

Handout – Map Coloring

Part 1 – General Maps

All of you would have seen a map before. For example, a map of the US with state boundaries or a map of the world with national boundaries. Imagine looking at a map of the US where all states had the same color. It would be hard to figure out where they ended and started without looking carefully. So, we would like to color in the states so that states which share a border have different colors. However, we are also being very stingy – we want to use the least possible number of colors in order to do this.

Let us take a simple map first. London, like many cities around the world, is built on a river. What we might want to do is to use colors to separate south of the river from north of the river Thames. Clearly, we need two colors, and two colors are all we need. For example, we can color north of the river green and south of the river red.



Let us try another, slightly more challenging example:



Clearly, even for this map, we can color it with 2 colors. For example, Colorado and Arizona can be colored green while New Mexico and Utah can be colored red.

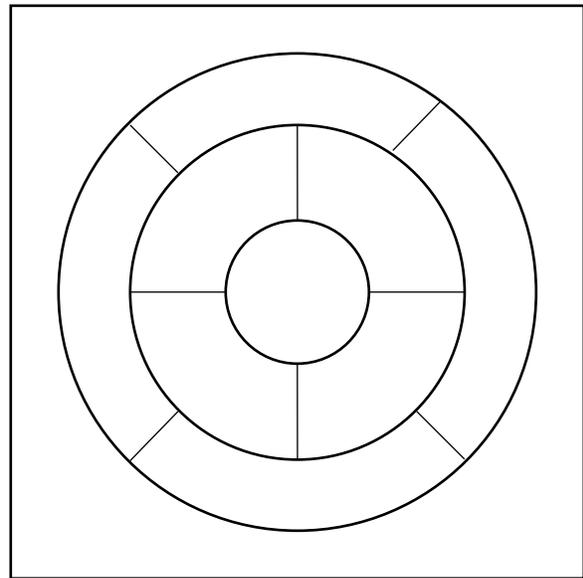
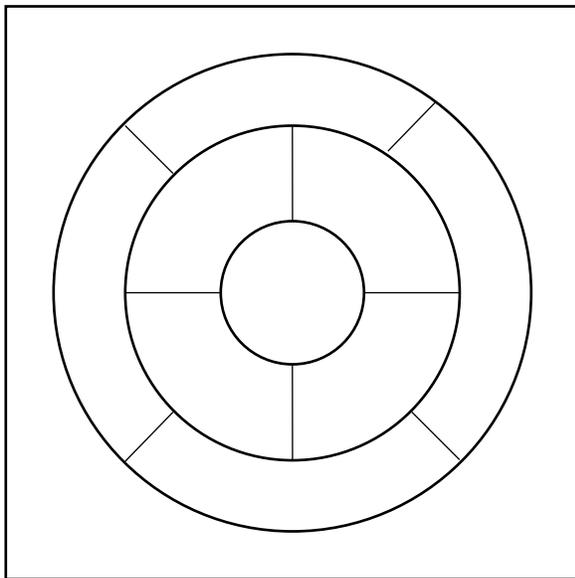
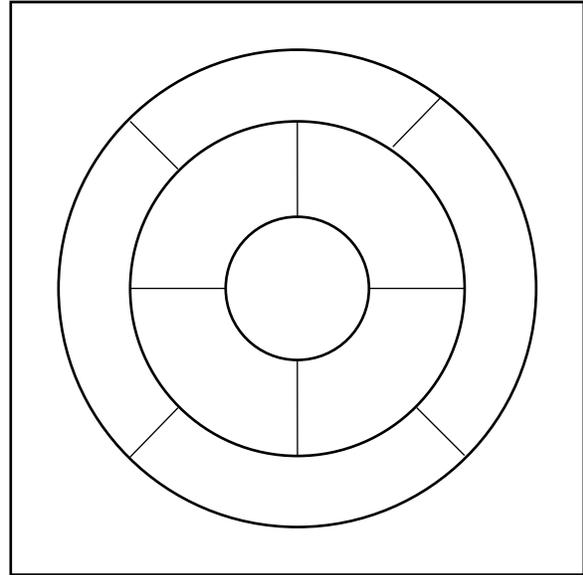
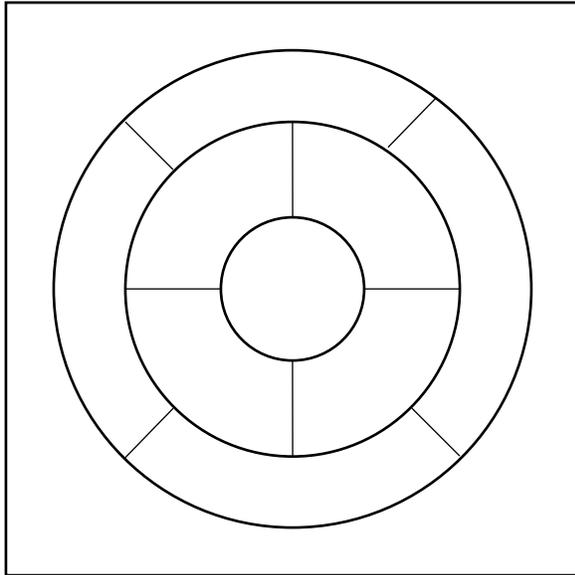
Exercise 1: So far, both the maps we have seen can be colored with 2 colors. For your first exercise, try to create a map which requires 3 colors.

I am going to make the following claim:

Claim: All maps require at most 3 colors.

Exercise 2: The first thing you should do when presented with any claim is to look for 'counter-examples.' What is a counter-example? A counter-example is an example which opposes the claim being presented. For the claim above, a counter-example would be a map which cannot be colored by 3 colors – which requires 4 or more colors. A single counter-example is enough to prove a claim to be false. Try to find one.

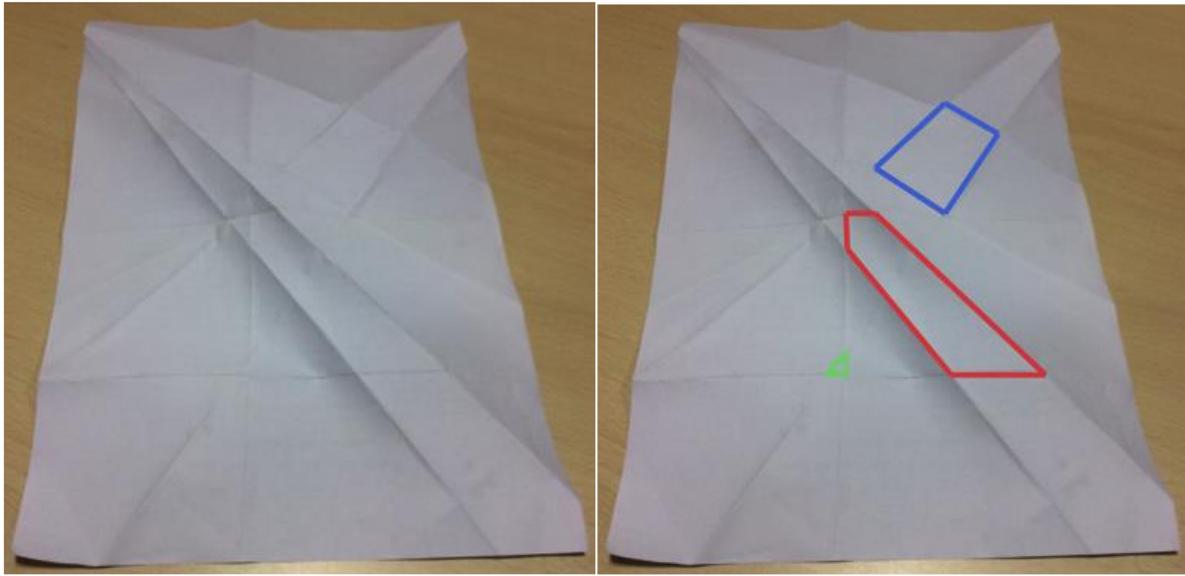
Exercise 3: The obvious next thing to do would be to find maps which require 5 colors. However, before we do that, I'm going to give you a map which is colored with 5 colors. Your job is to figure out if it can be colored with less than 5. Below are blank versions of the 5-colored map – you will be shown the colored in version.



Exercise 4: Try to create your own maps which require 5 or more colors. Do this in pairs. Once you create it, give it to your neighbor so that they can find a coloring with 4 or fewer colors.

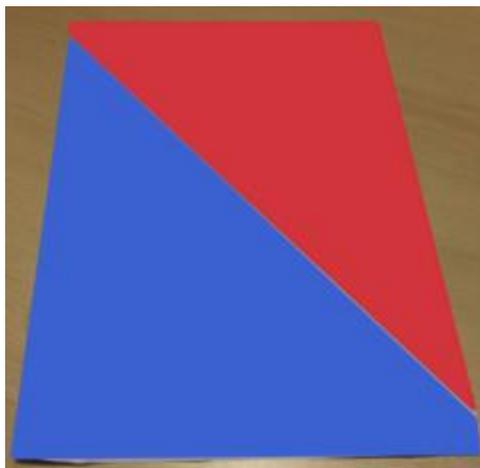
Part 2 – Paper Folding Maps

The maps we have worked on so far have no real restrictions. Anything you can draw on a flat piece of paper is allowed. However, now we will move on to specific kinds of maps. The first type of map we will consider can be obtained by folding a piece of paper using a straight fold, and then folding again and again. You will get something like this:



The regions of the map are the spaces in between the folds. See the figure on the right for examples of what regions look like.

We ask the same question about these ‘Folded Paper Maps.’ What are the minimum amount of colors required to color these maps? Clearly, you need at least 2 – here is an example of a map requiring 2 colors:



It just has one fold.

Exercise 5: Can you create a folded paper map which required 3 colors? Do this with your neighbor once again. You create a map and get them to color it with the minimum number of colors. Don't get too crazy with the folding, and once you have opened the folds, use a pencil to draw lines where the folds are.