

# GUJARAT TECHNOLOGICAL UNIVERSITY

**SUBJECT NAME: Complex Variables and Partial Differential Equations**

**SUBJECT CODE:2910002**

**1<sup>st</sup> Semester**

**Type of course:** Basic Science Course

**Prerequisite:** Advanced Engineering Mathematics

**Rationale:** This subject is a powerful tool for solving a wide array of applied problems. It is related to several traditional mathematical disciplines such as real analysis, differential equations, algebra and topology. Its useful for common applications include wave propagation phenomena such as those occurring in many engineering branches like electrodynamics, fluid mechanics and quantum mechanics, diffusion problems such as heat and contaminant diffusion, signal processing and communication theory etc.

**Teaching and Examination Scheme:**

Teaching Scheme			Credits	Examination Marks				Total Marks
L	T	P		Theory Marks		Practical Marks		
			ESE (E)	PA (M)	ESE (V)	PA (I)		
3	2	0	5	70	30	0	0	100

**Content:**

Sr. No.	Content	Total Hrs	% Weightage
01	Polar Form of Complex Numbers, Powers and Roots, Complex Variable – Differentiation : Differentiation, Cauchy-Riemann equations, analytic functions, harmonic functions, finding harmonic conjugate; elementary analytic functions (exponential, trigonometric, logarithm) and their properties; Conformal mappings, Mobius transformations and their properties.	10	24 %
02	Complex Variable - Integration : Contour integrals, Cauchy-Goursat theorem (without proof), Cauchy Integral formula (without proof), Liouville's theorem and Maximum-Modulus theorem (without proof);	08	18 %
03	Sequences, Series, Convergence Tests, Power Series, Functions Given by Power Series, Taylor and Maclaurin Series, Uniform Convergence.	04	10 %
04	Laurent's series; Zeros of analytic functions, singularities, Residues, Cauchy Residue theorem (without proof), Residue Integration Method, Residue Integration of Real Integrals.	04	10 %
05	First order partial differential equations, solutions of first order linear and nonlinear PDEs.	06	14 %
06	Solution to homogeneous and nonhomogeneous linear partial differential equations second and higher order by complementary function and particular integral method. Flows, vibrations and diffusions, second-order linear equations and their classification, Initial and boundary conditions (with an informal description of well-posed problems), D'Alembert's solution of the wave equation; Duhamel's principle for one dimensional	10	24 %

	wave equation. Separation of variables method to simple problems in Cartesian coordinates. The Laplacian in plane, cylindrical and spherical polar coordinates, solutions with Bessel functions and Legendre functions. One dimensional diffusion equation and its solution by separation of variables. Boundary-value problems: Solution of boundary-value problems for various linear PDEs in various geometries.		
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**Suggested Specification table with Marks (Theory):**

Distribution of Theory Marks					
R Level	U Level	A Level	N Level	E Level	C Level
<b>7</b>	<b>14</b>	<b>14</b>	<b>14</b>	<b>14</b>	<b>7</b>

**Legends: R: Remembrance; U: Understanding; A: Application, N: Analyze and E: Evaluate C: Create and above Levels (Revised Bloom’s Taxonomy)**

Note: This specification table shall be treated as a general guideline for students and teachers. The actual distribution of marks in the question paper may vary from above table.

**Reference Books:**

- (1) Erwin Kreyszig, Advanced Engineering Mathematics, 10th Edition, John Wiley and Sons.
- (2) Peter O'Neill, Advanced Engineering Mathematics, 7th Edition, Cengage.
- (3) Dennis G. Zill, 4th edition, Advanced Engineering Mathematics, 4th Edition, Jones and Bartlett Publishers.
- (4) Dennis G. Zill, Patrick D. Shanahan, A First Course in Complex Analysis with Applications, Jones and Bartlett Publishers.
- (5) S. J. Farlow, Partial Differential Equations for Scientists and Engineers, Dover Publications, 1993.
- (6) Ian Sneddon, Elements of Partial Differential Equations, McGraw Hill.
- (7) J. W. Brown and R. V. Churchill, Complex Variables and Applications, McGraw Hill.

**Course Outcome:**

After learning the course, the students should be able to:

1. Find a polar form, powers and roots of a complex number.
2. Understand the basic concepts of a complex number and a function of complex variables, differentiability, Cauchy-Riemann conditions, analyticity, conformal mapping and Mobius transformation.
3. Evaluate complex line/contour integrals by using Cauchy-Goursat theorem and Cauchy Integral formula.
4. Understand the role of Liouville’s theorem and Maximum-Modulus theorem for a function of complex variables.
5. Observe the behavior of complex sequence and series.
6. Expand the functions of complex variables by using Taylor, Maclaurin series and Laurent series.
7. Find the zeros, singularities and residues of complex functions.
8. Apply Cauchy Residue theorem in evaluating some real integrals.
9. Solve first order linear and nonlinear partial differential equations.
10. Find the solution of homogeneous and nonhomogeneous linear partial differential equations of second and higher order.
11. Apply the method of separation of variables to solve some problems related to flows, vibrations and diffusions.

**List of Open Source Software/learning website:**

MIT Opencourseware.