	13	72	13.1111	72.1481	0.85	0.21
18	2	3	3	15	79	15.0000	80.1204	0.00	1.42
19	3	1	1	7	67	7.1389	68.1204	1.98	1.67
20	3	1	2	10	73	9.6944	73.6481	3.06	0.89
21	3	1	3	11	81	11.2500	79.9537	2.27	1.29
22	3	2	1	8	74	8.0000	73.6481	0.00	0.48
23	3	2	2	11	79	10.8889	79.9259	1.01	1.17
24	3	2	3	13	87	12.7778	86.9815	1.71	0.02
25	3	3	1	9	78	8.8611	77.6204	1.54	0.49
26	3	3	2	12	85	12.0833	84.6481	0.69	0.41
27	3	3	3	14	93	14.3056	92.4537	2.18	0.59

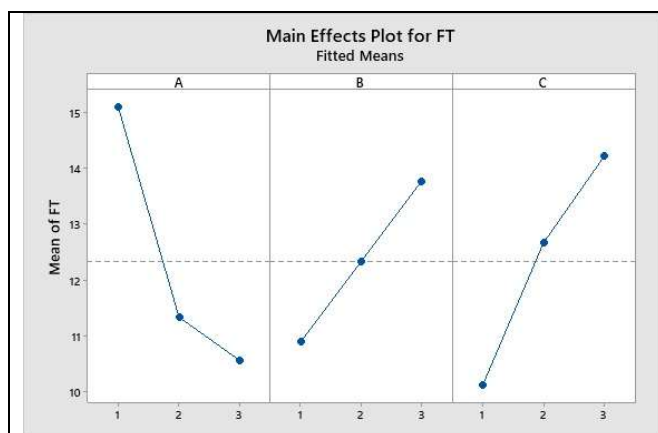


Fig.3 Main effective Plots for FT

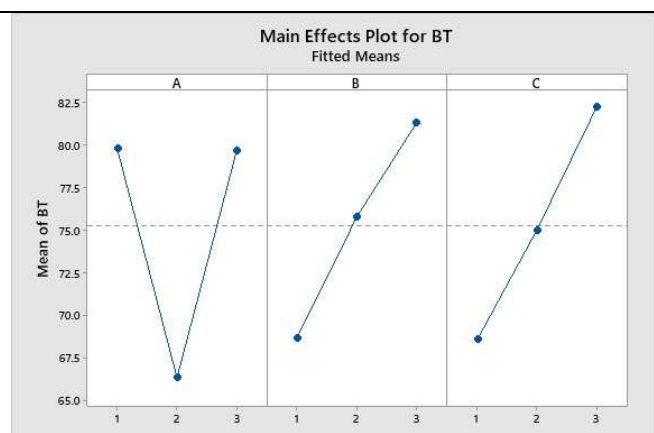


Fig. 4 Main effective plots for BT

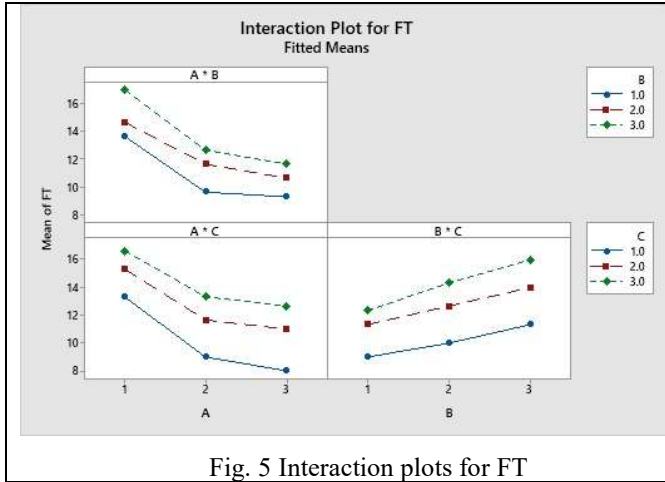


Fig. 5 Interaction plots for FT

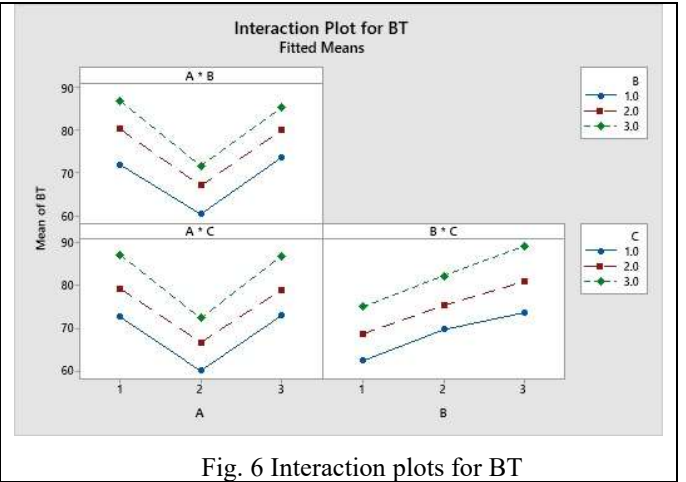


Fig. 6 Interaction plots for BT

6.4. Analysis of Variance (ANOVA)

Analysis of variance [34], [35], [36] was used to evaluate the effects of axial load, speed, and rib convexity on friction torque (FT) and bearing temperature (BT), and to assess the adequacy of the developed regression models. Two-way interaction terms were included to enhance model predictability. The ANOVA results for FT and BT are presented in Tables 4 and 5. The regression P-values for both models are 0, which are below the significance level ($\alpha = 0.05$), confirming that the models are statistically significant. Factors with P-values greater than 0.05 are considered insignificant. In Table 4, terms B and B² are insignificant for FT, while in Table 5, terms B², C², and A×C are insignificant for BT.

Table 4 ANOVA table for FT					
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	9	225.417	25.0463	164.82	0.000
A	1	21.042	21.0419	138.47	0.000
B	1	0.482	0.4818	3.17	0.093
C	1	2.187	2.1867	14.39	0.001
A*A	1	13.500	13.5000	88.84	0.000
B*B	1	0.000	0.0000	0.00	1.000
C*C	1	1.500	1.5000	9.87	0.006
A*B	1	0.750	0.7500	4.94	0.040
A*C	1	1.333	1.3333	8.77	0.009
B*C	1	1.333	1.3333	8.77	0.009
Error	17	2.583	0.1520		
Total	26	228.000			

Table 5 ANOVA table for BT					
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	9	2658.08	295.34	293.58	0.000
A	1	786.01	786.01	781.33	0.000
B	1	27.26	27.26	27.10	0.000
C	1	4.99	4.99	4.96	0.040
A*A	1	1075.57	1075.57	1069.17	0.000
B*B	1	3.63	3.63	3.61	0.075
C*C	1	0.91	0.91	0.90	0.356
A*B	1	8.33	8.33	8.28	0.010
A*C	1	0.33	0.33	0.33	0.572
B*C	1	6.75	6.75	6.71	0.019
Error	17	17.10	1.01		
Total	26	2675.19			

The Pareto charts shown in Figures 7 and 8 further confirm that these terms, located to the left of the reference line, have negligible effects and can be omitted in MINITAB 19. The normal probability plots of residuals for FT and BT (Figures 9 and 10) show Anderson–Darling P-values greater than 0.05, indicating normally distributed residuals and validating the models. The close agreement between experimental and predicted results is also evident from Figures 11 and 12 with maximum percentage errors of 5.56% for FT and 2.49% for BT, demonstrating strong correlation and good predictive accuracy of the regression models.

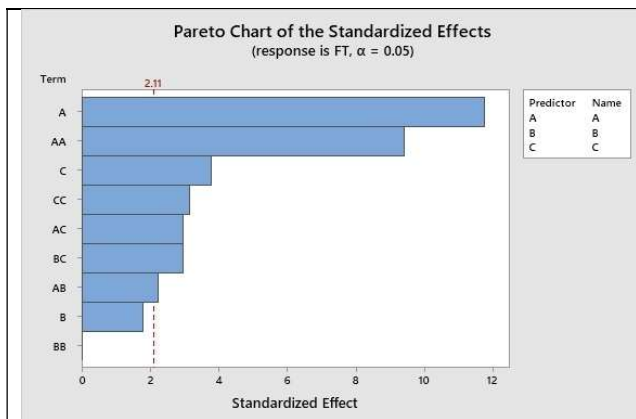


Fig. 7 Pareto Chart of the standardized effect for FT

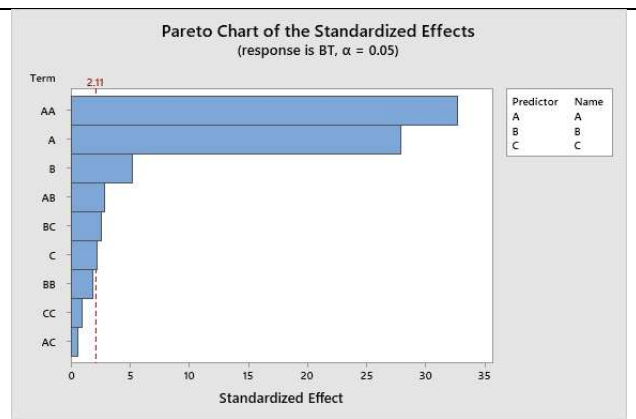


Fig. 8 Pareto Chart of the standardized effect for BT

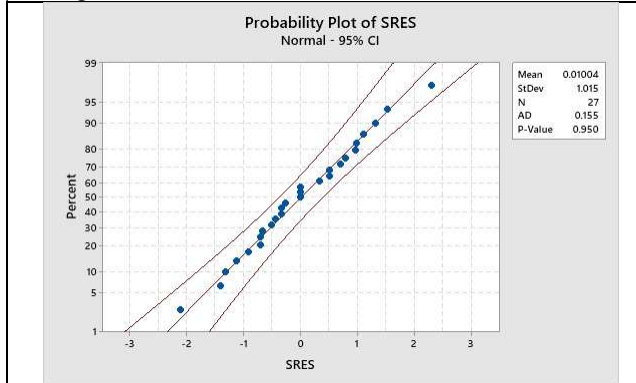


Fig. 9 Normal Probability Plot for residuals a) Friction Torque

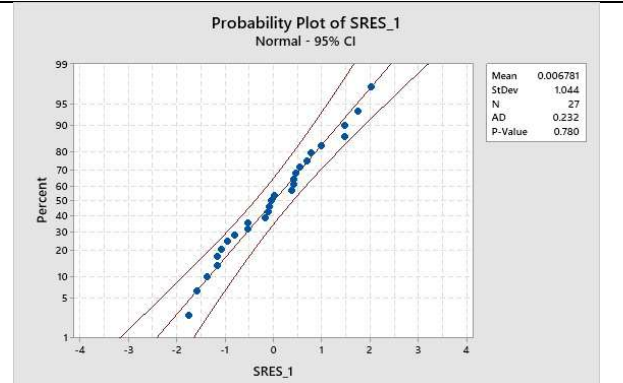


Fig. 10 Normal Probability Plot for residuals for Bearing Temperature

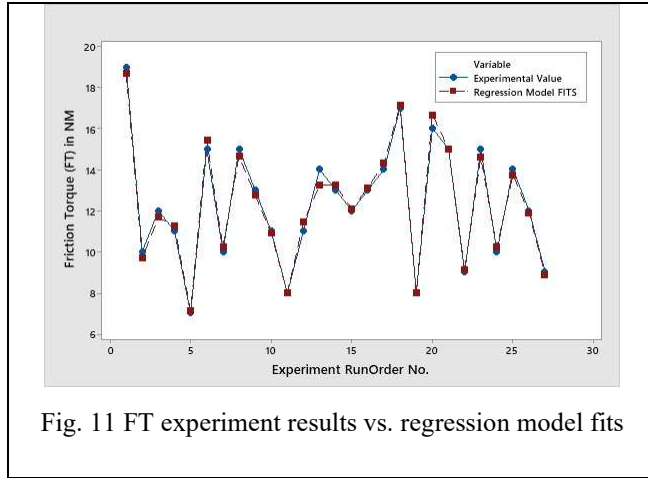


Fig. 11 FT experiment results vs. regression model fits

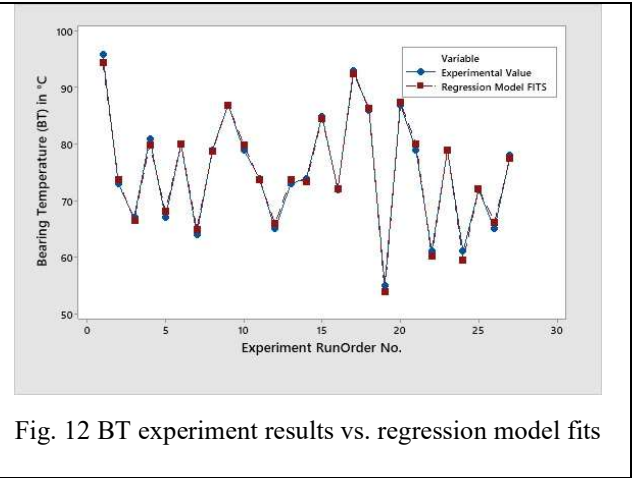


Fig. 12 BT experiment results vs. regression model fits

Table 6 Regression Model for FT and BT using FFD

Responses	Correlation
Friction Torque	$= 14.89 - 8.444 A + 1.278 B + 2.722 C + 1.500 A * A - 0.500 C * C - 0.250 A * B + 0.333 A * C + 0.333 B * C$
	$R^2 = 98.87\%, R^2(\text{adj}) = 98.27\%$
Bearing Temperature	$= 91.37 - 51.61 A + 9.61 B + 4.11 C + 13.389 A * A - 0.778 B * B + 0.389 C * C - 0.833 A * B - 0.167 A * C + 0.750 B * C$
	$R^2 = 99.36\%, R^2(\text{adj}) = 95.15\%$

FFD based regression analysis is carried out using Minitab 19 with 95% confidence level. This developed regression models for FT and BT are as shown in Table 6. Here R^2 shows the variation measures in regression model and $R^2(\text{adj})$ shows the variation measures for multiple regression model. Both are used to check the integrity of the model. R^2 value can change or may remain same as the predictor add in the model while $R^2(\text{adj})$ can adjust the value only if the new predictor improve the model. So, for multiple regression analysis $R^2(\text{adj})$ is considered as more reliable for the goodness of fit of the model. Here as shown in Table 6 FT model show $R^2 = 98.87\%$ and $R^2(\text{adj}) = 98.27\%$ and BT models shows $R^2 = 99.36\%$ and $R^2(\text{adj}) = 95.15\%$. Both R^2 and $R^2(\text{adj})$ values are higher than 95% and hence it represents that both models are adequately fit with the experimental results.

6.5. Response Surface Methodology (RSM)

Among various DOE techniques Response Surface Methodology (RSM) are widely applied in engineering and manufacturing studies [37], [38], [39] particularly effective for modeling, analyzing, and optimizing processes involving multiple variables, enabling efficient experimentation and reliable interpretation of results [37], [38], [39], [40]. RSM establishes a mathematical relationship between the response variable and the input parameters using a second-order polynomial model which includes linear, interaction, and quadratic terms to capture curvature effects and identify optimal conditions [33]. RSM designs mainly include Central Composite Design (CCD) and Box–Behnken Design (BBD). The Box–Behnken design employs three-level factors (−1, 0, +1) and combines factorial and incomplete block designs, offering efficient experimentation with fewer runs

for higher-order models [32], [41], [42], [43]. In the present study, a Box–Behnken Design was adopted, and the coded design matrix with corresponding responses is presented in Table 7.

Table 7 BBD with coded factors with responses

Run Order	A	B	C	Friction Torque (Nm)	Bearing Temperature (°C)
1	-1	-1	0	14	72
2	1	-1	0	10	73
3	-1	1	0	17	86
4	1	1	0	12	85
5	-1	0	-1	13	74
6	1	0	-1	8	74
7	-1	0	1	16	87
8	1	0	1	13	87
9	0	-1	-1	8	55
10	0	1	-1	10	64
11	0	-1	1	11	65
12	0	1	1	15	79
13	0	0	0	12	78
14	0	0	0	12	74
15	0	0	0	12	76

Table 9 ANOVA Table for Box Behnken Design

Source	DF	Friction Torque		Bearing Temperature	
		SS	P-Value	SS	P-Value
Model	9	96.1500	0.000	1161.68	0.000
Linear	3	83.2500	0.000	625.25	0.000
A	1	36.1250	0.000	0.00	1.000
B	1	15.1250	0.000	300.13	0.000
C	1	32.0000	0.000	325.13	0.000
Square	3	10.6500	0.000	529.18	0.000
A*A	1	6.9808	0.000	290.83	0.000
B*B	1	0.0577	0.332	127.44	0.000
C*C	1	2.8269	0.001	70.67	0.002
2-Way Interaction	3	2.2500	0.006	7.25	0.369
A*B	1	0.2500	0.076	1.00	0.495
A*C	1	1.0000	0.007	0.00	1.000
B*C	1	1.0000	0.007	6.25	0.125
Error	5	0.2500		9.25	
Total	14	96.4000		1170.93	
R-square		99.74%		99.21%	
R-square (adj)		99.27%		97.79%	
R-square (pred)		95.85%		96.75%	

Table 8 Response Surface Regression (RSR) Model

Responses	Correlation
Friction Torque	$= -12.63 - 2.292 A - 0.000187 B + 0.0925 C + 0.1528 A*A - 0.000000 B*B - 0.000088 C*C - 0.000042 A*B + 0.001667 A*C + 0.000002 B*C$
Bearing Temperature	$= -80.6 - 5.58 A + 0.01194 B + 0.4763 C + 0.9861 A*A - 0.000001 B*B - 0.000437 C*C - 0.000083 A*B - 0.000000 A*C + 0.000006 B*C$

Table 8 represents the RSR model for friction torque and bearing temperature, obtained through the RSM method using MINITAB 19. These response models signify the effect of independent variables on responses by establishing a correlation between them. ANOVA was used to evaluate the effects of axial load (A), speed (B), and rib convexity (C) on friction torque and bearing temperature, as well as to assess model adequacy as shown in Table 9. The models include two-way interaction terms to enhance predictive accuracy. For friction torque, the model explains 99.74% of the variation, with adjusted R² values of 99.27%, indicating high reliability. Factors A, B, and C are significant (p < 0.05), while interaction and second-order terms are insignificant. For bearing temperature, the model shows an R² of 99.21%, with adjusted R² values of 97.79%, confirming excellent fit. Speed (B) and

rib convexity (C) are significant factors, whereas other terms have negligible effects. Overall, both models effectively explain the variation in the responses.

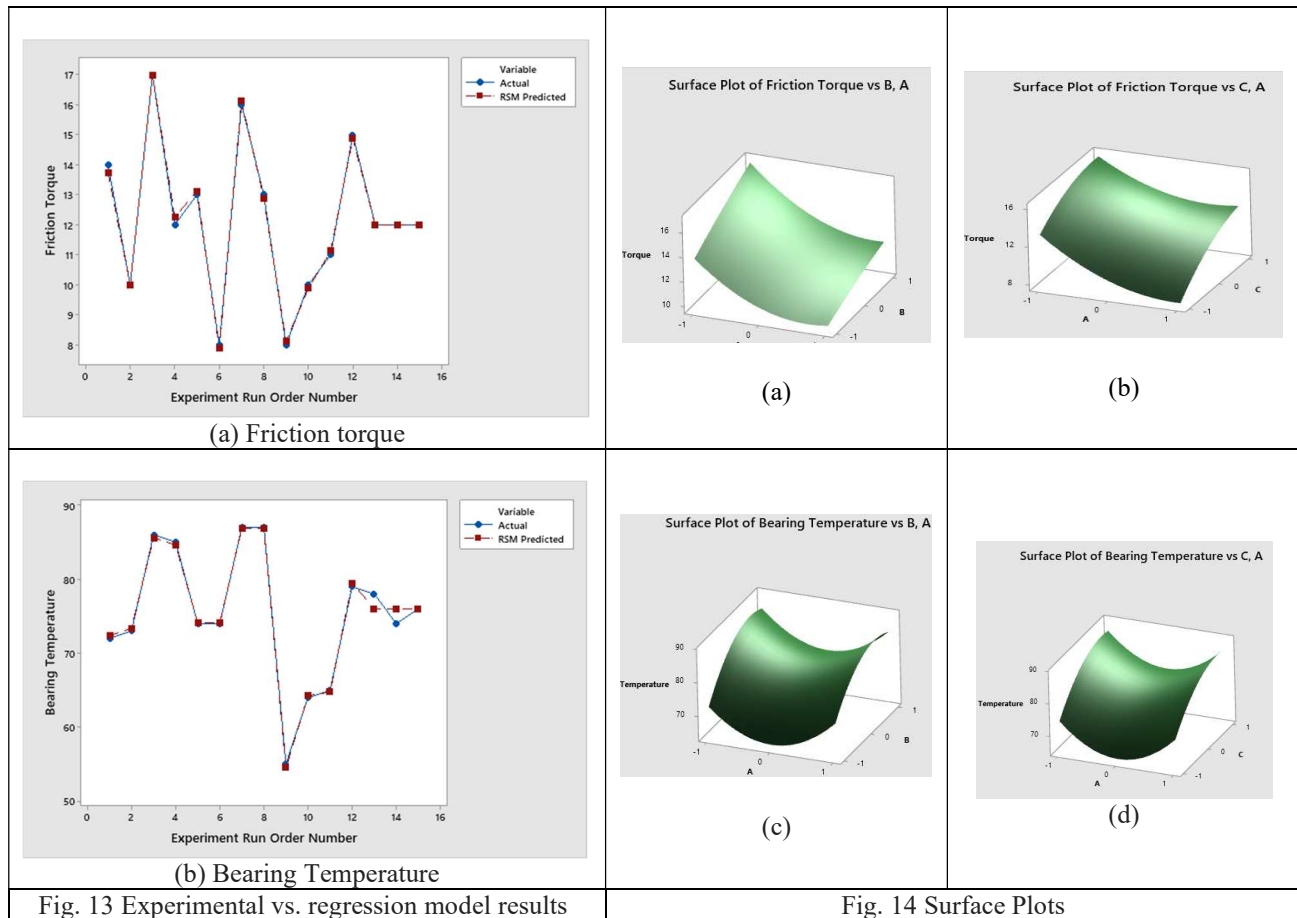


Figure 13 provides a graphical comparison, illustrating how well the model predictions align with the experimental results. Figure 14 presents surface plots illustrating the effects of process parameters on friction torque and bearing temperature. These plots aid in identifying extreme response values and optimal factor combinations. Figures 14 (a) and (b) show that friction torque increases at 0 μm rib convexity with increasing axial load (2000–6000 kgf) and speed (up to 600 rpm), while appropriate combinations of rib convexity and axial load minimize torque. Figures 14 (c) and (d) indicate that bearing temperature remains minimum at a rib convexity of 3 μm and a speed of 400 rpm.

6.6. Multi Response Optimization Methods and Comparisons:

In this study, four different optimization approaches were employed to achieve simultaneous minimization of friction torque and bearing temperature. These include Full Factorial optimization, Desirability Function Analysis (DFA), Response Surface Methodology based Box–Behnken Design (RSM-BBD) optimization, and RSM-BBD–based Grey Relational Analysis (GRA). The Full Factorial and RSM-BBD optimizations were performed using the response optimizer in Minitab 19 as shown in Figure 15 and Figure 16 respectively, while DFA and GRA were applied as multi-response decision-making techniques. DFA converts multiple responses into a single composite

desirability function as shown Table 10, whereas GRA evaluates overall system performance using the Grey Relational Grade Table 11. The comparative results of these methods are summarized in Table 12. Full Factorial optimization and DFA resulted in the same optimal factor combination (A2B1C1), demonstrating consistency between the approaches. Among all methods, RSM-BBD optimization yielded the highest composite desirability value ($D = 1.000$), indicating the best compromise between the responses. The RSM-BBD-based GRA method produced a comparable solution with a slightly lower Grey Relational Grade. The close agreement among the optimization results obtained from Minitab 19, DFA, and GRA confirms the robustness and reliability of the selected optimal parameter settings.

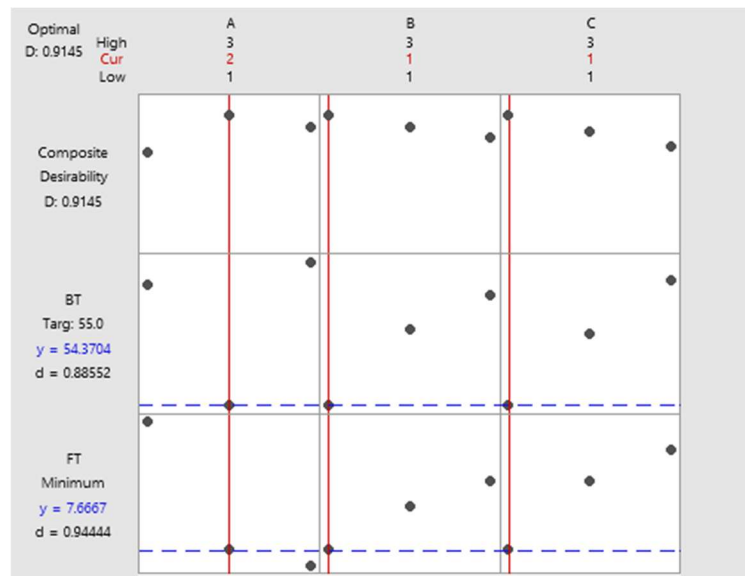


Fig. 15 FFD Optimization using MINITAB19

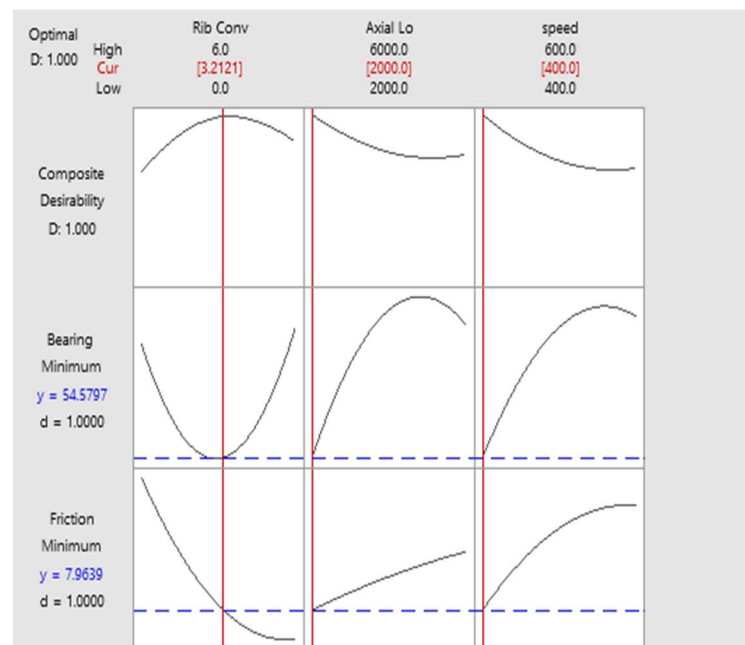


Fig. 16 BBD RSM Optimization Output

Table 10 Optimization using DFA

Ex. No	Factors			Experiment Value		Desirability Index		Overall Desirability (D)	Rank
	A	B	C	FT (Nm)	BT (°C)	FT	BT		
1	1	1	1	11.8611	66.2315	0.76899	0.83356	0.89478	9
2	1	1	2	13.7500	72.0926	0.65413	0.74235	0.83477	16
3	1	1	3	14.6389	78.7315	0.59242	0.62312	0.77947	19
4	1	2	1	13.2222	73.4259	0.68816	0.71999	0.83898	13
5	1	2	2	15.4444	80.037	0.53033	0.59688	0.75008	22
6	1	2	3	16.6667	87.4259	0.41890	0.41841	0.64704	24
7	1	3	1	14.5833	79.0648	0.59646	0.61653	0.77873	20
8	1	3	2	17.1389	86.4259	0.36689	0.44676	0.63629	25
9	1	3	3	18.6944	94.5648	0.00000	0.00000	0.00000	27
10	2	1	1	8.0000	53.787	0.96202	1.00000	0.99037	1
11	2	1	2	10.2222	59.4815	0.85626	0.92755	0.94403	4
12	2	1	3	11.4444	65.9537	0.79209	0.83764	0.90252	7
13	2	2	1	9.1111	60.1481	0.91067	0.91870	0.95639	2
14	2	2	2	11.6667	66.5926	0.77985	0.82823	0.89648	8
15	2	2	3	13.2222	73.8148	0.68816	0.71334	0.83704	15
16	2	3	1	10.2222	64.9537	0.85626	0.85215	0.92423	5
17	2	3	2	13.1111	72.1481	0.69511	0.74144	0.84729	12
18	2	3	3	15.0000	80.1204	0.56543	0.59517	0.76165	21
19	3	1	1	7.1389	68.1204	1.00000	0.80529	0.94730	3
20	3	1	2	9.6944	73.6481	0.88252	0.71620	0.89164	10
21	3	1	3	11.2500	79.9537	0.80264	0.59859	0.83255	17
22	3	2	1	8.0000	73.6481	0.96202	0.71620	0.91108	6
23	3	2	2	10.8889	79.9259	0.82188	0.59916	0.83770	14
24	3	2	3	12.7778	86.9815	0.71555	0.43124	0.74531	23
25	3	3	1	8.8611	77.6204	0.92248	0.64462	0.87814	11
26	3	3	2	12.0833	84.6481	0.75638	0.49314	0.78150	18
27	3	3	3	14.3056	92.4537	0.61628	0.22753	0.61193	26

Table. 12 Comparisons of Optimizations Techniques

Method	A	B	C	BT Fit	FT Fit	Composite Desirability / GRG	Rank	Remarks
Full Factorial Optimization (Using MINITAB 19)	3	2000	400	54.3704	7.6667	0.91451	3	
DFA	3	2000	400	53.787	8.000	0.99037	2	
RSM-BBD Optimization (Using MINITAB 19)	3.2121	2000	400	54.5797	7.9639	1.00000	1	Best composite value
RSM-BBD based GRA	3	2000	400	54.5	8.125	0.97403	4	

Table 11 Optimization- RSM Based GRA

Factors and Its Levels			Experimental		RSM Regression Model		Normalization		Deviation Sequence		Grey Relational Coefficient		GRG	Rank
A	B	C	FT	BT	FT FITS	BT FITS	FT	BT	FT	BT	FT	BT		
-1	-1	0	14	72	13.75	72.375	0.3562	0.4479	0.6438	0.5521	0.4371	0.4752	0.4562	10
1	-1	0	10	73	10	73.375	0.7671	0.4170	0.2329	0.5830	0.6822	0.4617	0.5720	5
-1	1	0	17	86	17	85.625	0.0000	0.0386	1.0000	0.9614	0.3333	0.3421	0.3377	15
1	1	0	12	85	12.25	84.625	0.5205	0.0695	0.4795	0.9305	0.5105	0.3495	0.4300	11
-1	0	-1	13	74	13.125	74.125	0.4247	0.3938	0.5753	0.6062	0.4650	0.4520	0.4585	9
1	0	-1	8	74	7.875	74.125	1.0000	0.3938	0.0000	0.6062	1.0000	0.4520	0.7260	2
-1	0	1	16	87	16.125	86.875	0.0959	0.0000	0.9041	1.0000	0.3561	0.3333	0.3447	14
1	0	1	13	87	12.875	86.875	0.4521	0.0000	0.5479	1.0000	0.4771	0.3333	0.4052	12
0	-1	-1	8	55	8.125	54.5	0.9726	1.0000	0.0274	0.0000	0.9481	1.0000	0.97403	1
0	1	-1	10	64	9.875	64.25	0.7808	0.6988	0.2192	0.3012	0.6952	0.6241	0.6597	3
0	-1	1	11	65	11.125	64.75	0.6438	0.6834	0.3562	0.3166	0.5840	0.6123	0.5981	4
0	1	1	15	79	14.875	79.5	0.2329	0.2278	0.7671	0.7722	0.3946	0.3930	0.3938	13
0	0	0	12	78	12	76	0.5479	0.3359	0.4521	0.6641	0.5252	0.4295	0.4773	6
0	0	0	12	74	12	76	0.5479	0.3359	0.4521	0.6641	0.5252	0.4295	0.4773	6
0	0	0	12	76	12	76	0.5479	0.3359	0.4521	0.6641	0.5252	0.4295	0.4773	6

6.7. Confirmation of Experiments

Confirmation experiments were conducted to validate the optimal parameter settings obtained from Full Factorial Design (FFD) and Response Surface Methodology (RSM) optimization as shown in Table 13 and Table 14 respectively. For the FFD-based optimum solutions, experimental results closely matched the predicted values of friction torque (FT) and bearing temperature (BT), with

deviations limited to 1.57–4.25% for FT and 2.78–3.65% for BT. Similarly, the RSM-based optimal solutions exhibited good agreement between predicted and experimental responses, with deviation percentages ranging from 0.45–1.64% for FT and 2.54–3.99% for BT. The low deviation values observed for both optimization approaches confirm the adequacy, reliability, and predictive accuracy of the optimized parameter settings. Overall, the confirmation tests validate that the proposed optimization methodologies are effective in minimizing friction torque while maintaining bearing temperature within the acceptable operating range.

Table.13 Conformation of Experiments for FFD Optimum Solutions

FFD Optimum Solutions	A (μ)	B (kgf)	C (rpm)		FT (Nm)	BT ($^{\circ}$ C)
1	2.8	2000	400	Predicted	7.66	54.37
				Experimental	8	56
				Deviation %	4.25	2.91
2	6.1	2000	400	Predicted	7.11	67.37
				Experimental	7	65
				Deviation %	1.57	3.65
3	3	4000	400	Predicted	9.22	61.25
				Experimental	9	63
				Deviation %	2.44	2.78

Table. 14 Conformation of Experiments for RSM BBD based predicted

RSM BBD based	A (μ)	B (kgf)	C (rpm)		FT (Nm)	BT ($^{\circ}$ C)
1	3.2	2000	400	Predicted	7.96	54.57
				Experimental	8	56
				Deviation %	0.45	2.54
2	4.3	6000	400	Predicted	8.86	65.84
				Experimental	9	68
				Deviation %	1.60	3.18
3	0.6	2000	400	Predicted	10.82	59.53
				Experimental	11	62
				Deviation %	1.64	3.99

7. Achievements with Respect to Objectives

Objective	Achievement
1. To experimentally evaluate the effect of axial load and rotational speed on frictional torque of tapered roller bearings with varying rib convexity.	Experimental investigations using Full Factorial DOE and RSM-BBD successfully quantified the influence of axial load, speed, and rib convexity on friction torque and bearing temperature. Results showed a significant increase in friction torque with axial load and speed, while rib convexity strongly influenced frictional and thermal behavior under high-load conditions.
2. To propose a predictive and optimization model for frictional torque within an acceptable bearing temperature range.	Accurate predictive models were developed using Full Factorial DOE and RSM in Minitab 19, achieving over 98% prediction accuracy. Multi-response optimization using DFA and GRA identified the optimal condition at rib convexity $\approx 3\text{--}3.21\text{ }\mu\text{m}$, axial load 2000 kgf, and speed 400 rpm, yielding minimum friction torque and bearing temperature.
3. To validate the proposed friction torque model through confirmation experiments.	Confirmation experiments conducted at optimal and selected parameter combinations showed close agreement with predicted results, with deviations within $\pm 5\%$, confirming the robustness and reliability of the developed models.

8. Conclusion

The present experimental investigation systematically evaluated the influence of operating parameters—axial load and bearing speed—together with the geometrical parameter of inner race rib convexity on friction torque (FT) and bearing temperature (BT) in tapered roller bearings (TRBs). A Full Factorial Design and Response Surface Methodology (RSM) of experiments was employed to develop reliable regression models, analyze parameter interactions and determine optimal conditions for minimizing power loss and heat generation. Both the models successfully predicted the bearing performance, identifying the optimal parametric combination at a rib convexity of approximately 3–3.21 μm , an axial load of 2000 kgf, and a bearing speed of 400 rpm. Under these conditions, minimum friction torque and bearing temperature were achieved. To address the simultaneous optimization of multiple responses, the desirability function analysis (DFA) and Grey Relational Analysis (GRA) methods was applied, which confirmed the same optimal combination within the selected experimental range. The predicted optimal results were validated through confirmation experiments, including additional parametric combinations, with deviations within $\pm 5\%$ from experimental observations. This close agreement demonstrates the robustness and accuracy of the

proposed statistical approach. Overall, the study confirms that integrating full factorial DOE, RSM, and DFA, GRA provides an effective framework for predicting and optimizing frictional performance in TRBs, and highlights the critical role of rib convexity in reducing friction torque and thermal losses under high axial load and high-speed operating conditions.

9. Limitations and Future Scope of Work

Limitation:

The study is conducted for a specific TRBs with specific material, geometry and operating ranges under various assumptions and considerations. Hence, direct generalization of the results to different bearing sizes, profiles, or materials may require additional work.

Future Scope of the Work:

1. Future studies can extend the friction torque analysis with combine axial and radial loads on TRBs to enhance model applicability in other real-world applications.
2. Coupling friction torque models with bearing dynamic models can help analyze instability, skidding, and vibration behaviour, especially under high-speed and light load conditions.
3. Hybrid approaches combining DOE with machine learning techniques (e.g., ANN- Artificial Neural Network, SVR-Support Vector Regression) may enhance prediction capability and reduce experimental effort compared to conventional DOE.

10. List of Paper Published

- [1] A. B. Khant and J. K. Dadhaniya, "Influence of Operating Parameters and Rib Convexity on Friction Torque of Taper Roller Bearings," SSRG Int. J. Mech. Eng., vol. 11, no. 11, pp. 34–46, 2024. (Scopus Indexed, ISSN: 2348-8360)
- [2] A. B. Khant and J. K. Dadhaniya, "A Review on Friction Torque of Tapered Roller Bearings," Int. J. Eng. Res. Technol., vol. 14, no. 05, 2025. (ISSN: 2278-0181)
- [3] A. B. Khant, J. K. Dadhaniya, and H. G. Chothani, "Parametric Optimization of Friction Torque and Bearing Temperature in Tapered Roller Bearings Using Response Surface Methodology," J. Eng. Technol. Ind. Appl., vol. 11, no. 54, pp. 120–130, 2025. (Scopus Indexed, ISSN/ISBN: 2447-0228)

References

- [1] T. A. Harris and M. N. Kotzalas, *Essential Concepts of Bearing Technology*. 2006.
- [2] T. A. Harris and M. N. Kotzalas, *Advanced Concepts of Bearing Technology*. Taylor & Francis Group, LLC, 2006.
- [3] T. A. Harris, "Rolling Bearing Analysis," *CRC Press. New York*, p. pp 147–154, 2001.
- [4] T. N. Wang, H. T. Li, and J. X. et al Wei, "Evaluating Stress Impact on Roller Bearing Within Disc Cutters of Tunnel Boring Machines," *Strength Mater.*, vol. 56, pp. 1179–1192, 2025.
- [5] Palmgren A, "Ball and roller bearing engineering," *Burbank, Philadelphia*, p. pp 30–39, 1959.
- [6] H. G. Chothani, D. J. Marsonia, N. N. Jadedda, and S. H. Zala, "LUBRICATED WORM GEARBOX ON CHURNING POWER LOSSES," *J. Eng. Technol. Ind. Appl.*, vol. 10, no. 49, pp. 137–143, 2024, doi: DOI: <https://doi.org/10.5935/jetia.v10i49.1123>.
- [7] S. Aihara, "A New Running Torque Formula for TRB under axial load," *J. Tribol. ASME*, vol. 109, 1987.
- [8] R. S. Zhou and M. R. Hoeprich, "Torque of Tapered Roller Bearings," *J. Tribol. ASME*, 1991, [Online]. Available: <http://asme.org/terms>
- [9] M. Manjunath, D. Fauconnier, and W. Ost, "Experimental Analysis of Rolling Torque and Thermal Inlet Shear Heating in Tapered Roller Bearings," *Machines*, vol. 11, 801, pp. 1–22, 2023.
- [10] D. C. Witte, "Operating torque of tapered roller bearings," *ASLE Trans.*, vol. 16, no. 1, pp. 61–67, 1973, doi: 10.1080/05698197308982705.
- [11] D. C. Witte and H. E. Hill, "Tapered roller bearing torque characteristics with emphasis on rib-roller end contact," *SAE Tech. Pap.*, 1987, doi: 10.4271/871984.
- [12] I. Bercea, N. Mitu, I. Damian, M. Mercea, and S. Cretu, "A theoretical and experimental study on friction losses in tapered roller bearing," *Tribol. Ind.*, vol. 18, no. No 1, 1996.
- [13] H. Matsuyama, S. Kamamoto, and K. Asano, "The analysis of frictional torque for tapered roller bearings using EHD theory," *SAE Tech. Pap.*, no. 724, 1998, doi: 10.4271/982029.
- [14] H. Matsuyama and S. Kamamoto, "Analysis of Frictional Torque in Raceway Contacts of Tapered Roller Bearings," *KOYO Eng. J. English Ed.*, vol. 159, no. 159E, pp. 53–60, 2001.
- [15] L. Houpert, "Ball bearing and tapered roller bearing torque: analytical, numerical and experimental results," *Tribol. Trans.*, vol. 45, no. 3, pp. 345–353, 2002, doi: 10.1080/10402000208982559.
- [16] J. J. Blake and C. E. Truman, "Measurement of running torque of tapered roller bearings," *Proc. Inst. Mech. Eng. Part J J. Eng. Tribol.*, vol. 218, no. 4, pp. 239–249, 2004, doi: 10.1243/1350650041762659.
- [17] T. Cousseau, B. Graça, A. Campos, and J. Seabra, "Experimental measuring procedure for the friction torque in rolling bearings," *Lubr. Sci.*, vol. 22, no. 4, pp. 133–147, 2010, doi: 10.1002/lis.115.
- [18] M. Zander, M. Otto, T. Lohner, and K. Stahl, "Evaluation of friction calculation methods for rolling bearings," *Forsch. im Ingenieurwes. Springer*, vol. 87, no. 4, pp. 1307–1316, Dec. 2023, doi: 10.1007/s10010-023-00715-1.
- [19] W. E. Jamison, J. J. Kauzlarich, and E. V. Mochel, "Geometric effects on the rib-roller contact in tapered roller bearings," *ASLE Trans.*, vol. 20, no. 1, pp. 79–88, 1977, doi: 10.1080/05698197708982820.
- [20] C. L. Karna, "Performance characteristics at the rib roller end contact in tapered roller bearings," *ASLE Trans.*, vol. 17, no. 1, pp. 14–21, 1974, doi: 10.1080/05698197408981434.
- [21] Y. Liu, W. Kang, Y. Zhu, K. Yan, and J. Hong, "Effects of Defect on Roller-Raceway Contact State and Friction Torque of Tapered Roller Bearings," *J. Tribol.*, vol. 142, no. 11, Nov. 2020,

- doi: 10.1115/1.4047194.
- [22] S. Wirsching, M. Marian, M. Bartz, T. Stahl, and S. Wartzack, "Geometrical optimization of the ehl roller face/rib contact for energy efficiency in tapered roller bearings," *Lubricants*, vol. 9, no. 67, pp. 1–19, 2021, doi: 10.3390/lubricants9070067.
 - [23] L. Liu, W. Liu, K. Wang, D. Wang, and Y. Wang, "Effects of Raceway Convexity on Friction Moment of Tapered Roller Bearings," *J. Phys. Conf. Ser. Pap.*, 2022, doi: 10.1088/1742-6596/2174/1/012047.
 - [24] P. Wingerts Zahn, O. Koch, L. Maccioni, F. Concli, and B. Sauer, "Predicting Friction of Tapered Roller Bearings with Detailed Multi-Body Simulation Models," *Lubricants*, vol. 11, no. 9, 2023, doi: 10.3390/lubricants11090369.
 - [25] P. L. Wu *et al.*, "Theoretical calculation models and measurement of friction torque for rolling bearings: state of the art," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 44, no. 9, 2022, doi: 10.1007/s40430-022-03726-1.
 - [26] I. M. Klebanov, A. M. Brazhnikova, and K. A. Polyakov, "Tapered Roller Bearing Rib-Roller End Interaction at Hydrodynamic Contact," *J. Frict. Wear*, vol. 43, no. 6, pp. 391–397, 2022, doi: 10.3103/S106836662206006X.
 - [27] S. Lee and H. G. An, "Evaluation of Friction Torque in Tapered Roller Bearings," *SAE Int.*, p. 6, 2024, doi: <https://doi.org/10.4271/2024-01-3047>.
 - [28] X. Zhou, Z. Ge, X. Li, M. Rao, and Q. Han, "Thermal Analysis of Grease-Lubricated Double Row Tapered Roller Bearings," *Lubricants*, vol. 13(2), 2025, doi: <https://doi.org/10.3390/lubricants13020084>.
 - [29] B. Martin and H. E. Hill, "Design and selection factors for automatic transaxle tapered roller bearings," *SAE Tech. Pap.*, 1992, doi: 10.4271/920609.
 - [30] M. Gärtner *et al.*, "Evaluation of hybrid spindle bearings with nitriding raceway steels under high rotational speeds," *Tribol. Mater.*, vol. 2, no. 3, pp. 88–98, 2023, doi: 10.46793/tribomat.2023.012.
 - [31] SKF, "SKF GENERAL CATALOGUE," SKF: Gothenburg, Sweden, 2003. [Online]. Available: https://imparayaycia.com/SKF_CATALOGO_GENERAL.pdf
 - [32] R. A. Fisher, "The Design of Experiments,," 1974. doi: 10.2307/2348207.
 - [33] D. C. Montgomery, *Design and analysis of experiments*. 2013. doi: 10.2307/2983009.
 - [34] L. Sthle and S. Wold, "Analysis of variance (ANOVA)," *Chemom. Intell. Lab. Syst.*, vol. 6, no. 4, pp. 259–272, Nov. 1989, doi: 10.1016/0169-7439(89)80095-4.
 - [35] T. J. Quirk, "One-Way Analysis of Variance (ANOVA)," *Excel 2007 Educ. Psychol. Stat.*, pp. 163–179, 2012, doi: 10.1007/978-1-4614-3725-3_8.
 - [36] P. Stoker, G. Tian, and J. Y. Kim, "Analysis of variance (Anova)," *Basic Quant. Res. Methods Urban Planners*, pp. 197–219, Jan. 2020, doi: 10.4324/9780429325021-11.
 - [37] E. R. Ziegel, "Response Surfaces: Designs and Analyses," *Technometrics*, vol. 39, no. 3, p. 342, 1997, doi: <https://doi.org/10.1080/00401706.1997.10485140>.
 - [38] C. M. Anderson-Cook, C. M. Borror, and D. C. Montgomery, "Response surface design evaluation and comparison," *J. Stat. Plan. Inference*, vol. 139, no. 2, pp. 629–641, 2009, doi: 10.1016/j.jspi.2008.04.004.
 - [39] A. I. Khuri and J. A. Cornell, "Response Surfaces: Designs and Analyses Second Edition (2nd ed.)," *CRC Press. Boca Rat.*, 1996, doi: <https://doi.org/10.1201/9780203740774>.
 - [40] A. I. Khuri and Siuli Mukhopadhyay, "Response surface methodology," *WIREs Comput. Stat.*, vol. 2, no. 2, pp. 128–149, 2010, doi: <https://doi.org/10.1002/wics.73>.
 - [41] A. Dean, V. D., and D. Draguljić, *Response Surface Methodology. Design and Analysis of Experiments*. 2017. doi: 10.1007/978-3-319-52250-0.

- [42] A. Yıldız, L. Uğur, and İ. E. Parlak, “Optimization of the Cutting Parameters Affecting the Turning of AISI 52100 Bearing Steel Using the Box-Behnken Experimental Design Method,” *Appl. Sci.*, vol. 13, no. 1, 2023, doi: 10.3390/app13010003.
- [43] S. Bhivsane, A. Chel, and S. Sayyed, “Machining Parameters Optimization in Wet Turning of En31 Material Using Box-Behnken Approach of Rsm,” *J. Eng. Technol. Ind. Appl.*, vol. 10, no. 50, pp. 103–111, 2024, doi: 10.5935/jetia.v10i50.1354.