Introduction to Logics

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In this session, I introduced the students to the logics of rational inquiry. I would like to share my account of the session, (i) to sketch how it can be done in a classroom, adaptable as a video lesson that students and teachers can learn from, and (ii) for your feedback.

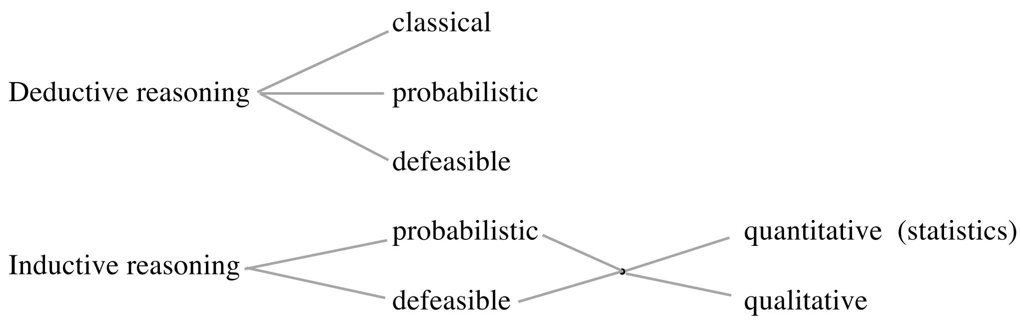
1. Reasoning in academic inquiry: a syllabus

The students in the class had already come across examples of mathematical, scientific, and conceptual inquiries, and worked through them. I began the class by sketching a syllabus for reasoning in the context of these three modes of inquiry. Reasoning is the *process of arriving at inferences from the grounds* that are available to us. It is a central thread that runs through the concepts of *proof*, *evidence*, *arguments*, *justification*, and *debate*. I summarised the core features of the three inquiries, and indicated the relevant types of reasoning: at this stage, the terms were simply pegs for them to hang their understanding on. Here is what emerged on the white board.

> *Axioms* and *definitions* form the grounds for *mathematical inquiry*. Their *logical consequences*, called *theorems*, are derived through CLASSICAL DEDUCTIVE REASONING.
>
> *Data points* form the grounds for *scientific inquiry*. Samples of data points yield *observational generalizations* through INDUCTIVE REASONING. Such generalizations include qualitative and quantitative statements on the distribution of the values of a variable, or the correlation between two variables.
>
> Observational generalizations, once established, need to be *explained*, for which scientists invent theories, made up of *theoretical concepts, laws,* and *models*. The logical consequences of theories are called *predictions*. When the predictions of a theory match the observational generalizations, we say that the theory explains those generalizations. Predictions are derived from theories through CLASSICAL, PROBABILISTIC, and DEFEASIBLE DEDUCTIVE REASONING.
>
> A theory and its components have to be rationally justified, in terms of an 'explanation-based' argument that uses SPECULATIVE-DEDUCTIVE REASONING.
>
> Speculative-deductive reasoning is used to establish either a theory as a whole, or a particular component of the theory. To defend a specific theory-internal explanation of an observational generalization, or an interpretation of a data point, we use ABDUCTIVE REASONING. It appeals to a combination of data points and an established theory as its grounds. It is crucially used in medical diagnosis, and in historical inquiry (including forensics, paleontology, archeology, historical linguistics, and the prosecution lawyer's arguments to establish the guilt of the accused).
>
> *Shared judgments on examples* form the grounds for *conceptual inquiry*. As in theoretical science, we explain these judgements by showing that they can be derived from the propositions of the theory through one of the three forms of deductive reasoning mentioned above.
>
> An account of a typology of reasoning in inquiry must include CAUSAL REASONING. There is a specific class of theories that use *causal* explanations, and require causal reasoning.

The items given in SMALL CAPS above form the syllabus topics for a module or course on the logics of rational inquiry. At this stage, not surprisingly, they are just words, not concepts that the students can understand. Understanding of the concepts would happen gradually, by the end of the course/module.

The diagram given below indicates the structure of the syllabus topics.



Speculative deductive reasoning
Abductive reasoning
Causal reasoning

The intended learning outcomes of the module can now be stated as:

The module on logics seeks to help students

(a) *understand* the concepts associated with the words in the diagram; and
(b) develop the *reasoning abilities* associated with each mode of reasoning, as a subset of inquiry and critical thinking abilities.

2. Good and bad arguments, premises, and conclusions

As mentioned above, reasoning is the *process of arriving at inferences from grounds*. Logic is the *study of reasoning*: it allows us to *distinguish between good reasoning and poor reasoning*.

When talking to a friend, suppose we observe that he has stopped smiling, his face has turned red, and his voice is very tense. We conclude that he is angry. The grounds for this conclusion are our observations. Likewise, suppose we are told that Zeno's pet has a long beak. We conclude that Zeno's pet is a bird. The grounds for this conclusion is the statement that it has a beak. Reasoning is the process of deriving the conclusion from such grounds.

When the grounds of an argument are formulated as propositions, logicians call them *premises*. Thus, the English sentence, "Zeno's pet has a beak," expresses a premise-proposition, while the sentence, "Zeno's pet is a bird," expresses a conclusion-proposition. [Asking students to identify the premises for the conclusion that the friend we are talking to is angry would be a good practice activity.]

When we are given inferences of this kind, we judge the reasoning in some of the examples to be good or legitimate. This means that the conclusion follows from the given grounds. There may also be examples whose reasoning we judge to be poor or illegitimate. Here we mean that the conclusion does not follow from the grounds. For instance, we judge the reasoning in example 1 to be good, in contrast to the reasoning in example 2, which we judge to be bad:

Example 1	Example 2
Zeno is a human being.	Zeno is a human being.
Therefore, he has exactly one heart.	Therefore, Zeno has exactly three mouths.

In 1, we can see that the conclusion follows from the grounds, but not in 2.

3 Implicit premises and sound arguments

Why are our judgements on the arguments in examples 1 and 2 different? Here is why. Let us note that the reasoning in 1 carries the implicit assumption that every human being has exactly one heart:

Example 1 (expanded)

Premise 1:	*Zeno is a human being.*	(explicitly stated)
Premise 2:	*Every human being has exactly one heart.*	(implicit)
Conclusion:	*Therefore, Zeno has exactly one heart.*	
	(The conclusion logically follows from premises 1 and 2.)	

We automatically supply the missing premise without even being aware of it. We accept this premise as correct. This is why we judge the reasoning in the unexpanded version of example 1 to be good even though the premise is not explicitly stated.

The reasoning in 2 carries the implicit assumption that every human being has exactly three mouths:

Example 2 (expanded)

Premise 1:	*Zeno is a human being.*	(explicitly stated)
Premise 2:	*Every human being has exactly three mouths.*	(implicit)
Conclusion:	*Therefore, Zeno has exactly three mouths.*	
	(The conclusion logically follows from premises 1 and 2.)	

In example 2, we can't even imagine that someone would assume something as blatantly false as premise 2. Hence, in the unexpanded version of example 2, we can't see how the conclusion follows, and so we judge the argument to be flawed.

In short, we accept premise 2 in example 1 as true, but reject premise 2 in example 2 as false. We now understand why we judge these two instances of reasoning differently. In the literature on logic, the technical terms for 'good' and 'bad' arguments are ***sound*** and ***unsound***. The argument in example 1 is sound, but that in example 2 is unsound.

Some implicit premises are false, but this may not be obvious until they are stated explicitly. Therefore, when we critically evaluate arguments, it is important to unearth and explicitly state all the missing premises. Take the following argument:

Zeno has higher student feedback scores than Athena.
Therefore, Zeno is a better teacher than Athena.

To an untrained mind, this argument might look sound. But notice that the conclusion is based on the implicit assumption that:

Student feedback scores are a direct measure of the quality of a teacher.

If this assumption is true, then it follows that:

If X has higher student feedback scores than Y, then X is a better teacher than Y.

Only when we have unearthed and examined the implicit asssumption can we evaluate it. Is the assumption acceptable? Are student feedback scores a direct indicator of the teacher's quality of teaching, or of her popularity? Suppose teacher X in a university offers a tough course, and teacher Y offers a diluted course. The quality of X's teaching may be higher, but who do you think would get a higher score? If we reject the idea of student feedback scores as a direct measure of teaching quality, it becomes clear that the initial impression of the soundness of the above argument was without basis.

Similar remarks apply to the following example:

Zeno has higher IIT JEE scores than Athena.
Therefore Zeno is more intelligent than Athena.

This carries the implicit assumption that *IIT JEE scores are a direct measure of a person's intelligence*, leading to the implicit premise that *for any X and Y, if X's IIT JEE scores are higher, then X is more intelligent*. How acceptable is this position?

> Note: We discussed the IIT JEE issue at length, and arrived at a collective decision that it tests information, guesswork, and speed in mechanical calculation acquired through repeated practice, but is not a measure of intelligence.
>
> It became clear that students need **a lot** of practice in unearthing and evaluating hidden assumptions in arguments. This is a strand of critical thinking ability that is as important as the ability to detect logical contradictions. I would recommend a diet of at least one or two tasks a week to unearth and evaluate hidden assumptions.

4 Valid and invalid reasoning

As we can gather from the above examples, the first step in the evaluation of an argument is the evaluation of the premises: *it is only if the premises can be accepted as correct that a given argument can be judged as sound*. If one of the premises is judged to be false, the argument is unsound.

But is that all there is to soundness? Consider the following example:

Example 3
Premise 1: *Zeno is a human being.*
Premise 2: *Every human being has exactly one heart.*
Conclusion: *Therefore, Zeno has exactly one mouth.*

We accept both premises in this example as true. In fact, we even judge the conclusion to be true. Yet, we judge the argument to be unsound. This is because the conclusion does not follow logically from the premises.

The technical terms for "conclusion follows from the given grounds," and "conclusion doesn't follow from the given grounds," are ***valid***, and ***invalid*** respectively. The reasoning in the argument in the expanded versions of examples 1 and 2 is valid, but that in example 3 is invalid.

It is important to note that *validity of reasoning* does not depend on the truth of either the premises or the conclusion. We accept the premises and the conclusion in example 3 as correct, but still judge the argument to be invalid. Conversely, in example 2, we reject premise 2 and the conclusion as false; but we still judge the argument to be valid. [This point is extremely important.]

Based on these examples, we can define soundness as follows:

An argument in support of a conclusion is judged to be sound if and only if
the grounds are accepted as correct; and
the reasoning is valid.

Notice that we have been using the terms 'reasoning' and 'argument' with slightly different meanings. An argument is the rational justification (proof, defence) advanced in support of a conclusion or claim. It has three parts: the grounds, the reasoning, and the conclusion. Reasoning is one of the components of an argument, the steps leading from the grounds to the conclusion.

The structure of an argument: grounds → reasoning → conclusion

We can now say that *logic is the study of the validity of reasoning in rational justification*.

5. Rules of Inference in Logic

Consider the following examples:

Example 4

Premise 1: *If Zeno was in Delhi on 12 May 2013, then he was in Mumbai on 7 May 2013.*
Premise 2: *Zeno was in Delhi on 12 May 2013.*
Conclusion: *Therefore, Zeno was in Mumbai on 7 May 2013.*

Example 5

Premise 1: *If Zeno was in Delhi on 12 May 2013, then he was in Mumbai on 7 May 2013.*
Premise 2: *Zeno was in Mumbai on 7 May 2013.*
Conclusion: *Therefore, Zeno was in Delhi on 12 May 2013.*

We judge the reasoning in example 4 to be valid, and the reasoning in example 5 to be invalid. Why do we say that the conclusion follows logically from the premises in 4 but not in 5?

Systems of logic are constructed to allow us to derive logical consequences, and arrive at conclusions through valid means. Central to this pursuit is the concept of ***rules of inference*** that sanction the steps of reasoning in an argument.

To see what this means, let us construct the relevant rule of inference that distinguishes 4 from 5. But first, we need a symbolic notation. Let us represent atomic propositions as upper case letters P, Q, R, etc. In examples 4 and 5, we can break down premise 1 into two atomic propositions. let us use P to represent *Zeno was in Delhi on 12 May 2013*, and Q to represent *Zeno was in Mumbai on 7 May 2013*:

If	*Zeno was in Delhi on 12 May 2013,*	*then*	*he was in Mumbai on 7 May 2013.*
	P		**Q**

If so, premise 1 in examples 4 and 5 can be represented as:

If P, then Q.

Using the arrow → as a symbol to represent "if…then…" we can represent premise 1 as:

$P \rightarrow Q$

Using this notation, we can symbolically represent the arguments in 4 and 5 as follows:

	Example 4		Example 5
	$P \rightarrow Q$		$P \rightarrow Q$
	P		Q
therefore	Q	therefore	P

We may now state the relevant rule of inference, called *Modus Ponens* in the literature, as follows:

Modus Ponens

Given the premises $(P \rightarrow Q)$ and P
it is resonable to conclude that Q

This rule sanctions the reasoning in example 4, but not in example 5. Suppose we state the essence of "X follows from Y"as the validity condition given below:

An argument is valid in a logical system S if and only if
the conclusion and every step leading from the premises to the conclusion
are sanctioned by a rule of infernce in S.

Since Modus Ponens as stated above is the only rule of inference we have so far, it follows that the reasoning in example 4 is valid in the system we have developed so far (the conclusion is sanctioned by Modus Ponens). In contrast, the conclusion is not sanctioned by any rule of inference in any system of logic we have developed so far, and hence the reasoning is invalid.

Obviously we need to set up more rules of inference. We also need to set up other systems of logic. That is what we will do next.

Remaining time in class

In the remaining time, I covered the following topics:

- Classical deductive reasoning
  - Propositional calculus: the symbols for ‘and’, ‘or’ and negation
  - Predicate calculus: the unpacking of propositions
    - (represented as atomic symbols in propositional calculus) into predicates and arguments.)
    - The students were familiar with all this, probably from their compuer science courses.
- Probabilistic deductive reasoning and probabilisic modus ponens
- Defeasingle deductive reasoning and defeasible modus ponens
- Inductive reasoning

Since (a) this document is already six pages and (b) these details of the rest of the topics are covered in the lengthy document on logics (distributed to the band members earlier), I am not going to go into how I covered the above topics.

I made brief mention of frequentist and Baysean approaches to probability in the class. In the next class, we will go on to the distinction between probabilistic and defeasible inductive reasoning, and also speculative-deductive reasoning, abductive reasoning, and causal reasoning.

Plan for youtube videos

I want to use the structure in sections 2-5 as an entry point into a set of video lessons on logics. The general perspective in section 1 will have to be replaced by something that does not refer to the characteristics of the different modes of inquiry, because, unlike the CoEP students, the general audience may not have worked through exercises in these modes of inquiry. I will have to find an alternative intro for the videos.