



GUJARAT TECHNOLOGICAL UNIVERSITY

(Established by Government of Gujarat under Gujarat Act No. : 20 of 2007)

ગુજરાત ટેકનોલોજીકલ યુનિવર્સિટી

(ગુજરાત સરકારના ગુજરાત અધિનિયમ ક્રમાંક : ૨૦/૨૦૦૭ દ્વારા સ્થાપિત)

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Abstract of the Thesis



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Faculty: Mathematics

Discipline/Branch: Science_Mathematics

Title of the Thesis: Differential Geometry of Finsler Space with Special Metrics

Abstract

Finsler geometry has emerged as a powerful generalization of Riemannian geometry, with close ties to differential topology and the geometric aspects of differential equations, enabling analyses that go beyond traditional Riemannian methods. Within differential geometry, Finsler geometry extends the Riemannian concepts by permitting the metric to be more general than a quadratic form. A particularly important type of Finsler metric is the generalized (α, β) -metric, this being one that depends on a Riemannian metric α and on a 1-form β . These metrics provide flexibility when it comes to modeling geometric structures and physical phenomena.

Building on this foundation, necessary and sufficient conditions are established for projective transformations between the cubic (α, β) -metric and the Kropina metric $\bar{F}$, as well as between the square-root metric F and the Kropina metric $\bar{F}$, including conditions for isotropic S-curvature and isotropic Berwald curvature. A new exponential (α, β) -metric F is introduced, with conditions for it to be a Berwald, Douglas, and weakly Berwald space. Further, the cubic (α, β) -metric and Generalized Matsumoto metric are studied under locally projective flatness and constant Ricci curvature, yielding conditions for vanishing flag curvature. Finally, local dual flatness and projective flatness of the cubic and exponential (α, β) -metrics are investigated under isotropic S-curvature, proving their reduction to locally Minkowskian spaces.

The (α, β) -framework is extended to metrics involving a Riemannian metric α and two 1-forms β and γ , with the corresponding spray coefficients and Cartan connection derived, leading to new classes of Douglas metrics. The S-curvature and mean Berwald curvature are obtained for a class of homogeneous exponential (α, β) -metrics, including an explicit distortion formula. An $\mathbb{R}$ -complex Finsler metric is developed for a cubic (α, β) -metric, with its fundamental tensor, determinant, inverse, and Hermitian properties established. Finally, a new Kropina–Randers type metric based on a Riemannian metric p and two 1-forms β and η is introduced, with necessary and sufficient conditions for local projective flatness and Douglas classification.



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This thesis will help in advancing the study of (α, β) -metrics and their role in describing direction-dependent geometric structures in Finsler spaces. These findings may also support future applications in complex differential geometry, as well as in theoretical physics and cosmology, particularly in modeling anisotropic spacetimes and generalized gravitational theories.

List of Publications:

- B. K. Tripathi, D. Patel and T. N. Pandey, *On a Weakly-Berwald Space with Exponential Form of (α, β) -Metric*, Journal of Rajasthan Academy of Physical Sciences, **22** (2023), 97-113. (Web of Science/UGC-Care list). https://raops.org.in/epapers/june23_9.pdf
- B. K. Tripathi, D. Patel and V. K. Chaubey, *Projectively Flat Cubic (α, β) -Metric with Constant Flag Curvature $K=0$* , Surveys in Mathematics and its Applications, **19** (2024), 167-178. (SCOPUS/UGC-Care list). https://www.utgjiu.ro/math/sma/v19/p19_10.pdf
- B. K. Tripathi and D. Patel, *Berwald and Douglas Spaces of a Finsler Space With an Exponential form of (α, β) -metric*, Korean Journal of Mathematics, **32** (2024), 661-671. (Web of science/Scopus). <https://kkms.org/index.php/kjm/article/view/1531>
- B. K. Tripathi, V. K. Chaubey and D. Patel, *On R-Complex Finsler Space with Cubic (α, β) -metric*, Journal of Dynamical Systems and Geometric Theories, **22** (2024), 47-63. (Web of Science). <https://doi.org/10.47974/JDSGT-2024-003>
- B. K. Tripathi, V. K. Chaubey and D. Patel, *Locally dually flatness properties in Cubic (α, β) -metric*, Mathematical Foundations of Computing, **8**(4) (2025), 507-517. (Scopus, Web of Science). <https://doi.org/10.3934/mfc.2024027>
- D. Patel and B. K. Tripathi, *Projective change between Cubic (α, β) -metric and Kropina metric*, Bulletin of Mathematical Analysis and Applications, **17**(1) (2025), 1-16. (Scopus, Web of Science). <http://www.bmathaa.org/repository/docs/BMAA17-1-1.pdf>
- D. Patel and B. K. Tripathi, *Projective change between Square root metric and Kropina metric*, Bulletin of the Institute of Mathematics Academia Sinica, **21** (2026), 1-19. (Web of Science). <https://doi.org/10.21915/BIMAS.2026101>