Volatility vs. Tail Risk: Which One is Compensated in Equity Funds?
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Abstract

Research that has led to what is known as the “low volatility anomaly” in cross-sectional stocks from a similar universe indicates that volatility is not compensated with a “volatility” premium. We find evidence of a risk premium, but it depends on the definition or measure of risk. “Tail risk” measures the probability of having significant losses and should be what investors care about the most. We investigated several risk measures, including volatility and tail risk, and found that volatility is not compensated but tail risk is compensated with higher expected return in both U.S. and non-U.S. equity funds.
Volatility vs. Tail Risk: Which One is Compensated in Equity Funds

We are motivated to test whether volatility and tail risk are both compensated in the equity fund universe. We address both issues at the same time so our study can directly answer the question: if volatility is not compensated, what about tail risk? We choose equity funds, instead of individual stocks, since systematic risk is more relevant to investors’ portfolios because idiosyncratic risk is largely diversified away in a typical portfolio setting.

Extensive empirical evidences show that low-volatility or low-beta portfolios historically have offered higher realized average stock returns versus comparable high-volatility or high-beta portfolios constructed from the same starting universe of individual securities. Ang, Hodrick, Xing, and Zhang (2006, 2009) found that high-risk stocks have had abysmally low average returns in longer U.S. samples and in international markets and rekindled interest to the low-volatility anomaly. Blitz and van Vliet (2007) demonstrated its robustness across regions. Frazzini and Pedersen (2010) documented the low-volatility anomaly in global stock, Treasury, credit, and futures markets. Finally, Baker, Bradley, and Wurgler (2011) provide an excellent review on the low-volatility and low-beta anomalies.

This historical finding is counter to the fundamental principle that higher risk is compensated with higher expected return. Several behavioral models, including a preference for lotteries and limits on arbitrage, were proposed to explain this anomaly. For example, Baker, Bradley, and Wurgler (2011) suggest that the typical institutional investor’s mandate to beat a fixed benchmark such as S&P 500 Index could increase the demand for high-beta stocks and thus lower their expected returns. Hsu, Kudoh and Yamada (2013) provide an alternative explanation for the low-volatility puzzle, which hypothesizes that analysts inflate earnings forecasts more aggressively for volatile stocks. Because investors are known to overreact to analyst forecasts, this can lead to systematic overvaluation and low returns for high-volatility stocks.

We are interested in asking a different question: if volatility or beta is not compensated within a particular universe of similar securities, are there other types of risk that might be compensated? More specifically, in contrast to risk measures such as volatility or beta that penalize upside gains the same as downside losses and fail to account for non-normal return characteristics, perhaps a type of risk, such as tail risk, that is unambiguously viewed as “bad” by investors would garner a “tail risk” premium? In other words, volatility or beta may not be a relevant risk measure, whereas tail risk might be a candidate for higher compensation. A large negative event can significantly
reduce portfolio value. Examples include the stock market crash of 1929, Black Monday in 1987, the Asia Crisis in 1997, the dot-com bubble burst in 2000, and the financial crisis of 2008. Hence, investors might require a premium to hold assets that have high tail risk. Indeed, recent asset pricing studies have demonstrated that such a premium exists and is economically significant in explaining the cross-sectional stock returns.

Kraus and Litzenberger (1976) show that investors dislike stocks with high tail risk in terms of negative coskewness, and stocks with more negative coskewness tend to have higher expected returns. Harvey and Siddique (2000) show that coskewness is economically important and commends a risk premium, on average, of 3.6% per year for U.S. stocks.

Bawa and Lindenberg (1977) suggest that a natural extension of the CAPM that takes into account the asymmetric risk preference is to specify asymmetric downside and upside betas. Ang, Chen, and Xing (2006) estimate that the downside risk premium is approximately 6% per annum for U.S. stocks.

Most of the previous studies on the low volatility anomaly and downside risk premium were conducted on the U.S. stock universe. It appears that volatility is not compensated while a downside risk premium is economically significant in cross-sectional U.S. stocks. On the other hand, there is little related research reported on the equity fund universe. One exception is that Moreno and Rodríguez (2009) found evidence that adding a coskewness factor is economically and statistically significant in evaluating mutual funds.

The biggest difference between the fund universe and the stock universe is the idiosyncratic component. Funds have eliminated most individual stock idiosyncratic risks since a typical mutual fund holds more than 100 stocks and diversification significantly lowers the idiosyncratic component. However, funds still have systematic risks related to the nature of their investments, causing them to have different tail risks. Another difference for the fund universe is that transaction costs are fully considered since all of our analyses are based on net total returns for funds, whereas estimating transaction costs in the stock universe can be very challenging.

We show that the low volatility anomaly is indeed shown in both U.S. equity funds and non-U.S. equity funds. However, the tail risk premium is economically significant in both U.S. equity funds and non-U.S. equity funds.
Description of Data

We use Morningstar’s open-end equity mutual fund universe containing both alive and dead funds from January 1980 to September 2011. Only the oldest share class is collected for each fund. We include both U.S. equity mutual funds and non-U.S. equity mutual funds. The Morningstar categories include all funds from the nine size-valuation style boxes that form the U.S. equity universe, although when analyzing subsection of the U.S. equity universe we focus on the somewhat less granular valuation-based columns from the style box (value, core, and growth), size-based rows from the style box (large, mid, and small), plus the non-U.S. category, which helps to insure that each of composites contain a reasonably large number of funds. Most of the non-U.S. equity funds have inception dates later than 1990.

We collected 3,389 U.S. equity funds and 1,055 non-U.S. equity funds that have at least a five-year history. The Appendix shows that the majority of the equity funds exhibit non-normality, which indicates that tail risk is important to consider in addition to volatility.

Volatility vs. Tail Risk
In this section we test three risk metrics: volatility (VOL), skewness (SKEW), and excess conditional-value-at-risk (ECVaR). Volatility is simply the standard deviation of the fund’s total returns. It is not sensitive to the tail information, while both SKEW and ECVaR are tail risk measures.

SKEW is a measure of the asymmetry of the data around the sample mean. It is the third standardized moment. Negative skewness indicates the propensity to have large negative returns with greater probability.

The second tail risk measure, ECVaR, measures the left tail risk specifically. The right tail has impact on skewness, but it has no impact on ECVaR. ECVaR is based on Conditional Value-at-Risk (CVaR). A different name for CVaR is the expected tail loss. CVaR is defined as the fund’s CVaR in excess of the implied CVaR with a normal distribution with the same mean and standard deviation in a given period. In other words, the fund’s ECVaR is a normalized version of CVaR by controlling for the volatility of the fund. See Appendix for more details on ECVaR.

We compute the risk measures (VOL, SKEW, and ECVaR) on total returns for most of following analyses. Total return can be decomposed into systematic and idiosyncratic components. In the final section “Beta vs. Coskewness”, we investigate the systematic component of both volatility (beta) and tail risk (coskewness) to see whether beta or coskewness is compensated.
Exhibit 1 shows the average correlation among the three risk measures for both U.S. equity funds (Panel A) and non-U.S. equity funds (Panel B). The sample period is from January 1980 to September 2011. The correlation is measured across all the U.S. or non-U.S. equity funds using a rolling five-year window, and then averaged over time. VOL has relatively low correlation with both SKEW and ECVaR in both fund universes, indicating that VOL is capturing different information from the left tail. ECVaR depends on the tail level, and a smaller tail level corresponds to a more extreme left tail. The values for ECVaR in Exhibit 1 are computed with a tail level of 5%, i.e. the worst 5% of returns. The correlation between ECVaR and SKEW is 66% and 74% for U.S. equity funds and non-U.S. equity funds, respectively, indicating both ECVaR and SKEW capture much of the same left tail information. We will focus more on ECVaR since our empirical analyses show that ECVaR performs slightly better than SKEW.

**Exhibit 1: The Average Correlation for Volatility and Tail Risk Measures**
(January 1980 – September 2011)

<table>
<thead>
<tr>
<th>Panel A: U.S. Equity Funds</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL</td>
<td>1.00</td>
<td>0.13</td>
</tr>
<tr>
<td>SKEW</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>ECVaR</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Non-U.S. Equity Funds</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL</td>
<td>1.00</td>
<td>0.13</td>
</tr>
<tr>
<td>SKEW</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>ECVaR</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Volatility Premium vs. Tail Risk Premium**

Our portfolio formation methodology follows Harvey and Siddique (2000). Using the first 60 months of returns, we compute the VOL and ECVaR for each of the equity funds. We sort all the equity funds into quintiles based on VOL or ECVaR. The 61st month excess returns are averaged with equal weights for each quintile. The first quintile (Q1) is the proxy for the lowest VOL or lowest tail risk (ECVaR) quintile, and the fifth quintile (Q5) is the proxy for the highest VOL or highest tail risk (ECVaR) quintile. A more negative value of ECVaR means a more severe loss, hence higher tail risk.

One problem associated with tail risk estimates is the trade-off between statistical significance and survivorship bias. As we work with monthly data, we need at least 60 months of data to estimate tail risk, which unfortunately introduces some survivorship bias as some of the funds with the worst losses were shut down.

Exhibit 2 presents the summary statistics for the five VOL quintiles and five ECVaR quintiles for both U.S. and non-U.S. equity funds. All returns in this paper are annualized and in excess of T-Bills. We compute the volatility premium and tail risk premium as the difference in arithmetic mean for Q5 and Q1 (Q5-Q1) shown in Exhibit 2. At first glance, the volatility premium is positive for U.S. equity funds.
(1.64%), but on a risk-adjusted basis, the highest volatility quintile (Q5) has the lowest Sharpe ratio, significantly lower than the other four quintiles. The geometric mean for Q1 and Q5 is almost the same from January 1985 to September 2011 (both are 4.03%). This provides evidence that the volatility is not compensated on a risk-adjusted basis. For non-U.S. equity funds, the volatility premium (Q5-Q1) is even negative (-1.35%), which strongly supports the low volatility anomaly.

However, the tail risk premium (Q5-Q1) for both U.S. equity funds (2.67%) and non-U.S. equity funds (2.50%) are positive even after the volatility is controlled for, suggesting that tail risk is compensated.

Exhibit 2: Summary Statistics for the Five VOL Quintiles and Five ECVaR Quintiles (January 1985 – September 2011)

Panel A: U.S. Equity Funds

<table>
<thead>
<tr>
<th>VOL</th>
<th>Low Volatility 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High Volatility 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ari. Mean</td>
<td>4.87%</td>
<td>5.70%</td>
<td>6.41%</td>
<td>6.36%</td>
<td>6.51%</td>
</tr>
<tr>
<td>Geo. Mean</td>
<td>4.03%</td>
<td>4.57%</td>
<td>5.09%</td>
<td>4.76%</td>
<td>4.03%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>12.57%</td>
<td>14.54%</td>
<td>15.64%</td>
<td>17.23%</td>
<td>21.46%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.39</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Panel B: Non-U.S. Equity Funds

<table>
<thead>
<tr>
<th>VOL</th>
<th>Low Volatility 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High Volatility 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ari. Mean</td>
<td>6.21%</td>
<td>5.39%</td>
<td>5.75%</td>
<td>5.27%</td>
<td>4.86%</td>
</tr>
<tr>
<td>Geo. Mean</td>
<td>5.18%</td>
<td>4.05%</td>
<td>4.22%</td>
<td>3.58%</td>
<td>2.46%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>13.80%</td>
<td>15.77%</td>
<td>16.85%</td>
<td>17.69%</td>
<td>21.22%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.45</td>
<td>0.34</td>
<td>0.04</td>
<td>0.30</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Exhibit 3 shows the risk-return relationship for the five VOL quintiles and five ECVaR quintiles for U.S. equity funds presented in Exhibit 2, Panel A. The VOL quintiles have a positive slope from Q1 to Q3. It is then flat from Q3 to Q5. The positive slope from Q1 to Q3 can be partly explained by the possibility that the equity funds in Q1 hold relatively higher allocations to cash and bonds than the funds in Q3 so that Q1 has lower volatility and lower arithmetic return. On the other hand, the five ECVaR quintiles present a very different picture. They have a similar level of standard deviations by construction, but the expected excess returns are significantly lower for the lowest tail risk quintile, indicating that the tail risk premium is significantly positive.
The ECVaR is measured on a five-year period and the left tail has only three monthly data points when the tail level is 5% (0.05*60 = 3). To address the concerns on the small number of data points, we also plotted the tail risk frontier with a tail level of 10% in Exhibit 3 (triangles). We found that the tail risk frontier is similar to the one for the 5% tail level except that the tail risk premium for the 10% level is a little lower.  

**Exhibit 3: The Risk-Return Relationship for the Five VOL Quintiles and Five ECVaR Quintiles (Both 5% and 10% Levels)**

**U.S. Equity Funds (January 1985 – September 2011)**

Exhibit 4 shows that the risk-return relationship for the five VOL quintiles and five ECVaR quintiles for both 5% tail level and 10% tail level for non-U.S. equity funds. The five VOL quintiles have a negative slope, which confirms the low volatility anomaly. On the other hand, the five ECVaR quintiles for both 5% tail level and 10% tail level confirm that the tail risk premium (Q5-Q1) is positive.

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1 Unfortunately, a higher tail level in the ECVaR measure will capture a higher portion of the bulk distribution, hence capture the volatility effect. Daily returns will mitigate the small sample issue, but they are not available for a longer period of time.
Exhibit 4: The Risk-Return Relationship for the Five VOL Quintiles and Five ECVaR Quintiles (Both 5% and 10% Levels)

Non-U.S. Equity Funds (January 1985 – September 2011)

Exhibit 5 shows the growth of value for Q1 and Q5 for both VOL and ECVaR (5% tail level) for U.S. equity funds. For the entire period, the highest tail risk composite dominated the lowest tail risk composite. However, the highest volatility composite only dominated the lowest volatility composite for the first half of the period (mostly during the technology bubble period). The highest volatility composite underperformed the lowest volatility composite after the technology bubble burst, so they end up with almost the same value. This result is consistent with the observation of Baker, Bradley, and Wurgler (2011) that the low volatility anomaly has widened after the technology bubble burst.
Exhibit 5: Growth of $1 for Selected Volatility and Tail Risk Quintiles
(ECVaR for 5% Tail Level)

U.S. Equity Funds

Exhibit 6 shows the growth of value for Q1 and Q5 for both VOL and ECVaR (5% tail level) for non-U.S. equity funds. For the entire period, the highest tail risk composite dominated the lowest tail risk composite. In contrast, the lowest volatility composite dominated the highest volatility composite for the entire period. Therefore, both Exhibit 5 and 6 support the thesis that volatility is not compensated but tail risk is compensated in both U.S. and non-U.S. equity mutual funds.
Exhibit 6: Growth of $1 for Selected Volatility and Tail Risk Quintiles  
(ECVaR for 5% Tail Level)

Non-U.S. Equity Funds

Portfolios Performance of (Q5-Q1)

As an important element in performance evaluation, we are interested in measuring the alpha of the long/short (Q5-Q1) portfolio on a risk-adjusted basis for both VOL (high volatility minus low volatility risk) and 5% tail level ECVaR (high tail risk minus low tail risk). The regression results are shown in Exhibit 7. The alpha of Q5-Q1 is measured against two benchmarks from January 1985 to September 2011: the Carhart (1997) four-factor model (the three Fama-French factors plus momentum) and the fund category average. The t-stat of alpha is simply the t-stat of the regression intercept. The differences between Panel A and Panel B of Exhibit 7 are striking.

For VOL in Panel A, 13 of the 16 alphas are negative, none of the alphas are significantly positive, and the core category is even significantly negative at the 1% level. The results are consistent with our main thesis that volatility is not compensated.

In contrast, for ECVaR in Panel B, all 16 alphas are positive and half of the alphas are significant at the 5% level. The Carhart alpha of the Q5-Q1 portfolio for all U.S. equity funds is 2.65% per year with t-stat of 2.82. It confirms that the tail risk premium is robust after adjusting for the Carhart (1997) four-factors.

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2 For U.S. equity funds, the three Fama-French factors and momentum factor are used and downloaded from the Kenneth French Data Library. For non-U.S. equity funds, the three global Fama-French factors and global momentum factor are used. The Market factor is adjusted for average fund expense.
Exhibit 7: Volatility Risk Alpha and Tail Risk Alpha for U.S. and Non-U.S. Equity Funds
(January 1985 – September 2011)

Panel A: Volatility Risk Alpha

<table>
<thead>
<tr>
<th>Category</th>
<th>Carhart Alpha</th>
<th>t-stat</th>
<th>Category Average Alpha</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>All U.S. Equity</td>
<td>-0.25%</td>
<td>-0.26</td>
<td>-1.59%</td>
<td>-0.88</td>
</tr>
<tr>
<td>Large</td>
<td>0.86%</td>
<td>0.83</td>
<td>-0.71%</td>
<td>-0.56</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.24%</td>
<td>-0.18</td>
<td>-0.27%</td>
<td>-1.39</td>
</tr>
<tr>
<td>Small</td>
<td>-1.71%</td>
<td>-1.27</td>
<td>-4.77%</td>
<td>-2.32</td>
</tr>
<tr>
<td>Core</td>
<td>-1.75%</td>
<td>-3.03</td>
<td>-1.66%</td>
<td>-1.62</td>
</tr>
<tr>
<td>Growth</td>
<td>-0.13%</td>
<td>-0.13</td>
<td>-1.61%</td>
<td>-1.02</td>
</tr>
<tr>
<td>Value</td>
<td>0.24%</td>
<td>0.33</td>
<td>0.43%</td>
<td>0.45</td>
</tr>
<tr>
<td>Non-U.S. Equity</td>
<td>-1.61%</td>
<td>-0.92</td>
<td>-3.61%</td>
<td>-2.21</td>
</tr>
</tbody>
</table>

Panel B: Tail Risk Alpha

<table>
<thead>
<tr>
<th>Category</th>
<th>Carhart Alpha</th>
<th>t-stat</th>
<th>Category Average Alpha</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>All U.S. Equity</td>
<td>2.65%</td>
<td>2.82</td>
<td>2.44%</td>
<td>2.63</td>
</tr>
<tr>
<td>Large</td>
<td>1.87%</td>
<td>2.42</td>
<td>1.31%</td>
<td>1.70</td>
</tr>
<tr>
<td>Medium</td>
<td>2.14%</td>
<td>1.95</td>
<td>1.08%</td>
<td>1.59</td>
</tr>
<tr>
<td>Small</td>
<td>2.25%</td>
<td>1.89</td>
<td>3.12%</td>
<td>2.35</td>
</tr>
<tr>
<td>Core</td>
<td>1.67%</td>
<td>2.45</td>
<td>1.89%</td>
<td>2.51</td>
</tr>
<tr>
<td>Growth</td>
<td>1.41%</td>
<td>1.57</td>
<td>1.01%</td>
<td>1.12</td>
</tr>
<tr>
<td>Value</td>
<td>1.82%</td>
<td>2.48</td>
<td>2.10%</td>
<td>2.57</td>
</tr>
<tr>
<td>Non-U.S. Equity</td>
<td>2.82%</td>
<td>1.70</td>
<td>1.79%</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Fama-MacBeth Regressions

Now we examine the influence of tail risk on the cross-section of fund returns through standard Fama-MacBeth (1973) regressions. In each month, starting from January 1985 to September 2011, we run a simple cross-sectional regression of future fund returns on past volatility and tail risk. The past 60 months of data are used to estimate the volatility or tail risk for each fund for each month. For example, January 1980 to December 1984 are used to estimate the volatility or tail risk for each mutual fund for the first holding period, i.e. January 1985. This gives us 321 monthly estimates of the slope coefficients along with the associated standard errors. We then aggregate these slope coefficient estimates across time.

The average slopes and associated t-statistics (in parentheses) are presented in Exhibit 8. As expected, none of the VOL coefficients are positively significant. The ECVaR is statistically significant at the 5% level for the ALL U.S. equity funds category and three other categories.3 Note that the negative coefficient for ECVaR is interpreted as a more severe of ECVaR (i.e. higher tail risk) corresponds with a higher expected returns.

3 In unreported results, the t-stats are largely unchanged for ECVaR, when we add beta and momentum as independent variables into the regression.
Exhibit 8: Fama-MacBeth Regressions for U.S. Equity Funds and Non-U.S. Equity Funds
(January 1985 – September 2011)

<table>
<thead>
<tr>
<th></th>
<th>All U.S.</th>
<th>Large</th>
<th>Medium</th>
<th>Small</th>
<th>Core</th>
<th>Growth</th>
<th>Value</th>
<th>Non-U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL</td>
<td>0.04</td>
<td>0.06</td>
<td>0.05</td>
<td>-0.03</td>
<td>-0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td>(0.96)</td>
<td>(0.82)</td>
<td>(-0.43)</td>
<td>(-0.36)</td>
<td>(0.80)</td>
<td>(0.93)</td>
<td>(-0.62)</td>
</tr>
<tr>
<td>ECVaR</td>
<td>-0.08</td>
<td>-0.06</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.09</td>
<td>-0.01</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>(-2.79)</td>
<td>(-2.17)</td>
<td>(-1.46)</td>
<td>(-1.81)</td>
<td>(-3.55)</td>
<td>(-0.48)</td>
<td>(-2.41)</td>
<td>(-1.51)</td>
</tr>
</tbody>
</table>

In summary, our Fama-MacBeth analyses are consistent with previous results. We find that overall the tail risk is significantly related to future fund returns. It is largely significant when size, value, fund beta, and fund momentum are controlled for.

\[
\text{Coskewness} = \frac{E[\varepsilon_{i,t}^2 \varepsilon_{M,t}]}{\sqrt{E[\varepsilon_{i,t}^4] E[\varepsilon_{M,t}^2]}},
\]

\[
\varepsilon_{i,t} = r_{i,t} - r_{ft} - \beta_i (r_{M,t} - r_{ft})
\]

\[
\varepsilon_{M,t} = r_{M,t} - \text{avg}(r_{M,t})
\]

where the residual \( \varepsilon_{i,t} \) is computed from the regression of the excess return on the contemporaneous market excess return for security \( i \) in period \( t \). \( \varepsilon_{M,t} \) is the market return in period \( t \) in excess of average market return. A negative coskewness means that the security is adding negative skewness to the portfolio. Harvey and Siddique (2000) suggest that a stock with more negative coskewness should have a higher expected return.

Exhibit 9 plots the risk-reward tradeoff for fund beta and coskewness for U.S. equity funds. On a risk-adjusted basis, fund beta is not compensated since the Sharpe ratio for Q5 is the lowest. On the other hand, coskewness premium (Q5-Q1) is 1.85%, a little lower than the ECVaR premium (2.67%). Exhibit 10 shows the growth of value for Q1 and Q5 for both VOL and Coskewness for U.S. equity funds. The findings in Exhibit 9 and 10 are largely consistent with Exhibit 3 and Exhibit 5, respectively. Beta and coskewness analyses for non-U.S. equity funds yield similar results.
Exhibit 9: The Risk-Return Relationships for the Five Beta Quintiles and Five Coskewness Quintiles
U.S. Equity Funds (January 1985 – September 2011)

Exhibit 10: Growth of $1 for Selected Beta and Tail Risk (Coskewness) Quintiles
U.S. Equity Funds
Conclusions

We investigated whether volatility and tail risk are compensated in both U.S. and non-U.S. equity mutual funds. We have provided evidence that volatility is not compensated on a risk-adjusted basis for both U.S. equity funds and non-U.S. equity funds. Hence, our research using the fund universe largely supports similar findings in cross-sectional stocks. On the other hand, the tail risk premium in both U.S. and non-U.S. equity mutual funds is economically significant. Funds that have higher tail risk have higher expected returns. This tail risk-return relationship is consistent with an economy where agents demand a premium to compensate for tail risk.

We introduced a new left tail risk measure, excess conditional-value-at-risk (ECVaR), and also used a standard measure: skewness (SKEW). The cross-sectional premium for bearing tail risk is approximately 2.67% per annum for U.S. equity funds, and 2.50% for non-U.S. equity funds with the ECVaR measure. The tail risk premium remains statistically significant after controlling volatility as well as size, value, fund beta, and fund momentum for equity funds.

Finally, analyses on fund beta and coskewness, the systematic variables for volatility and tail risk, respectively, confirmed the main thesis that volatility is not compensated but tail risk is compensated.
Appendix

Non-Normality of Equity Mutual Funds
It is well known that the returns of many financial asset classes do not follow a normal distribution. Mandelbrot (1963) found that the changes in cotton prices did not follow a normal distribution but instead a stable distribution. Fama (1965) confirmed the non-normality in stock prices. Since then, there was extensive literature on non-normality of financial assets. See, for example, Rachev, Menn and Fabozzi (2005) for an introduction.

The normal distribution can underestimate the tail risk considerably. We can easily test if equity mutual funds behave the same. Exhibit A1 shows that U.S. equity funds and non-U.S. funds clearly exhibit non-normality. The median skewness and kurtosis for U.S. equity funds are -0.59 and 4.35, respectively, well beyond that for a normal distribution. The Jarque–Bera (1980) tests show that the normal distribution is rejected for about 84% of both U.S. equity funds and non-U.S. funds that have at least a five-year history. For each particular style or size, about 80% to 90% of funds exhibit non-normality.

Exhibit A1: Non-Normal Tests for U.S. and Non-U.S. Equity Mutual Funds

<table>
<thead>
<tr>
<th>Number of Funds</th>
<th>Median Skewness</th>
<th>Median Kurtosis</th>
<th>Non-Normal Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>All U.S. Equity</td>
<td>3389</td>
<td>-0.59</td>
<td>4.35</td>
</tr>
<tr>
<td>Large</td>
<td>1992</td>
<td>-0.62</td>
<td>4.28</td>
</tr>
<tr>
<td>Medium</td>
<td>668</td>
<td>0.59</td>
<td>4.69</td>
</tr>
<tr>
<td>Small</td>
<td>729</td>
<td>-0.52</td>
<td>4.24</td>
</tr>
<tr>
<td>Core</td>
<td>1275</td>
<td>-0.64</td>
<td>4.32</td>
</tr>
<tr>
<td>Growth</td>
<td>1392</td>
<td>-0.49</td>
<td>4.19</td>
</tr>
<tr>
<td>Value</td>
<td>722</td>
<td>-0.67</td>
<td>4.67</td>
</tr>
<tr>
<td>Non-U.S. Equity</td>
<td>1055</td>
<td>-0.62</td>
<td>4.54</td>
</tr>
</tbody>
</table>

ECVaR
We are interested in a measure that only measures the left tail risk. Our new measure is based on Conditional Value-at-Risk (CVaR). The precursor to CVaR and a slightly better known measure of downside risk is the standard value-at-risk (VaR) measure. VaR estimates the loss that is expected to be exceeded with a given level of probability over a specified time period. CVaR is closely related to VaR and is calculated by taking a probability-weighted average of the possible losses conditional on the loss being equal to or exceeding the specified normal distribution VaR. Other terms for CVaR include mean shortfall, tail VaR, and expected tail loss. CVaR is a comprehensive measure of the entire part of the tail that is being observed, and for many, the preferred measurement of tail risk.
For most of our study, we fixed the probability level for the CVaR at 5 percent (corresponding to a confidence level of 95 percent). For a normal distribution with mean of $\mu$ and standard deviation of $\sigma$, the CVaR has a closed-form formula (see Rockafellar and Uryasev (2000)):

$$\text{CVaR}_{\text{normal}} = (\mu - 2.06*\sigma) \quad (2)$$

ECVaR for a non-normally distributed fund is defined as:

$$ \text{ECVaR} = \text{CVaR}_{\text{fund}} - (\mu - 2.06*\sigma) \quad (3)$$

It is the difference between the fund’s $\text{CVaR}_{\text{fund}}$ and the $\text{CVaR}_{\text{normal}}$ under the assumption that the fund’s return distribution is normally distributed. ECVaR controls for the volatility or beta of the fund by subtracting $\text{CVaR}_{\text{normal}}$. The constant coefficient 2.06 in Equation 2 and 3 is replaced by 1.76 for the tail level of 10%.
References


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