GUJARAT TECHNOLOGICAL UNIVERSITY

Program Name: Bachelor of Engineering
Level: UG
Branch: Robotics & Automation
Subject Code: BE04041041
Subject Name: Numerical Methods & Computations

w.e.f. Academic Year:	2024-25
Semester:	4
Category of the Course:	Basic Science Course

Prerequisite:	Basic knowledge of Linear Algebra and Calculus
Rationale:	To understand the methods which are applicable when analytic methods fail to solve a real problem, this course will play significant role.

Course Outcomes: At the end of the course, students will be able to

Sr.No.	CO statement	BT Level	Marks % weightage
CO-1	**Explain** mathematical modeling concepts and **analyze** different numerical errors affecting computational accuracy in engineering simulations.	Understand/Analyze	15
CO-2	**Apply** numerical techniques to solve linear and nonlinear equations, and **evaluate** the convergence and accuracy of these methods for engineering and robotics problems.	Apply/Analyze	30
CO-3	**Perform** least-squares curve fitting and **develop** regression models (linear, polynomial, and nonlinear) for engineering datasets.	Apply/Evaluate	15
CO-4	**Construct** interpolation and spline-based approximations (polynomials, cubic splines, trajectory interpolation) and **estimate** their errors for robotic motion planning.	Apply/Create	20
CO-5	**Compute** eigenvalues using iterative methods and **implement** numerical approaches to solve forward and inverse kinematics of robotic arms with minimization of pose errors.	Apply/Analyze	20

Teaching and Examination Scheme:

Teaching / Learning Scheme (in Hours per semester)					Total Credits	Assessment Pattern and Marks					Total Marks
						Theory		Tutorial / Practical			
L	T	P	PBL*	Total no of hours per semester		ESE (E)	PA / CA (M)	PA/CA (I)	TW/SL (I)	ESE (V)	
45	0	30	15	90	3	70	30	20	30	50	200

*** Problem Based Learning (PBL) aims to accommodate learning beyond syllabus as per clause 9.4 of NBA manual.**

Content:

Sr. No.	Content	Total Hrs
1	**Mathematical Modeling and Error Analysis:** Mathematicalmodeling,Accuracyandprecision,Modelingerror,Blunder,Inherent error, Round-off and truncation errors, Absolute error, True percentage relative error, Approximate percentage relative error.	**04**
2	**Systems of Linear Equations:** Numerical simulation: Using Gauss Elimination Method, Gauss Jordan Method and matrix inversion, techniques for improving solutions,Gauss-SeidelMethod,Gauss-JacobiMethod,Convergencecriteria,Case Studies.	**06**
3	**Solution of Nonlinear Equations:** Bisection Method,Iteration methods based on first degree – Secant and Regula-Falsi methods, Newton- Raphson Method, modified Newton-Raphson Method, Iteration methods based on second degree equation– Muller's method, Chebyshev method, Roots of polynomials, Computing with polynomials, Bairstow's method to extract a quadratic factor, Case studies applicable to robotics.	**10**
4	**Principles of Least-Squares:** Fitting a straight line, fitting asecond-degree curve, fitting an exponential curve, calculation of the sum of the squares of the residuals for the line fit and a parabolic fit. Linearization of nonlinear relationships (Exponential Model, Power Law, Saturation-growth-rate equation), Polynomial regression, Multiple linear regression, Case studies.	**08**
5	**Polynomial Interpolation:** Introduction to interpolation, Trajectory generation using Cubic interpolation, Newton interpolating Polynomial, Lagrange interpolating Polynomial, Inverse Interpolation, Extrapolation. Error and approximation properties, Estimation of deviation between actual and interpolated motion.	**06**
6	**Spline and Piecewise Interpolation:** Introduction to splines, Linear splines, Quadratic splines, Cubic splines, multi-dimensional interpolation.	**04**
7	**Iterative method for Eigen values:** Eigne value problem, The polynomial method, The power method, Ordinary Differential Equations & Eigen values, Jacobi's method for finding eigen values, Case studies.	**07**
8*	**Numerical solution of 2D and 3D robotic arms:** Forward and inverse kinematic solutions, Numerical solution to minimize the error between the forward kinematic solution and the desired end effector pose. To be dealt in the Lab.	
	Total Hrs.	**45**

Suggested Specification table with Marks(Theory):(ForB.E.only)

DistributionofTheoryMarks					
R Level	U Level	A Level	N Level	E Level	C Level
7	7	14	0	42	0

R: Remembrance; U: Understanding; A: Application; N: Analyze; E: Evaluate; C: Create and above Levels (Revised Bloom's Taxonomy)

Note:This specification table shall be treated as a general guide line for students and teachers.The actual distribution of marks in the question paper may vary slightly from above table.

Reference Books:

1. S.C.ChapraandR.P.Canale,"NumericalMethodsforEngineers",McGrawHillInternational Edition.
2. P. Kandasamy, K. Thilagavathy, K. Gunavathi, "Numerical Methods", S. Chand Publications.
3. John H. Mathews, Kurtis D. Fink, "Numerical Methods Using MATLAB", 4th Ed., Pearson
4. Dr.B.S.Grewal,"NumericalMethodsinEngineeringandScience",Khanna Publications.
5. M.K.Jain,S.R.K.Iyengar,R.K.Jain,"NumericalMethodsforScientificandEngineering Computation", New Age International Publishers.
6. S.S.Sastry,"IntroductoryMethodsofNumericalAnalysis",PrenticeHallof India.

List of Experiments:

1. **Error Analysis and Floating-Point Precision**
   Write a program to compute true error, absolute error, relative error, round-off error, and truncation error for common functions (sin, exp, log).Plot how errors change with step size or precision.
2. **Gauss Elimination and Gauss–Seidel Comparison**
   Implement Gauss elimination and Gauss–Seidel iterative method to solve Ax = b.Compare iterations, convergence rate, and solution error.
3. **Nonlinear Equation Solving using Newton–Raphson**
   Solve multiple nonlinear equations using Newton–Raphson, including:
   - f(x)=0 form
   - Robotics-inspired equations like forward positioning errors.

   Plot convergence behaviour.
4. Bisection vs Secant vs Regula-Falsi Methods
   Implement all three bracketing/open methods.Compare number of iterations and robustness for different test functions.
5. Curve Fitting using Least Squares (Linear + Quadratic)
   Write a program to fit:
   - straight line
   - parabola
   - exponential model (after linearization)

   Compute residuals, RMSE, and R^2 for comparison.
7. Implement Lagrange and Newton Divided Difference interpolation.Evaluate interpolated values and plot interpolation error.
8. Cubic Spline Interpolation for Smooth Trajectory Generation
   Generate a robot joint trajectory using cubic splines.Plot position, velocity, and acceleration profiles.
9. **Power Method for Dominant Eigenvalue**
   Implement the Power Method to find the largest eigenvalue of a matrix.Test with matrices from vibration or robotic Jacobian data.
10. Forward Kinematics Simulation (2R or 3R Robot Arm)
    Write a program to compute and plot forward kinematics:
    - Input: joint angles

- Output: end-effector position
- Plot workspace of the robot.

10. Numerical Inverse Kinematics using Error Minimization
Numerically solve IK by minimizing the error:

$$\| x_{desired} - x_{FK}(\theta) \|$$

Use Newton method / gradient-based update.
Show iterative convergence.

MajorEquipment:

NA

*** List of Open Source Software/learning website:**

1. Python Ecosystem (Free & Open Source)

- **Python** – general scientific computing
- **NumPy** – numerical computation
- **SciPy** – optimization, interpolation, ODEs, linear algebra
- **Matplotlib** – plotting and visualization
- **SymPy** – symbolic mathematics
- **Jupyter Notebook / JupyterLab** – interactive coding
- **Pandas** – data analysis

2. Octave (Open-source MATLAB alternative)

- GNU **Octave** – almost complete MATLAB-like environment
  Website: https://octave.org

3. Scilab

- Numerical computation, simulations, GUI building
  Website: https://www.scilab.org/

4. OpenModelica

- Open-source system modeling& simulation
  Website: https://openmodelica.org/

Free learning websites:

1. MIT Open Course Ware

https://ocw.mit.edu/
Free courses: Numerical Methods, Linear Algebra, Robotics, Dynamics.

2. Coursera (Audit for free)

https://www.coursera.org/
Courses on Python, Machine Learning, Robotics, Controls.

3. edX (Audit free)

https://www.edx.org/
Courses from MIT, Harvard on computing and engineering.

4. NPTEL
https://nptel.ac.in/
Excellent lectures for Numerical Methods, Robotics, Python, and more.

5. Khan Academy
https://www.khanacademy.org/
Basics of mathematics, linear algebra, calculus.

6. Brilliant.org
https://brilliant.org/
Interactive learning of math and algorithms.

7. Rosalind / Project Euler

- Algorithmic problem solving
  https://projecteuler.net/
  https://rosalind.info/

- **List of suggested activities for Term Work / Self Learning:**

Sl. No.	Name of the activity	No. of hours	Evaluation Criteria
1.	Industry/Research laboratory visit	Visit = 5hrs., Report preparation = 5hrs. Total = 10hrs.	Based on report submitted. Report should contain observations and calculations based on industry/ lab data.
2.	Technical Video based learning related to the subject	Duration of video = 5hrs. Report preparation = 5hrs. Total = 10hrs.	Report /presentation based on the video learning outcomes.
3.	Assignment writing. Numericals based assignment is preferable.	5 assignments of 4hrs. each. Total = 20hrs.	Based on the correctness of submittedassignment.
4.	Problem solving/Coding using C, C++, MATLAB, Python, SCILAB,modeling and Analysis software or any other software	5 small coding-based assignment of 2hrs. each. Total = 10hrs.	Based on the coding solution submitted.
5.	Self-learning online course	Minimum duration of the course should be 10hrs.	Examination based assessment at the end of course. Based on the certificate produced.
6.	Identification and solution of Complex problem	Maximum 2 problems. Study of the problem and solution finding, Total = 10hrs.	Based on the depth of the solution submitted.
7	Videos on Industrial safety/Disaster Management aspects based on subject	Duration of video = 5hrs. Report preparation = 5hrs. Total = 10hrs.	Based on quiz/report submitted
8	Technical paper reading and summarization of research papers based on relevant subject	5 research papers = 20 hrs.	Summarize research paper and evaluation critical parameters

9.	Poster/chart/power point preparation on technical topics	Duration = 6 hrs.	Based on poster/chart preparation and presentation skills
10	Working/non-working model on technical topics	Working = 12 hrs. Non- working = 8 hrs.	Based on inter department/external evaluation
11	Industrial exposure for 2-3 days to observe and provide tentative solutions on society/environment/health/sustainability/any other issue	Duration = 15 hrs. for industrial exposure Problem identification and tentative solution = 10 hrs. Total = 20 hrs.	Based on evaluation of critical problems and solutions
12	Group Discussion on emerging/trending technical topics based on subject	Duration = Min. 1 hr.per subject. Max. 3 hrs. per subject	Based on performance in group discussion, technical depth, knowledge etc.
13.	Real world case studies-based learning	Duration of data collection/study = 5hrs. Report preparation = 5hrs. Total = 10hrs.	Based on in-depth study, technical depth, data collected, fact finding, etc.
14.	Application/Software development	Duration = 10 hrs.	Depending on the complexity of the Application/Software
15.	Research paper publication	Duration = 10 hrs.	Based on submission of proof of publication
16.	Upgradation/Reverse engineering studies of existing equipment of the laboratory	Duration 10 hrs.	Based on the performance of the equipment
17.	Expert lecture/session	Duration 3 hrs. For attending the lecture/session– 2 hrs. and for report writing 1 hr.	Based on the proof of attendance and report submitted
18.	Annotated Video Explanation of Concept/Problem	10h (Preparation + Recording + Submission)	Based on accuracy of explanation, clarity, and presentation style.
19.	Patent Search and Innovation Gap Identification	10h (Search + Report)	Based on number of relevant patents analyzed and identification of innovation scope.

Note:
1. All the suggested activity should be related to the subject.
2. The number of hours are suggestive. Faculty can sub-divide the number of hours based on the activity. However, total number of hours is fixed.
3. Rubrics for the evaluation can be prepared by the faculty.
4. Subject teacher can add the relevant activities other than those listed above, with the consent of head of the department and DQAC.

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