Sense Perception, Counting, Measuring and Calculating

Perceiving and Counting

Suppose I ask you how many circles are there in the picture below, you would simply look at it and say 'three", without actually counting:

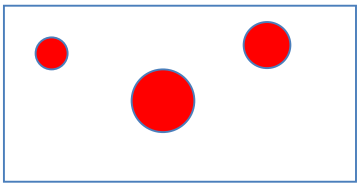


Figure 1

This would be a way of using your ***sense perception***. But if you were asked the same question about figure 2, you won't be able to answer the question merely by looking: you will have to ***count t***he circles.

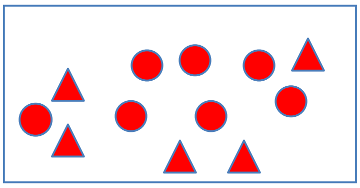


Figure 2

Likewise, if someone walked into your room and said, "Please let me know if I sneeze before I leave this room", you will be able to do it simply by listening, but if that person said "Please let me know how many times I sneeze before I leave this room" you will have to count. When a doctor looks at your tongue and says, "Hm, you have a coated tongue.", she is making an observation without counting, but when she feels your pulse and says, "Hm, your pulse rate is about 65 beats per minute." she is counting the number of pulses.

Measuring and Calculating

If a child asks you, "What is my height?" you may be able to say something like "Between a hundred and hundred and thirty centimeters", but mere sense perception may not allow you to respond with any more precision. Counting won't help you either. Yowever, you find the answer the question by asking the child to stand straight against a wall, making a mark on the wall against the top of the child's head, and ***measuring*** the distance from the mark to the ground. The result of measurement constitutes evidence for the answer "Your height is 123 cms."

What would you do if you wanted to find the height of a tree, and climbing the tree to measure the height is out of question?

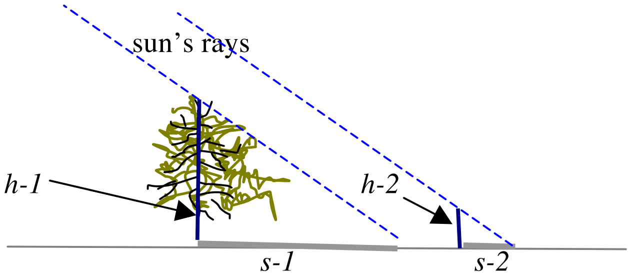
Tree

height of the tree

One solution would be to ***calculate*** the height. For instance, you may measure the length of the shadow of the tree, simultaneously measuring the length of the shadow of a vertical stick whose height is known. Let us suppose that:

The height of the tree is *h-1*

The height of the stick is *h-2,*
The length of the shadow of the stick is *s-1*, and
The length of the shadow of the stick is *s-2*,



The height of the tree can be ***calculated*** from the following equation:

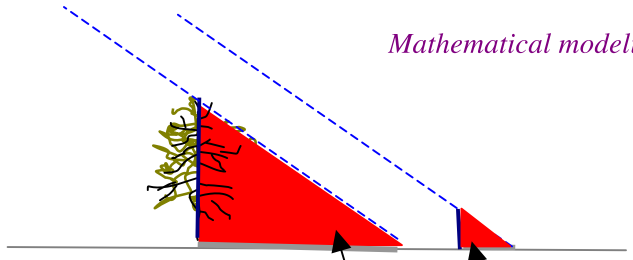
$$h\text{-}1 \,/\, s\text{-}1 = h\text{-}2 \,/\, s\text{-}2$$

That is,

$$h\text{-}1 = (h\text{-}2 \,/\, s\text{-}2)\; s\text{-}1$$

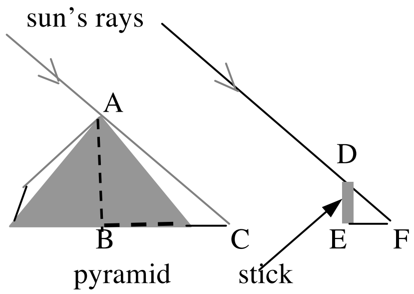
Since *h-2, s-2* and *s-1* are known, we can calculate the value of *h-1*.

The formula comes from the application of geometry, to be specific, the theorems on similar triangles. The use of the mathematics of similar triangles in this example is legitimized because of the assumption that the relations holding among the tree, stick, their shadows, and sun's rays can be ***modeled*** as similar triangles.



According to one of the stories about ancient Greek Philosopher-mathematician Thales (624–546 BCE), an Egyptian Pharaoh asked him to find out the height of pyramid in which his grandfather was buried. Thales couldn't directly measure the height of the pyramid. What he did was to place a straight stick on the ground, and when the length of its shadow cast by the sun was the same as the height of the stick, he measured the length of the shadow of the pyramid. On the basis of his knowledge of the mathematics of similar triangles, Thales concluded that the pyramid's height was the same as its shadow.[1]

[1] Two triangles are *similar* if their corresponding angles are identical. Two triangles are *congruent* if their corresponding sides are identical. Congruent triangles are necessarily similar, but the reverse is not true.



pyramid's shadow: BC

stick's shadow: EF

Thales needn't have waited till the shadow of the stick was of the same length as its height: he could have measured the shadow of the stick and the shadow of the pyramid at any time, and ***calculated*** the height of the pyramid from the height of the stick and the lengths of shadows of the pyramid and stick, using the formula:

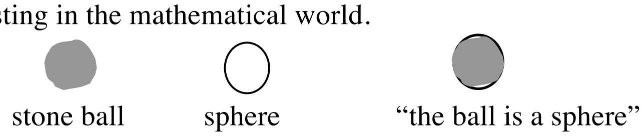
$$AB : BC = DE : EF$$

Implicit in such calculations is the idea of mathematical modeling. Pyramids, sticks, shadows and sun's rays are entities of the real world, while triangles are entities in the world of mathematics. What Thales did was to use strategy of ***mathematical modeling***. He superimposed the mathematical object on reality, viewing real objects as mathematical objects.

The nature of mathematical modeling will become clearer if we take another example. What would you do if someone gave you a stone ball and asked you what its volume was? One way of finding out would be fill a glass with water (to the brim), put the ball into it, and measure the quantity of overflowing water (as Archimedes did.) An alternative would be to measure the diameter d of the ball, and calculate its volume v using the following formula for the volume of a sphere:

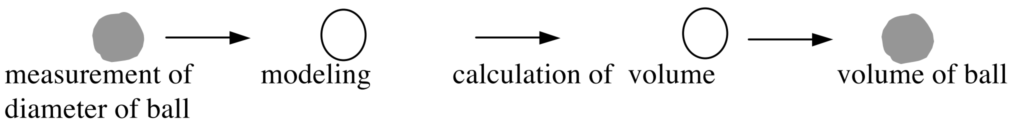
$v = 4/3\ \pi\ r^3$ $\quad$ v = volume, r = radius (= half the diameter)

Now, a stone ball is a real object in the world. In contrast, a sphere is mathematical object, an abstraction existing in the mathematical world.



A stone ball has weight, you can see it and touch and feel the texture. If you drop it, it makes a sound. And no real stone ball is perfectly spherical. As a mathematical object, however, a sphere has no weight, you cannot see it (even though we draw pictures to visualize it), you cannot feel it with your hands, and you cannot drop it. Unlike stone balls, spheres are perfectly symmetrical.

In using the formula for the volume of a sphere, we implicitly assume that the ball can be modeled as the mathematical object sphere, take a measurement from the real world, enter the world of mathematical objects with that measurement, make the calculation, and come back to the real world with the result.

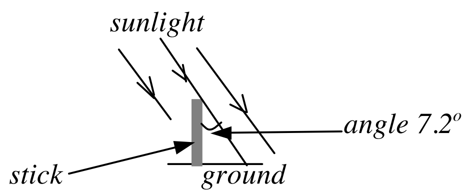


Mathematics is a product of human imagination and reasoning. For those who have reflected on the relation between mathematics and reality, it is a mystery that the products of human imagination and reasoning yield accurate results when applied to the real world: the volume of the stone ball arrived at through mathematical modeling, measurement and calculation matches the volume arrived at through the measurement of the displaced water. In a famous article on this topic, physicist Eugene Wigner referred to this puzzle as the "unreasonable effectiveness of mathematics." School students who are taught to make mathematical calculations without an understanding of the modeling behind the calculations take this activity for granted, thereby becoming incapable of appreciating Wigner's puzzle.

Ingenious calculations

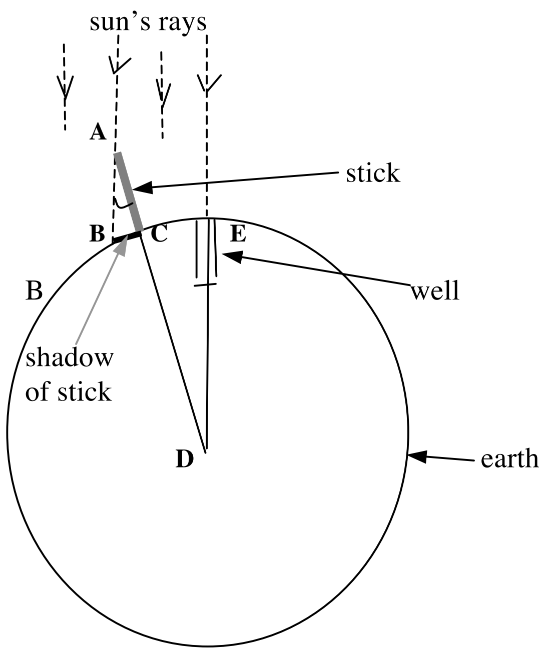
The value of the methodology of modeling-measuring-calculating illustrated above becomes clearer when we examine its use in finding out such things as the size of the earth, the moon and the sun, and the distance from the earth to the moon and the sun. In his book *Big Bang*, Simon Singh narrates the story of how Eratosthenes (b. 276 BCE in Cyrene, now in Libyia) calculated the circumference of the earth.[2] Eratosthenes knew that in Syene, a town in Egypt, the sun was directly above us at noon on 21 June each year, such that sunlight illuminated all sides of a famous well without casting a shadow. This did not happen in Alexandria, several hundred kilometers to the north of Syene. He also knew, on the basis of the work done by his predecessors, that the earth was round, not flat. This suggested a possible way of calculating the size of the earth.

At noon on 21 June, Eratosthenes placed a stick vertically on the ground in Alexandria. From the shadow cast by the sunlight, he measured the angle between the stick and the sun's rays as 7.2°.



Given the angle, and the distance from Syene to Alexandria, Eratosthenes calculated the circumference of the earth.

Here is how he did it. It was clear to the scholars of his time that the earth was spherical in shape (that it was not flat.) He assumed that the Sun is so far away from the earth that as far as someone on the earth is concerned, the rays of the sun are parallel. This yields the following picture:



[2] I would strongly recommend this book for anyone interested in the origin of the universe, or in the nature of scientific inquiry.

Here is how the calculations can be made. Angle BAC (the angle between the stick and the rays of the sun in Alexandria) equals angle CDE (the angle between the extension, to the earth, of the stick in Alexandria and that of the sun's rays in Syene.) Since the distance between Alexandria and Syene is the arc on the earth corresponding to this angle (7.2º), the total circumference (360º) can be calculated as:

Earth's circumference = Distance between Syene and Alexandria x 360/7.2

As Singh (2004:13) puts it, "It proved that all that was required to measure the planet was a man with a stick and a brain. In other words, couple an intellect with some experimental apparatus and almost anything seems achievable."

Taking the size of the earth as the starting point, Erastothenes, Aristarchus and Anaxagoras proceeded to extend the strategy of modeling-measuring-calculating to find out the size of the moon and the sun, and their distances from the earth. How they accomplished it is an impressive and riveting story, but I won't summarise it here. Those who are interested will find a clear and well-narrated account in Singh (2004:13-20).