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Bayesian Priests and Girls

In classical deductive reasoning you cannot conclude anything about Q -> P from P -> Q.

For example, given the premises

All humans have exactly one heart

The object in front of me has one heart

You cannot conclude that the object in front of me is human. It could be a chimpanzee or an alligator or even a refrigerator with human one human heart waiting to be transplanted.

Probabilistic reasoning, on the other hand, can allow you to draw inferences about the probability of P given Q being true from the probability of Q given P being true.

Lets take some cases of probabilistic reasoning.

Consider the following scenarios:

Case 1:

Premise 1: Christian priests are very likely to wear crosses

Premise 2: The person I see in front of me is wearing a cross

Conclusion 1: The person I see in front of me is a Christian priest

Case 2:

Premise 3: Girls are very likely to have long hair (as opposed to short hair)

Premise 4: The person I see in from of me has long hair

Conclusion 2: That person is a girl

In both these cases, the reasoning is of the form

P -> Q

Q

Therefore, P

In classical deductive reasoning, we can say nothing about P given P -> Q and Q.

However, in reality the conclusion in Case 2 seems like a good conclusion while the conclusion in Case 1 doesn't. Why?

Clearly, Conclusion 2 does not follow from only premises 3 and 4. If it did, Conclusion 1 would follow from premises 1 and 2. So, we need to add in some premises which are true in reality and we are assuming to be true in our mind while making the judgement that conclusion 2 makes sense:

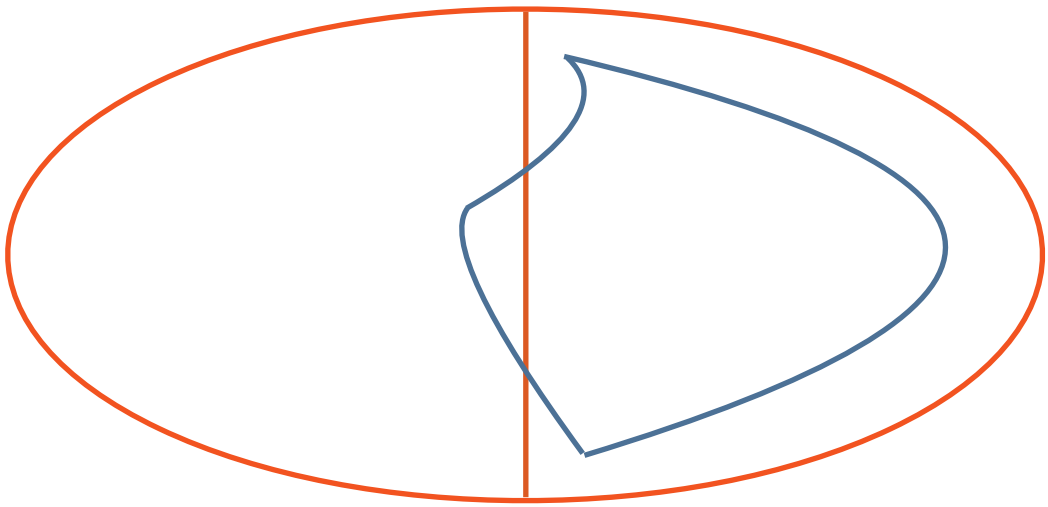
Premise 3: Girls are very likely to have long hair (as opposed to short hair)

Premise 4: The person I see in from of me has long hair

Premise 5: Very few non-girls have long hair

Premise 6: There are approximately an equal number of girls and non-girls

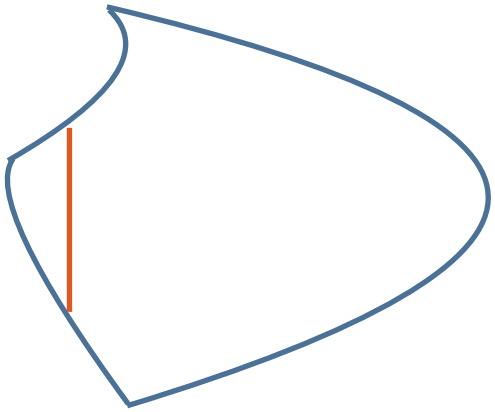
Are these premises enough to arrive at our conclusion? Lets represent this argument in the form of a diagram to find out.



Let the oval above represent people. From premise 6 we know that there are an equal number of girls and non-girls. Let the left half of the oval above represent non-girls and the

right half represent girls. The weird blue-bordered shape inside represents people with long hair. We know very few non-girls have long hair and almost all girls have long hair.

Now, the person in front of us has long hair. So, what we are interested in now is only the weird blue shape because we know that person is in there:



Now the left of the orange divider represents non-girls while the right represents girls. So, it is reasonable to conclude, with some doubt, that if you see a person with long hair, they are a girl.

Now, lets take the case of the priest. Here, is the argument with some missing premises similar to the ones for the girls

Premise 1: Christian priests are very likely to wear crosses

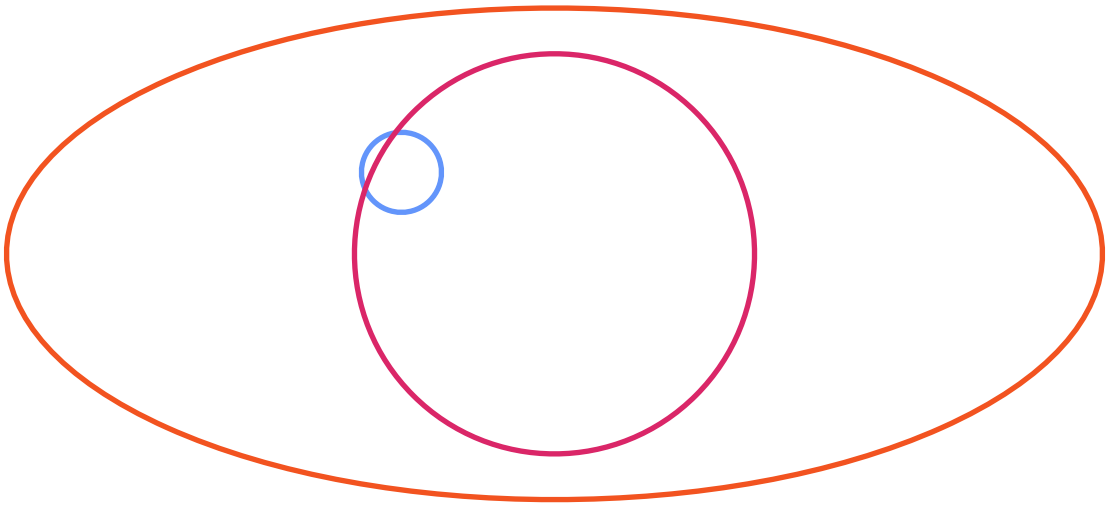
Premise 2: The person I see in front of me is wearing a cross

Premise 7: There are many Christian non-priests who wear crosses

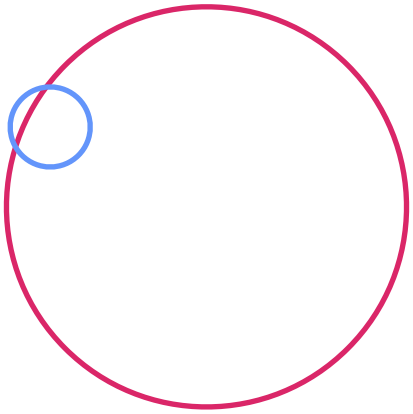
Premise 8: Christian priests make up a very small percentage of christians

Premise 9: No non-christians wear crosses

Premise 9 is only there for convenience. Given premise 9, we can restrict ourselves to Christians since the person in front of me is wearing a cross and no non-christian wears a cross.



The oval represents all Christians. The blue circle represents Christian priests and is very small which is due to Premise 8. Since most Christian priests wear crosses and many Christian non-priests wear crosses, the larger circle, represents the Christians who wear crosses. We know that the person in front of us is wearing a cross, so we need only concern ourselves with the larger circle



Here, as you can see, non-priests outnumber priests. So, the conclusion we can draw is the opposite of the one we had arrived at earlier. What we can say is that: 'it is reasonable to conclude, with some doubt, that the person in front of me is not a Christian priest.'

These are very simple examples of a type of reasoning called Probabilistic reasoning or Bayesian Reasoning. Here, we are using Bayesian reasoning to find out something about the Probability of P given Q from the Probability of Q given P. Bayesian Reasoning is also used to get the probability of a Hypothesis being true given evidence from the probability of evidence being found given the hypothesis. You can use this form of reasoning in order to proportion your belief in a hypothesis to the evidence you have for it. More on that later.