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The Logics of Scientific Inquiry

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[This paper may be viewed as material for a crash course in applied logic for science students.]

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1 The Purpose of this Learning Resource Unit

In broad terms, activities that are integral components of scientific inquiry include:

A. ***Noticing*** something interesting and worthwhile to pursue, and formulating a ***question*** about it.

B. Making ***particular observations*** and recording them, as part of a body of ***data***, to look for an answer to the question. [These data may be qualitative or quantitative, gathered in a laboratory or in a "field", with or without the aid of specialized instruments.]

Suppose we are gathering data on various aspects of human beings, for instance, their height, weight, age, gender, hair colour, skin colour, eye colour, blood type, smoking habits, and so on. The term ***variable*** refers to each of these aspects under study. Each variable has a ***value***. The variables and their values for two human subjects are given below by way of illustration:

Human subject	Height	Weight	Age	Gender	Hair colour	Skin colour	Eye colour	Blood type	Smoking habits
1	1.62m	67kg	54	male	red	pink	blue	B+	non-smoker
2	1.57m	64.3 kg	48	female	black	brown	black	B+	heavy smoker

The cluster of values of the relevant variables for an entity constitutes what we call a ***data point***. The information given above, for instance, constitutes two data points.

If we are studying the behaviour of pendulums, a data point would be the length, size, period (time taken for a complete back-and-forth cycle), weight, and so on of one pendulum. If we are studying the behaviour of planets, a data point would be the position of a particular planet at a given time.

C. Finding patterns in those observations/data/information, formulating them as *observational* ***generalizations***, and establishing them as reliable "facts".

Based on a large body of data points, we might arrive at observational generalizations (OG) like:
OG 1: The average height of a human being is 6.2 m.
OG 2: Male humans are taller than female humans.
OG 3: Smoking causes cancer.
OG 4: There are no humans with natural blue hair or natural blue skin.
OG 5: There is a correlation between the period of a pendulum and its length.
OG 6: All birds have two legs. (There are creatures with 4, 6, and 8 legs, but not birds.)

D. Constructing ***theories*** to *explain* those observational generalizations, and establishing them as reliable explanations.

Why and how does smoking cause cancer? Why are human males taller than human females? How come there are no four- or six-legged birds? Answers to such questions result in theories.

E. ***Interpreting*** particular observations in terms of existing theories.

Suppose we record a set of observations as telescope readings of the images of stars for a particular location. Based on these records, we might conclude that there must be a black hole at the center of that location. This conclusion is an interpretation within the existing theory of cosmology. In some cases, an interpretation may result in an explanation of those observations.

F. Arriving at a ***conclusion*** on the basis of the above activities.

G. ***Justifying*** ("proving") that conclusion to the satisfaction of other inquirers.

H. ***Critically evaluating*** one's own and others' conclusions and justification.

Engaging in C-H requires us to use diverse modes of reasoning. The aim of what follows is to sketch a systematic framework of reasoning (with occasional dips into the relevant logic that underlies the reasoning), to help students engage in these inquiry tasks with greater understanding and expertise.

We expect that you already have some familiarity with some of the components of scientific inquiry in A-H. If not, it would be useful to go through the following write-ups (available at https://sites.google.com/site/sciediiserp/additional-materials):

Introduction to Academic Inquiry and Critical Thinking
Justification
Inquiry Strategies
Introduction to Trans-disciplinary Courses on Rational Inquiry
What is Science?

In addition, by way of preparation, we would also like to recommend the following notes (available at https://sites.google.com/site/eplsmohanans/main-text/part-1-patterns-and-puzzles-in-language/part-1-contents):

Note 1: Puzzles and Explanations in Everyday Life
Note 2: Randomness and Regularity: Patterns in Data
Note 3: Null Hypotheses and the Unhabituated Mind
Note 8: Explanations and Theories
Note 9: Frameworks and Constructs
Note 12: Ways of Knowing
Note 13: Scientific Thinking in Everyday Life

2 Introduction to Reasoning

2.1 What is 'Reasoning'?

If we see dark clouds in the sky, we infer that it is likely to rain. If we hear laughter behind a closed door, we infer that there is either a person or a device (tape recorder/TV set…) behind that door. The experience of seeing dark clouds and of hearing laughter is an instance of the ***grounds*** that lead us to the ***conclusion*** about the likelihood of rain or about a person or device behind the door. The process of ***inference*** connects the grounds to the conclusion.

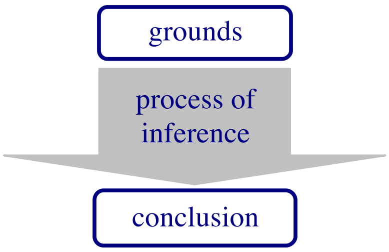


figure 1

In some cases, the grounds and the steps leading from the grounds to the conclusion can be expressed (either verbally or using some other symbolic system) in such a way that someone else can scrutinize the process and assess its merit. For instance, told that Zeno is a spider, we can infer that Zeno has eight legs. We can express this inference verbally as follows:

Zeno is a spider.
Therefore, it is reasonable to conclude that Zeno has eight legs.

How exactly does the conclusion follow from Zeno being a spider? Our reasoning would be:

Zeno is a spider.
All spiders have eight legs.
Therefore, it is reasonable to conclude that Zeno has eight legs.

Given the information that Leda's room is 10 ft wide and 15 ft long, we can reasonably conclude that its area is 150 sq ft: multiplying length by width gives us the area.

This is not to say that all forms of inference can be overtly expressed. Suppose you walk into an auditorium with a friend, look around at the people seated there, and tell your friend, "There seem to be almost a hundred people here." You didn't arrive at this conclusion by actually counting the number of people; rather, you made an inference based on your visual experience, through estimation. If your friend were to challenge you to articulate the steps of your inference in words or numbers, you may not be in a position to do so, because your conclusion is not open to step-by-step unpacking.

Reasoning is a particular form of inference, one that can be expressed in words or other symbols such that the legitimacy of the process can be evaluated.

2.2 Premises and Conclusions

The grounds that we appeal to in our inferences, expressed in words, numbers, or a combination of the two, are called ***premises***. Take a look at the premises in our examples of Zeno's legs and the area of Leda's room:

(**1**)	Zeno is a spider.	Premise 1
	All spiders have eight legs.	Premise 2
	Therefore, it is reasonable to conclude that Zeno has eight legs.	Conclusion

(**2**)	The width of Leda's room = 10 ft	Premise 1
	The length of Leda's room = 15 ft	Premise 2
	Hence, the area of Leda's room = 150 sq.ft	Conclusion

Take another example to illustrate the concept of premises. Suppose we tell you that all gleeps are dovines, all dovines have green eyes, and Limpy is a gleep. What is the colour of Limpy's eyes? Even though you have no idea what gleeps and dovines are, or who Limpy is, you will have no trouble inferring that Limpy's eyes are green. The reasoning here can be expressed as:

(**3**)	All gleeps are dovines.	Premise 1
	All dovines have green eyes.	Premise 2
	Limpy is a gleep.	Premise 3
	Therefore, Limpy has green eyes.	Conclusion

Reasoning then is *the process of arriving at a conclusion from a given set of premises:*

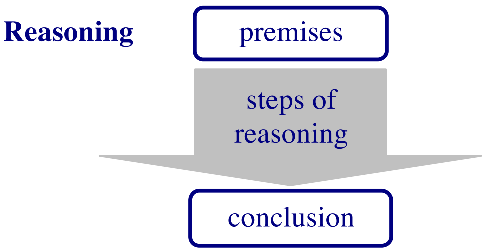


figure 2

2.3 Spelling out the Chain of Reasoning

Notice that example (3) leaves out an intermediate step in its reasoning. Given that all gleeps are dovines, and that Limpy is a gleep, we conclude that Limpy is a dovine. It is only then that, given that all dovines have green eyes, we can conclude that Limpy has green eyes. Here is a version of (3) that spells out the implicit steps:

(**4**)	All gleeps are dovines.	Premise 1
	Limpy is a gleep.	Premise 3
	Therefore: Limpy is a dovine.	Conclusion (intermediate)
	Limpy is a dovine.	
	All dovines have green eyes.	Premise 2
	Therefore: Limpy has green eyes.	Conclusion (final)

2.4 Implicit and Explicit Premises

Intuitively, we recognize the reasoning in example (2) as legitimate. However, there are two crucial premises that are taken for granted in this reasoning, namely, that Leda's room is a rectangle, and that the area of a rectangle is the product of its length and width. To be explicit, therefore, we must spell out these implicit premises as follows:

(**5**)	Leda's room is a rectangle.	premise 1
	The length of Leda's room = 15 ft.	premise 2
	The width of Leda's room = 10 ft.	premise 3
	The area of a rectangle = length x breadth.	premise 4
	Therefore: The area of Leda's room = 150 sq ft.	conclusion

In actual practice, we often leave out premises that we take for granted as readers as well as writers. Though the inexplicitness created by missing steps and implicit premises is not a serious flaw in the examples we just saw, such inexplicit premises can sometimes be false or untenable, and implicit steps may involve an invalid derivation. For instance, though most rooms are rectangular in shape, it is possible that Leda's room is elliptical. If so, the conclusion in (2)/(5) would be false. When we evaluate a sample of reasoning, therefore, it is important to unearth the hidden premises and check if any of them is false or untenable.

3 Deductive Reasoning

3.1 Classical Deductive Reasoning

3.1.1 The "If-then" Conditional

As we said earlier, reasoning is the process of arriving at inferences from a given body of information. The discipline of logic systematizes reasoning. Our purpose of studying logic here is to:

- further develop and strengthen our reasoning capacity, and
- evaluate the legitimacy of the reasoning that we are exposed to.

To achieve this purpose, logic has to enable us to distinguish between good and bad reasoning. And how do we distinguish between good and bad reasoning? Take the following examples:

A All humans are mammals.
Zenta is a human.
Therefore Zenta is a mammal.

B All humans are mammals.
Xilo is a mammal.
Therefore Xilo is a human.

Intuitively, we recognize A to be an instance of good reasoning. We also recognize B to be an instance of bad reasoning: that all humans are mammals doesn't mean that all mammals are humans. So if Zeno is a horse, it is a mammal, but not a human.

We expect logic to tell us why A is good and B is bad. It does so by formulating **rules of inference**, the *abstract patterns that sanction a conclusion from a given set of premises*. To state these rules of inference, the first step is to express the propositions such that their logical structure becomes clear. The sentence, *All humans are mammals*, expresses the proposition: "For any x, if x is a human, x is a mammal." Translating sentences as propositions, let us restate examples A and B as A' and B':

A′
For any x, if x is a human, x is a mammal.
Zenta is a human.
Therefore Zenta is a mammal.

B′
For any x, if x is a human, x is a mammal.
Xilo is a mammal.
Therefore Xilo is a human.

Let us denote *x is a human* as P and *x is a mammal* as Q:

For any x, if x is a human (P), x is a mammal (Q).

We now see that the sentence, *All humans are mammals*, expresses a complex proposition consisting of two simple propositions P and Q, and has the form: If P, then Q.

Logicians use an arrow to denote the "if-then" relation, and write "If P, then Q" as: P → Q. A proposition of the form "P → Q" is called a ***conditional*** in logic. A conditional expresses a relation between the ***antecedent*** proposition (P) and the ***consequent*** proposition (Q). What it says is: *If P is true, then Q is also true*.

We can now express the logical structure of examples A′ and B′ as A″ and B″:

	Example A″	Example B″
Premise 1:	P → Q	P → Q
Premise 2:	P	Q
Conclusion:	Therefore Q	Therefore P

3.1.2 A Rule of Inference: Modus Ponens

Why is the structure in B″ bad, in contrast to that in A″? An answer calls for a *rule of inference*. The rule of inference relevant here is called *Modus Ponens* in deductive logic, and it says:

Rule of inference: ***(Classical) Modus Ponens***

Given that:	*If P is true, then Q is true*
and	*P is true*
	it is reasonable to conclude that: *Q is true*.

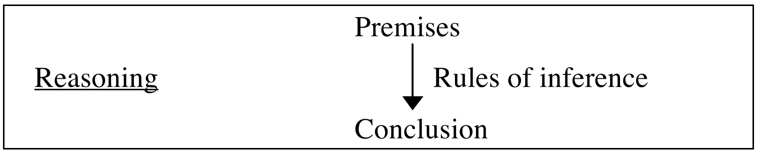
Let us express this rule of inference using the arrow notation:

Rule of inference: (**Classical) Modus Ponens**

If	(P → Q)	is true
	and P	is true
then	Q	is true.

This rule sanctions the reasoning in A″, but not in B″. The reasoning in B″ is bad because the rule of Modus Ponens does not sanction it, nor is there any other rule of inference that legitimizes it. (We will postulate more rules of inference as we proceed.)

The assumption that we are making above is that *each step in a chain of reasoning from premises to conclusions must be mediated by a legitimate rule of inference*:



3.1.3 Negation, Conjunction, and Disjunction

It would be useful to expand the notation introduced above to include three more concepts, those of negation ("not"), conjunction ("and"), and disjunction ("or"). Compare the sentences in (6) with those in (7):

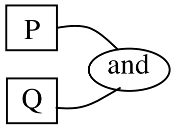
(6) a. Zeno is at home now.
b. Zeno's sister is making tomato soup.
c. Zeno's mother is happy.

(7) a. Zeno is not at home now.
b. Zeno is at home now and Zeno's sister is making tomato soup.
c. Either Zeno is at home now or Zeno's sister is making tomato soup.

Notice that if (6a) is true, (7a) cannot be true, and vice versa. The propositions expressed by these sentences, in other words are *logically inconsistent* (unless they are uttered at different times, or the name "Zeno" refers to different people, and so on). We can express this relation by representing (6a) as P and (7a) as not-P. Logicians use the symbol ¬ or ~ to notate negation (i.e., "it is not the case that"). Paraphrasing (7a) as: "It is not the case that Zeno is at home now," they represent it as ¬P or ~P. We will just use 'not-P' for convenience of typing.

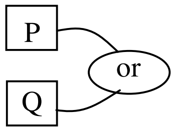
(7b) is a combination of (6a) and (6b). Taking (6a) as P (6b) as Q, and using "and" to represent conjunction, we can represent (7b) as:

P-and-Q   or pictorially as:



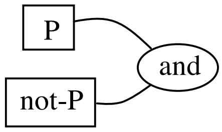
Likewise, representing disjunction as "or", we can represent (7c) as:

P-and-Q   or pictorially as:



Using this notational system, we can represent Aristotle's *prohibition of logical contradiction* as:

Any proposition of the form



is false.

3.1.4 Propositional Network

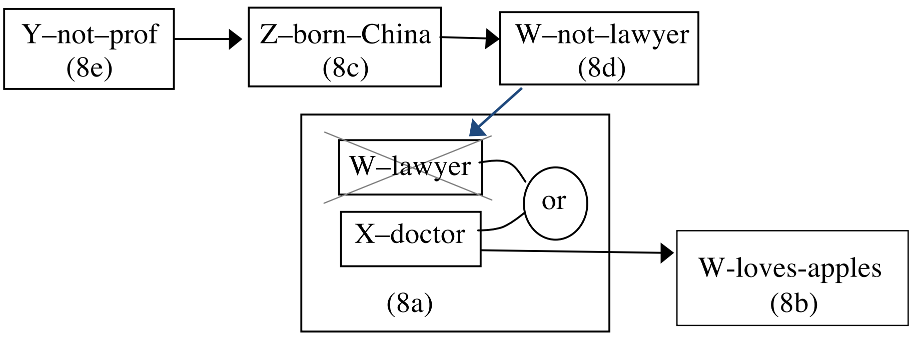
Another useful notation for exploring logical relations amongst the propositions of a theory is that of a network of conditionals ("If P, then Q") enhanced by negation, conjunction, and disjunction. Consider the following logic problem:

(**8**) a. Either W is a lawyer or X is a doctor.
b. If X is a doctor, then W loves apples.
c. If Y is not a professor, then Z was born in China.
d. If Z was born in China, then W is not a lawyer.
e. Y is not a professor.

Does W love apples?

It is much easier to answer the question if we represent (8a-e) as a logic network, like this:

(**9**)



Given [Y-not-prof] (8e) as the starting point, we can follow the steps in this diagram to infer [W loves-apples] (the answer to the question).

The diagrammatic notation of logical networks might strike some of you as both useful and fun; and it may strike others as offering nothing more substantial than its corresponding verbal statements illustrated in (8a-e). When we move from propositional networks to causal networks in section 6.2, however, its benefits will become evident to all of us.

The rule of modus ponens lies at the heart of what has been called classical deductive logic. There are other kinds of deductive logics, and other rules of modus ponens that correspond to these logics. We will discuss two of them here, namely,

probabilistic modus ponens in probabilistic deductive logic, and
defeasible modus ponens in defeasible deductive logic.

3.2 Probabilistic Deductive Reasoning

Consider the following examples of reasoning. Example (10) is found in most introductory books on logic:

(**10**) Every human being is mortal.
Socrates was a human being.
Therefore, Socrates was mortal.

(**11**) All human beings have their heart on the left side of the body and their liver on the right.
Plato was a human being.
Therefore, Plato had his heart on the left side of his body and his liver on the right.

The reasoning in both these examples appeals to the rule of Modus Ponens discussed in the previous section. We will refer to this as ***classical modus ponens***. It has the following property:

If the premises are true, and
the rule is applied correctly,
the conclusion is necessarily true.

If the statements that every human being is mortal and that Socrates was a human being are both true, it cannot be the case that the statement that Socrates was mortal is not true. Thus:

*If the premises are **totally certain**, and the application of the rule is correct, Classical Modus Ponens yields a **totally certain** conclusion.*

We have seen no evidence to believe that the statement that every human being is mortal is incorrect: we have not come across any immortal human being yet. So (10) creates no problem. However, in (11), the first premise is inaccurate. There is a rare genetic condition in healthy human beings called *situs inversus* in which the heart is on the right side of the body and the liver is on the left: apparently, less than 1 out of 10, 000 individuals have this condition. The point is, human beings with their heart on the right side of the body and their liver on the left do exist. Hence, we should revise the reasoning in (11) as (12):

(**12**) 99.99% of human beings have their heart on the left side and the liver on the right side.
Plato was a human being.
Therefore, the probability of Plato having his heart on the left side and the liver on the right side is 0.9999.

As it happens, there are also people with the heart on the left, and yet the liver is also on the left. This condition is called *situs inversus incompletes*. It appears that 1 out of 22,000 cases of *situs inversus* are instances of *situs inversus incompletes*. Thus, 1 out of 220, 000, 000 human beings has both the heart and the liver on the left side of the body:

(**13**) 1 in every 220,000,000 human beings has both the heart and the liver on the left side of the body.
Plato was a human being.
Therefore, the probability of Plato having both his heart and his liver on the left side of his body is approximately 0.0000000045.

The reasoning illustrated in (12) and (13) is probabilistic. To see the distinction between (11) and (13) more clearly, let us re-formulate (11) in numerical terms as (11').

(**11'**) 100% of human beings have the heart on the left side and the liver on the right side.
Plato was a human being.
Therefore, the probability of Plato having his heart on the left side and his liver on the right side is 1.

(11') is an instance of classical deduction in which the truth value is either 1 (true) or 0 (false): conclusions are totally certain. (12) and (13) are examples of probabilistic reasoning that allow degrees of certainty, with truth values *ranging from* 1 to 0, where 0 is false (with total certainty) and 1 is true (with total certainty). Alternatively, we could choose truth values ranging from –1 to +1 where –1 is false (with total certainty) and +1 is true (with total certainty), and 0 is exactly between the two (undecided).

Note that classical logic is ***two-valued***: something is either true or false. The use of truth-values ranging from +1 to –1 gives us a ***multi-valued*** logic. Instead of values ranging from +1 to –1, if we use the values +1, 0 and -1, we get a ***three-valued*** logic.
(See http://en.wikipedia.org/wiki/Three-valued_logic)

What would a Probabilistic Modus Ponens look like? To answer this question, let us re-formulate Classical Modus Ponens in terms of numbers (quantitative terms):

Classical Modus Ponens (quantitative)

Given that:	*If P is true, then the probability of Q is +1.*
and	*P is true*
it is reasonable to conclude that:	*the probability of Q is +1.*

It is now easy to see that the probabilistic version of Modus Ponens can be formulated as follows, replacing +1 with *n*, where n ranges from +1 to –1.

Probabilistic Modus Ponens (quantitative)

Given that:	*If P is true, then the probability of Q is n.*
and	*P is true*
it is reasonable to conclude that:	*the probability of Q is n.*

Probabilities are expressed qualitatively in ordinary language in terms of adjectives like *likely*, *most likely*, *almost certainly*, and so on. So the qualitative version of the Probabilistic Modus Ponens would be:

Probabilistic Modus Ponens (qualitative)

Given that:	*If P is true, then Q is (most) likely to be true.*
and	*P is true*
it is reasonable to conclude that:	*Q is (most) likely to be true.*

Instead of using numbers as in mathematical probability theory, we could use a probabilistic conditional, diagrammed in terms of a dotted arrow:

P ----------▶ Q

The dotted arrow in this notation is to be read as: "If P is true, then Q is probably true." (Instead of expressing probability in terms of numbers, we express it qualitatively.) Using this notation, we can formulate the rule as follows:

Rule of inference: ***Probabilistic Modus Ponens***

If	(P ---▶ Q)	is true
	and P	is true
then	Q	is likely to be true.

3.3 Defeasible Deductive Reasoning

Suppose we are told that Xena goes to work every weekday, and that 2 January 2007 was a weekday. We will conclude that Xena went to work on 2 Jan '07.

Now we get the additional information that Xena doesn't go to work when she is ill, and that she was ill on 2 Jan '07. We would most probably conclude that Xena didn't go to work on 2 Jan '07, overriding our earlier conclusion that she went to work that day.

Now we are told that there was a crisis at Xena's workplace on 2 Jan '07, and that whenever there is a crisis, she gets there. Did Xena go to her workplace on 2 Jan '07? Our answer would probably depend on whether the crisis was serious enough to override the illness factor.

This example with the shifts in its conclusions illustrates an important property of a large class of human reasoning, namely: w*hen additional information becomes available, a conclusion that was accepted earlier as correct can be rejected as incorrect*.

The mode of reasoning that has the above property is called ***defeasible reasoning***. Classical Modus Ponens is non-defeasible. To see this clearly, let us derive the conclusions in the example of Xena and her workplace using Classical Modus Ponens.

(**14**) a. If x is a weekday, Xena goes to work on day x.
b. If Xena is ill, she doesn't go to work that day.
c. 2 Jan '07 was a weekday.
d. Xena was ill on 2 Jan. 07.

---

e. By (a) and (c), Xena went to work on 2 Jan '07.
f. By (b) and (d), Xena didn't go to work on 2 Jan '07.
g. By (e) and (f), Xena went to work and didn't go to work on 2 Jan '07.

The reasoning in (14), using Classical Modus Ponens, yields a **logical contradiction**. This is because in this mode of reasoning, there is no way to cancel an earlier inference as long as the premises remain. To rectify the conclusion that Xena went to work on 2 Jan, we need a rule of inference that can suppress an otherwise valid conclusion. This rule is what Defeasible Modus Ponens provides:

Defeasible Modus Ponens

Given	(i) *if P is true, then Q is true*
	(ii) *if R is true then not-Q is true*
	(iii) *P is true.*
	(iv) *R is true*
and	(v) (ii) is stronger than (i)
it is reasonable to conclude that	not-*Q is true*. (*Q is false*)

Thus, (v) cancels the inference from the combination of (i)+(iii), namely, that Q is true.

Another way of expressing this idea is to say that:

If C-1 and C-2 are conclusions derived from premise sets P-1 and P-2 respectively,
P-2 is stronger than P-1, and
C-1 and C-2 are logically contradictory,
then C-2 overrides C-1.

Given such a situation we accept C-2 and reject C-1.

An instance where such defeasible reasoning is relevant outside scientific inquiry proper is moral reasoning. Reasoning from moral principles to moral judgments crucially needs Defeasible Modus Ponens.

Suppose we accept the following principles in our moral theory.

(**15**) Given an option between action and inaction,
   a. if action brings about death, choosing to act is immoral.
   b. if action saves a life, choosing to not act is immoral.

(**16**) Given an option between action and inaction,
   a. if action causes or increases suffering, choosing to act is immoral.
   b. if action prevents or alleviates suffering, choosing to not act is immoral.

(**17**) a. Given an option between telling a lie and not telling a lie, choosing to lie is immoral.
   b. Given an option between speaking and not speaking the truth, choosing to not speak the truth is immoral.

Now take the following scenario borrowed from philosopher Peter Singer as an illustration of moral dilemmas. Imagine that you are a Christian living in Germany during Hitler's time. In your basement is a Jewish family hiding from the Nazis. One day, the Nazi secret police knock on your door, and ask if you are harboring any Jews in your house. What is the right thing to do, to tell the truth and send the Jewish family to a prison camp, or to tell a lie and save the family?

We assume that most of us would opt for the latter. Given a moral theory that includes (15)-(17), with the specification that (15) and (16) are stronger than (17), the reasoning that underlies this conclusion can be given as follows.

(**18**) a. If action brings about death, given an option between action and inaction, choosing to act is immoral. (15a)
   b. If action causes or increases suffering, given an option between action and inaction, choosing to act is immoral. (16a)
   c. Given an option between telling a lie and not telling a lie, choosing to tell a lie is immoral. (17a)
   d. (15) and (16) are stronger than (17).
   e. Telling the Gestapo about the Jewish family hidden in the house would cause suffering and death for the family.
   f. Telling the Gestapo that there are no Jews hiding in the house would be telling a lie.

   ---

   g. Candidate: by (a), (b) and (e), it is immoral to tell the Gestapo that there are Jews hiding in the house.
   h. Candidate: by (c) and (f), it is immoral to tell the Gestapo that there are no Jews hiding in the house.
   i. By (d), (g) overrides (h). Therefore, we conclude that it is immoral to tell the Gestapo that there are Jews hiding in the house.

3.4 The Use of Deductive Reasoning in Mathematical and Scientific Inquiry

3.4.1 Deductive Reasoning in Mathematical Calculations and Proofs

What relevance do the three modes of deductive reasoning have for students of science and mathematics? Let us begin with classical deductive reasoning. Anyone who has taken a course in mathematics knows that learning math requires learning how to make calculations and how to prove theorems (at least, how to reproduce the proofs of theorems). The mode of reasoning used in both these activities is that of classical deduction.

In (5) earlier, we gave an example of the use of classical deduction in mathematical calculations, repeated below as (19) with some elaboration:

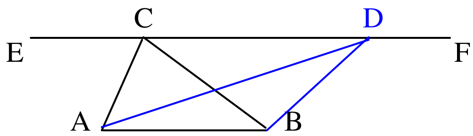
(**19**) Leda's room is a rectangle. — premise 1
The length of the room is 15 ft. — premise 2
The width of the room is 10 ft. — premise 3
The area of a rectangle = length x breadth. — premise 4

The product of 10 and 15 = 150. — intermediate conclusion
Therefore: The area of Leda's room = 150 sq ft. — conclusion

To see the use of classical deduction in mathematical proofs, consider the following conjecture:

Conjecture: Given a straight line AB and any two points C and D on a straight line EF parallel to AB, the areas of ABC and ABD are equal.

The diagram below gives a visual picture of the situation in the conjecture:



How do you prove this conjecture? Here is a deductive proof:

The area of a triangle is half the product of its base and its height.	Premise 1 (formula)
The heights of triangles ABC and ABD are the same.	Premise 2 (follows from the definition of parallel lines in Euclidean geometry)
ABC and ABD have the same base AB.	Premise 3 (given)
Hence, given premises 1-3, it follows that ABC and ABD have the same area.	Conclusion

You are unlikely to want more examples of the use of classical deductive reasoning in mathematical proofs. But if you do want more examples, take a look at: "Mathematical Knowledge and Inquiry," at https://sites.google.com/site/sciediiserp/additional-materials

3.4.2 Deductive Reasoning in Propositional Explanations

Suppose someone, call her Zina, showed you the following experiment:

She takes two metal bars, one with a red dot and the other with a green dot. She brings the red-dot metal bar close to a metal pin. Nothing happens. She brings the green-dot metal bar near the same pin, and it jumps towards the bar, and attaches itself to the bar.

Let us record what we observe as follows:

<u>Observation O1</u>: The pin (a) when close to the green-dot bar, moves towards the bar; but (b) when close to the red-dot bar, doesn't move.

Next she takes two more metal bars, one with a blue dot and the other with a black dot. She marks the ends of all four metal bars with numbers, as shown below:

<u>figure 3</u>

When she brings the ends of these bars near each other, we observe the following results:

<u>Observation O2</u>: The following ends of the metal bars:

a) move away from each other: 3 & 6; 4 & 5.
b) move towards each other: 3 & 5; 4 & 6;
1 & 3; 1 & 4; 1 & 5; 1 & 6; 2 & 3; 2 & 4; 2 & 5; 2 & 6;
7 & 3; 7 & 4; 7 & 5; 7 & 6; 8 & 3; 8 & 4; 8 & 5; 8 & 6.
c) remain motionless: 1 & 7; 1 & 8; 2 & 7; 2 & 8.

Observations O1 and O2 call for an explanation. In the terminology of philosophy, they are part of our ***explanandum*** — "that which needs to be explained." We now have to look for an explanation. Any student of physics can immediately see that we can offer an explanation by appealing to the theory of magnetism. The ***explanans*** — propositions that offer an explanation — can be stated as follows:

Explanans propositions

Theoretical propositions

I) A magnet consists of two poles, a south pole and a north pole.
The north (or south) poles of two magnets are 'like poles';
the north pole of one and the south pole of another are opposite poles.

II) (a) Like poles repel each other,
(b) opposite poles attract each other.

III) Magnets attract objects made of iron.

IV) (a) When X attracts Y, Y moves towards X, and
(b) when X repels Y, Y moves away from X.

Interpretation of observational entities within the theory

V) (a) The green-dot and blue-dot bars are magnets;
(b) the red-dot and black-dot bars are not magnets;
(c) the pin is not a magnet; and
(d) the pin is made of iron.

VI) Either 3 and 6 are north poles, and 4 and 5 are south poles;
Or 3 and 6 are south poles, and 4 and 5 are north poles.

Using classical deductive reasoning, we deduce the following logical consequence from (I)-(VI):

Given (III), (IVa), (Va) and (Vd), it follows that when the green-dot bar comes close to the pin, it would move towards the bar. This prediction matches observation O1(a).

Given (III), (IVa), (Vb) and (Vc), when the red-dot bar comes close to the pin, there is no reason to expect the pin to move towards the bar. This prediction matches observation O1(b).

We suggest that you work out for yourselves the rest of the derivations whose outcomes match the observations O2 (a-c). An additional exercise would be to work out the derivation of the following predictions as well, going beyond the particular bars we have talked about:

Additional predictions:

Any two bars that exhibit the property of repelling each other, as in O2(a), will also exhibit property O1(a), and vice versa.

Any two bars that do not exhibit the property of repelling each other, as in O2(b), will not exhibit property O1(a) either, and vice versa.

If, on further observation, we discover that there exist bars whose properties our theory rules out, then our predictions are inconsistent with our theory: the theory is wrong. To the extent that our theory makes such predictions and the predictions have not been proven wrong, we are justified in our confidence that the theory is correct. This is essentially what Karl Popper meant when he said that scientific theories must be falsifiable.

To get a firmer grasp of the nature of explanation in scientific theories, it would be useful to read the following notes (available at https://sites.google.com/site/eplsmohanans/main-text/part-1-patterns-and-puzzles-in-language/part-1-contents):

Note 1: Puzzles and Explanations in Everyday life
Note 2: "Randomness and Regularity: Patterns in Data, and
Note 8: Explanations and Theories, and

Note 13: Scientific Thinking in Everyday Life

In addition, it would be useful to go through the specifics of explanation provided by the heliocentric theory discussed in TDC102 Evolutionary Theory 1 at https://sites.google.com/site/sciediiserp/additional-materials

<u>Additional exercises</u>

Write down (a) the explanandum propositions; (b) the explanans propositions; and (c) the derivation of (a) from (b), for the following:

i) Brownian motion (if you don't remember what Brownian motion is, do a google search).

ii) Continental drift (if you don't remember what continental drift is, do a google search).

iii) Explanation of summer and winter.

3.4.3 The Place of Probabilistic and Defeasible Reasoning in Scientific Inquiry

What we have illustrated above is the use of classical deductive reasoning in deriving predictions from the propositions of a scientific theory. If you are familiar with thermodynamics and quantum mechanics, you know that the predictions of these theories are probabilistic, not absolute. If you are not familiar with these theories, consider weather predictions instead.

Though the use of probabilistic reasoning has become an accepted part of scientific prediction, defeasible deductive reasoning has not attracted as much attention. To see why we need defeasible reasoning in scientific explanation, let us go back to the Zina scenario. Suppose there is a brown patch on the smooth table where she is experimenting with pins and magnetic bars. By sheer chance, she places the pin on that patch, and then discovers that when she brings a magnet close to the pin, the pin doesn't move. More interestingly, when she places a magnet on the brown patch and brings her magnet close to that magnet, it neither moves towards nor away from the one coming towards it.

Given these observations, should we now conclude that the theory in (I)-(IV) is defective, and that its propositions should be revised? If our answer is no, then how do we derive the right predictions?

There could be a number of reasons why the pin and the magnet on the brown patch on the table don't move, even if the bar close to them is indeed a magnet. One possible reason is that the brown patch is a patch of glue, and that the pin and the magnet are stuck to the table. Another possible reason is that there is a more powerful magnet right under the brown patch that makes the pin and the magnet immobile. A third possible reason is that an electric current under the patch creates a magnetic field.

These are all possible explanations for the ***apparent*** violation of (I)-(IV). And they all involve defeasible deductive reasoning. Our conclusion from (I)-(IV) is that the pin/magnet will move when a magnet is brought close to it. But once new information comes along, and we know about the glue (or the hidden magnet), the earlier conclusion is no longer valid.

If you would like to have more examples of scientific explanation, see:

Aristotle's arguments for the proposition that the earth is round, not flat (at http://wiki.nus.edu.sg/display/aki/5.2.+Round+vs.+Flat+Earth)

The discussion of the explanandum and explanans of the heliocentric theory in "TDC 102 Evolution 1" (downloadable from https://sites.google.com/site/sciediiserp/additional-materials)

A comparison of the heliocentric and geocentric theories (at http://wiki.nus.edu.sg/display/aki/5.3.+Does+the+Earth+go+around+the+Sunx)

4 Inductive Reasoning

4.1 Making Inferences on the Population, Based on the Sample

4.1.1 Classical Deductive Reasoning vs. Inductive Reasoning

Most introductory textbooks on logic make a distinction between deductive and inductive reasoning. Consider the following example of classical deduction:

(**20**) Classical deductive reasoning

All human beings have exactly one heart.	Premise 1
Zeno is a human being.	Premise 2
Therefore, it is reasonable to conclude that Zeno has exactly one heart.	Conclusion

Suppose we ask: how do we know that every creature in the population of human beings has exactly one heart? How do we know that there are no humans with two or more hearts, or with no heart at all? In other words, what justifies the conclusion that premise 1 is true?

The answer to this question — our reason for the belief that human beings have exactly one heart — is given in terms of inductive reasoning, and can be articulated as follows:

(**21**) Inductive Reasoning

We have observed a large representative sample of human beings. Every human being in the sample has exactly one heart. Hence, until we find evidence to the contrary (until we find a human being with more than one heart or with no heart), it is reasonable to conclude that all human beings have exactly one heart.

Let us take a few more examples. Compare (22) with (23).

(**22**)

Every spider in the population of spiders has eight legs.	premise 1
Zeno belongs to the population of spiders.	premise 2
Therefore: (it is reasonable to conclude that) Zeno has eight legs.	conclusion

(**23**)

Every spider in this sample has eight legs.	premise 1
This is a sample of the population of spiders.	premise 2
Therefore, in the absence of evidence to the contrary, (it is reasonable to conclude that) all spiders in the population have eight legs.	conclusion

Example (22) employs classical deduction, while (23) uses induction.

To see what is going on in these examples, let us go through a thought experiment. Imagine a barrel of balls. You are told that all the balls in the barrel are red, instructed to close your eyes, and pick a hundred balls randomly from the barrel, and asked what colour the balls are. Your answer would be: red. Given that the colour of the balls in the ***population*** in the barrel is red, it follows logically that the balls in the ***sample*** from the barrel are also red.

(**24**) Population to sample (P-to-S)

All the balls in this barrel are red.	population
↓	
Therefore, the 100 balls in the sample from this barrel are red.	sample

The reasoning in this case is that of deduction, and the relevant rule is Classical Modus Ponens.

Now consider a different scenario. You are shown a barrel of balls, but are not told the colour of the balls. You are instructed to pick a hundred balls randomly from the barrel. You see that all the balls in the sample are red. You are now asked what colour the balls in the barrel are. Your answer would be that they are red: given that all the balls in the sample drawn from the barrel are red, it is most likely – though not completely certain – that the colour of the population of balls in the barrel is red.

(**25**) <u>Sample to population (S-to-P)</u>

The 100 balls in the sample from this barrel are red. sample

↓

Therefore, *in the absence of evidence to the contrary*, (it is reasonable to conclude that) all the balls in this barrel are red. population

Most logic textbooks use the term *induction* to refer to the mode of reasoning illustrated in (23) and (25), to contrast it with the *deduction* in (22) and (24).

Given the properties of a population (a set), P-to-S reasoning allows us to infer the properties of a particular sample (a proper subset) of the population. Given the properties of a sample of a population, S-to-P reasoning allows us to infer the properties the population. We could also say that P-to-S reasoning allows us to infer the particular from the general, while S-to-P reasoning allows us to infer the general from the particular.

4.1.2 Absence of Total Certainty

As in the case of Probabilistic reasoning and Defeasible reasoning, inductive reasoning also lacks total certainty. Even if we take a sample of a thousand balls from the barrel and find them all to be red, there is still a chance that one of the balls in the barrel is white, and that we happened to not notice it. In contrast, classical deductive reasoning is totally certain: if it is true that all the balls in the barrel are red, it cannot be the case that the balls in the sample are anything but red.

Thus, in (22), if the premises are true, it cannot be the case that the conclusion is false. In contrast, the conclusion in (23) (though reasonable) can be false even if the premises are true: there might be some spider that we have not observed with ten legs or six legs. Likewise, even though the conclusion in (25) is quite reasonable, there may be one white ball somewhere in the barrel that might not have been included in the sample. This possibility of error is acknowledged by the phrase *in the absence of evidence to the contrary*. One way to play it safe is to modify the conclusion to reflect the uncertainty, as in (26):

(**26**) All the balls in the sample of 1000 balls from this population are red.
Therefore, it is reasonable to conclude that all the 10,000 balls in this population are *most likely to be* red.

The inductive reasoning in (25), which is non-probabilistic, involves the risk of the conclusion being false. In contrast, the probabilistic inductive reasoning in (26) is cautious. From a different perspective, we could say that (25), unlike (26), employs "defeasible" reasoning: when contrary evidence turns up, the conclusion is "defeated" and hence abandoned (or modified).

4.1.3 Probabilistic and Quantitative Inductive Reasoning

As in the case of classical deduction, inductive reasoning can be either quantitative or qualitative. Compare the reasoning in (27a) with that in (27b) and (27c):

(**27**) a. 995 Martians in our representative random sample of 1,000 Martians have purple eyes. Therefore, it is reasonable to conclude that 99.5% Martians have purple eyes.

b. Every Martian in our large representative random sample of Martians has purple eyes. Therefore, it is reasonable to conclude that all Martians are likely to have purple eyes.

c. Every Martian in our representative random sample of Martians has purple eyes. Therefore, until we find Martians with non-purple eyes, it is reasonable to conclude that all Martians have purple eyes.

(27a) illustrates quantitative probabilistic inductive reasoning that we recognize as statistical reasoning. (27b) is its qualitative probabilistic counterpart. (27c) illustrates qualitative non-probabilistic inductive reasoning.

4.2 Data, Patterns in the Sample, and Patterns in the Population

In the preceding section, we laid out the nature of reasoning that takes a generalization in the sample as the grounds, and concludes that the same generalization holds in the population. Take an example:

> In my *sample*, the mean height of male humans is higher than that of female humans. Therefore I conclude that the mean height of males is higher than that of females in the human *population*.

Before arriving at this conclusion about the population, we need to arrive at and justify the conclusion about our sample. How do we do this?

If the data we are considering are quantitative, we arrive at and justify our conclusion about the sample by adopting the procedures and protocols of ***descriptive statistics***. If our data involve experimental observations, we would also need to adopt the procedures and protocols of experimental research. And the next step, that of arriving at a conclusion about the population on the basis of the generalization established in the sample, comes under ***inferential statistics*** in quantitative research.

Both deductive and inductive reasoning are used in various fields of academic inquiry. In university courses, quantitative probabilistic deductive reasoning is typically taught under probability theory in mathematics. Its qualitative counterpart is taught as probability logic in philosophy courses. Quantitative probabilistic inductive reasoning is taught as inferential statistics in mathematics, while its qualitative counterpart is taught under the broad rubric of inductive reasoning in philosophy. What we have covered in the previous section is the logic counterpart of inferential statistics.

If you are not familiar with the procedures and protocols of statistical and experimental inquiry, we suggest that you take a look at the following sites:

"Descriptive and Inferential Statistics" at
http://writing.colostate.edu/guides/research/stats/pop2b.cfm

"Descriptive and Inferential Statistics" at
https://statistics.laerd.com/statistical-guides/descriptive-inferential-statistics.php

"Experimental Methodology" at
http://www.cs.cmu.edu/afs/cs/project/jair/pub/volume8/finkelstein98a-html/node11.html

"Experimental Research Methods" at
aect.org/edtech/ed1/38.pdf

4.3 The Use of Inductive Reasoning in Scientific Inquiry

The conclusions arrived through inductive generalizations of the kind discussed above are *observational generalizations* that fall into three classes, namely,

(**28**) *A general property of a population*

a. All human beings have exactly one heart.
b. The average height of a human male is in the region of 1.7 meters
c. The height of a human male ranges from 2.5 meters to 1.2 meters.
d. The natural colour of human hair can be black, red, brown, yellow, or gray.
e. There are no humans with natural green hair.
f. The average boiling point of alcohol under atmospheric pressure is about 97.5ºC.

(**29**) *A correlational generalization between two or more properties of a population*

a. Human males are taller than human females.
b. Humans who have dark skin and black eyes are unlikely to have yellow hair.
c. The speed of a falling body on earth is directly proportional to the duration of the fall.
d. The period of a pendulum is directly proportional to its length.
e. The speed of a pendulum when plotted against time forms a sinusoidal wave.

(**30**) *A causal generalization that asserts a cause-effect relation in a correlation*

a. Obesity causes brain impairments.
b. Adding a few drops of lemon juice to boiling milk causes the milk to split.
c. Ajinomoto (monosodium glutamate), smoke, and red light cause migraine headaches.
d. Small pox is caused by a virus called variola.
e. Vaccination prevents small pox, polio, and TB.

Using the language of mathematics, we may say the generalizations in (28) are about the ***value*** of a single ***variable***. In (28a) the value of the "variable" is *invariant*, namely, one. In (28b), (28c), and (28f), the values exhibit variability. (28b) and (28f) are about the *center* of the variation (average), while (28c) is about its ***distribution***. In (28d) the values are given in *qualitative terms*, in contrast to that in (28b) and (28c). (28e) is about a ***gap*** in the distribution of the values of hair colour.

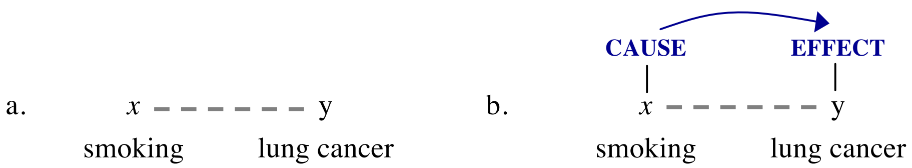
The generalizations in (29) are about the systematic relation between the values of two (or more) variables. (29a) states a relation between the variable of gender (males vs. females) and the variable of height. This is a ***correlation*** between gender and height in the population of humans, the values of gender being qualitative, and those of height being quantitative. (29b) states a negative correlation in qualitative terms between a particular combination of skin colour and eye colour on the one hand and hair colour on the other.

The remaining statements are also correlations, but while the correlation in (29c) and (29d) is linear, that in (29e) is a cyclic or periodic correlation. Many researchers use the term "correlation" to refer only to linear correlations (those expressed as a straight line), and not to non-linear correlations (those expressed as a curved line), let alone cyclic correlations. We will refer to any systematic relation between observational variables as correlations.

Like those in (29), the propositions in (30) express a relation between two observable variables, but the relation in these hypotheses goes beyond correlation to that of causation. While those in (29) say, "x and y are correlated," those in (30) say, "x causes y." If x causes y, we of course expect x and y to be correlated.

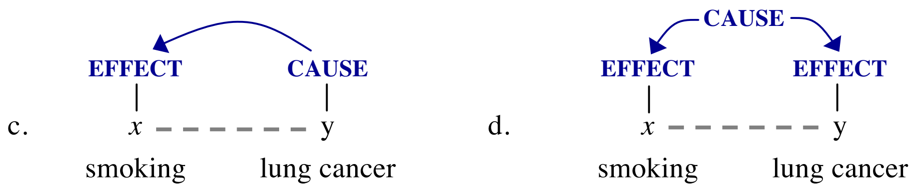
Here is an oft-used example of the distinction between plain correlation ("Smoking correlates with lung cancer." (31a)), and correlation+causation ("Smoking causes lung cancer." (31b)):

(**31**)



Suppose we take a very large random sample of human beings, and divide them into two groups, those who smoke and those who don't. We now divide each group into two, those who have lung cancer and those who don't. If we find that the incidence of lung cancer is significantly higher among those who smoke, we are justified in concluding that (31a) is true. However, even if (31a) is true, it doesn't follow that (31b) is true. The correlation, for example, could be because lung cancer induces/causes patients to smoke ((31c)), or because the tendency to smoke and the tendency to develop lung cancer are both caused by a gene ((31d)):

(31)



Without ruling out (31c) and (31d), we cannot establish the truth of (31b).

Now, what is the relation between the deductive and the inductive modes of reasoning within the context of scientific explanations? We can express the relation between scientific theories and the observational generalizations they predict as in figure 4:

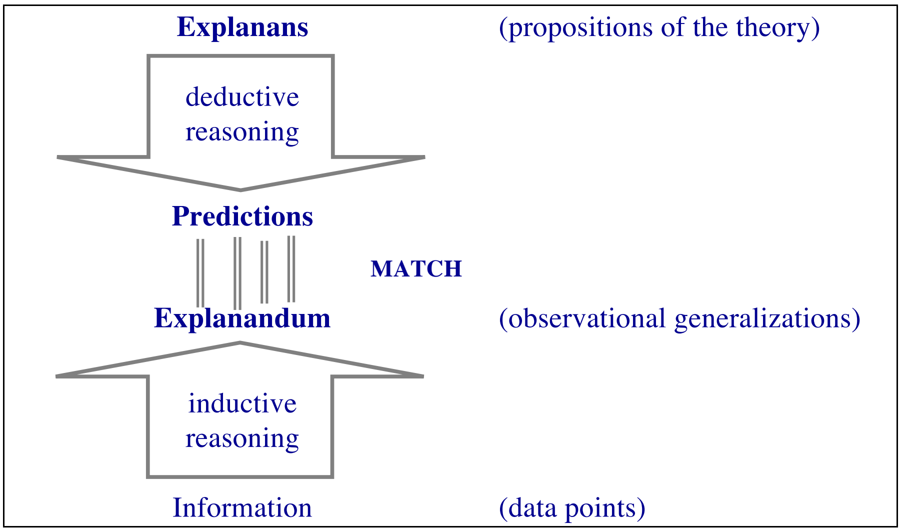


figure 4

4.4 Justification (and Refutation) in Scientific Inquiry: Hume and Popper

4.4.1 Induction and Falsifiability

The inductive arguments illustrated in the previous section are adequate to convince a skeptical inquirer, and hence can be judged as ***sound*** arguments. Yet, as already pointed out, the conclusions they offer cannot carry absolute certainty. When we conclude that all human beings have exactly one heart, we cannot rule out the possibility that, unknown to us, there exist a very small number of humans with two hearts. It is extremely implausible that there are humans with have no heart, but we cannot rule out even that situation with total certainty. The conclusion given above, in other words, is ***fallible***. Even though the premises are true, and the reasoning is valid, we cannot *guarantee* the conclusion.

To see the fallibility of induction, let us go back to the example of *situs inversus incompletes*. We know that 1 in 220,000,000 human beings who have their heart on the left have their liver also on the left. If you take two or three random samples of a thousand human beings each, chances are that you will observe that every human who has the heart on the left has the liver on the right. If you conclude on the basis of this evidence that every human being who has the heart on the left has the liver on the right, you would be wrong. The risk of being wrong is unavoidable in this mode of reasoning.

4.4.2 'Soundness' of arguments

Introductory textbooks and introductory courses on logic will tell you something along the following lines on "sound" arguments.

> A ***sound*** argument is one in which the premises are *true,* and the reasoning is *valid*.

The reasoning in an argument is ***valid*** if and only if it cannot be the case that if the premises are true, the conclusion cannot be false.

Only classical deductive logic meets this condition on validity. Probabilistic deductive logic, defeasible deductive logic, and inductive logic don't meet the requirement. Hence, adopting the above definition would force us to judge the reasoning in these modes of logic as invalid. As a consequence, we would also be forced to judge almost all instances of justification in science as invalid, and therefore unsound.

This textbook definition has further consequences. Take the conclusion that humans have exactly one heart. By this definition, the argument for this conclusion is unsound. This implies that for the proposition "Humans have exactly one heart," we are *not justified* in saying that it is true. It follows, then, that the deductive argument to conclude that Zeno has exactly one heart is also unsound. By the same token, the deductive argument to conclude that Socrates is mortal is unsound too. Yet, this is the classic textbook example for a sound deductive argument! Finally, since mathematical axioms are neither true nor false, requiring the premises to be true as a condition for sound argumentation leads to the consequence that mathematical proofs are also unsound.

If we go by the textbook criteria of soundness, we must conclude that there are no sound arguments in the world. Instead, consider the following criteria:

(32) A ***sound*** argument is one that convinces a community of skeptical rational inquirers that the conclusion advanced in the argument is correct.

The community of skeptical rational inquirers are convinced of the correctness of a conclusion if and only if the ***grounds*** and ***background assumptions*** of the argument are *acceptable* to them, and the ***reasoning*** from the grounds to the conclusion is *valid*.

The reasoning in an argument is ***valid*** in a given system of logic if and only if each step in the reasoning (except for the grounds) follows logically by a rule of inference in that system of logic.

The crucial differences in the criteria for soundness are:

	Textbook criteria	Criteria in (32)
(a)	Premises must be true (absolutist).	Grounds and background assumptions must be acceptable to skeptical inquirers.
(b)	If the premises are correct, the conclusion cannot be false.	The steps of reasoning must be sanctioned by the relevant system of logic. (The notion of logical consequence is relative to the relevant system of logic.)

4.3.3 Deductive and Inductive Reasoning: a historical perspective

Until recently, the role model for reasoning in academia was the kind of Classical Deductive Reasoning that the Greek mathematician Euclid introduced in his proofs for theorems, codified by philosopher Aristotle. Inductive reasoning lacks the certainty of classical deductive reasoning, and was therefore somehow seen as inferior. Perhaps as an extension of the same attitude, the term "inductive" gets equated with everything that is outside of classical deductive, and has been used for a rag-bag of different kinds of reasoning, covering not only the Sample-to-Population reasoning, but also what we have called Probabilistic Deduction, Defeasible Deduction, Abduction, and Speculative Deduction, rendering the concept of "inductive reasoning" not particularly useful for students of science.

In the philosophy literature, the idea called "Hume's Problem of Induction" reflects the same attitude. Philosopher David Hume, taking "justification" as having to satisfy the classical conditions of soundness and validity, claims that we are not justified in concluding that the sun will rise in the east tomorrow — because the conclusion is based not purely on classical deductive reasoning but also appeals to inductive reasoning.

Accepting Hume's claim, Karl Popper proceeded to "save" scientific inquiry by claiming that:

> Scientific conclusions cannot be justified, but they can be refuted (shown to be false).

In asserting that scientific conclusions can be refuted, and that for a claim in science to be taken seriously, it has to be refutable, or falsifiable, Popper was right. However, he made a mistake in assuming that a claim can only be justified through classical deductive reasoning, and that therefore, scientific theories cannot be justified. If we recognize probabilistic deductive reasoning, defeasible deductive reasoning, and inductive reasoning as legitimate modes of justification in science, we are forced to reject Popper's conclusion.

Instead of Hume's requirement of total certainty for justification, our demand is that a scientific claim must be *rationally justified* to be accepted as knowledge. This is a reasonable position to take; in addition, it avoids the damage Popper's position can do to scientific inquiry. The view that scientific claims cannot be justified is a detrimental position because, if taken seriously, it would lead to a form of research in which scientists are preoccupied with refuting claims, and do not pay attention to justifying claims.

5 Speculative Deductive Reasoning

We have seen how we arrive at and justify observational generalizations (explanandum) using inductive reasoning. We have also shown how we explain those generalizations using deductive reasoning. The next question is: how do we arrive at and justify the theoretical propositions (explanans) that explain those generalizations?

We do so via the following process:

> *we arrive* at the explanans propositions through ***speculation*** (guessing, imagining, etc.);
>
> *explain* the explanandum by showing that they are the predictions (the logical consequences) of the explanans propositions through ***deductive reasoning***; and
>
> *justify* the explanans propositions by showing that they yield the best available explanations.

As this involves both speculation and deduction, this mode of reasoning is called ***Speculative Deductive Reasoning.*** Here is an example:

(**33**) During a total solar eclipse, the positions of stars near the sun as observed from the earth appear farther away from the sun than their positions under normal conditions. This puzzling phenomenon can be explained by assuming that the gravitational pull of the sun causes the space around the sun to curve, thereby causing the light from the stars to take a curved path, as predicted by Einstein's theory of gravity. Since no other theory explains the observations, we accept Einstein's theory as correct.

Given below is another example of speculative deductive reasoning to support the theory of continental drift in geography proposed by Alfred Wegener in 1912:

(**34**) The shape of the east coast of South America fits neatly with the shape of the west coast of South Africa. Fossils of mesosaurus (small freshwater reptiles) are found only in two different parts of the earth, namely, Brazil (South America) and Western Africa. Dinosaur fossils are scattered in identical rock strata, again in the east coast of Brazil and west coast of Africa. We can provide an explanation for the above facts if we assume that South America and West Africa formed a single landmass at an earlier point in time, and drifted apart subsequently (continental drift hypothesis). In the absence of a better or equally good alternative explanation for the above observations, we accept the continental drift hypothesis as correct.

Our last example is a speculative deductive argument to establish gravity and the law of gravitation:

(**35**) Suppose we assume that

(a) there is such a thing as gravity, and

(b) it is governed by a law that gravitational attraction between any two bodies is proportional to the product of their masses and indirectly proportional to the square of the distance between them.

This law of gravity, combined with other laws, correctly explains the generalizations of falling bodies on the earth as well as the generalizations of planetary motion. In the absence of contrary evidence, and of a better or equally good alternative theory, it is reasonable to conclude that (a) and (b) are correct.

The basic structure of these arguments can be fleshed as follows:

Here is a body of data that illustrates a set of observational generalizations (a body of explanandum propositions) that call for an explanation (explanans propositions). And here is my explanation:

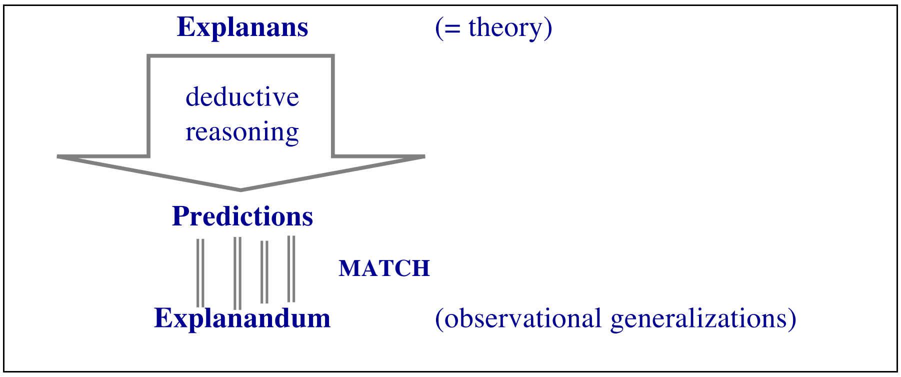


figure 5

This is the best explanation for the given body of explanandum.

Therefore, until a better or equally good explanation becomes available, we must accept my explanation.

As in the case of deductive and inductive reasoning, the rule of inference for Speculative Deductive Reasoning can be given as follows:

Rule of Inference: ***Speculative Deduction***

Given that: *Theory T is the best available explanation for Q*
and *Q is true*
it is reasonable to conclude that: *T is true*
in the absence of (i) evidence contrary to T, and
(ii) a better/equally good alternative explanation for Q.

Richard Feynman is pointing to Speculative Deductive Reasoning when he says:

"In general, we look for a new law by the following process. First, we **guess** it. Then we **compute** the consequences of the guess, to see what, if this is right, if this law we guess is right, to see what it would imply, and then we **compare** the computation results to nature or we say compare to **experiment** or **experience**, compare it directly with observations to see if it works.

If it disagrees with experiment, it's WRONG. In that simple statement is the key to science. It doesn't make any difference how beautiful your guess is, it doesn't matter how smart you are, who made the guess, or what his name is… If it disagrees with experiment, it's wrong. That's all there is to it."

Richard Feynman. Messenger Lectures, Cornell University, 1964.
Anthologized in *The Character of Physical Law*. Modern Library.
For a video recording, see http://www.youtube.com/watch?v=EYPapE-3FRw

Einstein and Infeld point to the same process when they say:

> "Physical concepts are **free creations of the human mind**, and are **not**, however it may seem, **uniquely determined by the external world**. In our endeavor to understand reality, we are somewhat like a man trying to understand the mechanism of a closed watch. He sees the face and the moving hands, even hears its ticking, but he has no way of opening the case. If he is ingenious he may form some picture of the mechanism which could be responsible for all the things he observes, but he may **never be quite sure his picture is the only one which could explain his observations**.
>
> He will never be able to compare his picture with the **real mechanism** and he cannot even imagine the possibility of the meaning of such a comparison." [Emphases ours. KPM and TM]
>
> Albert Einstein. *The Evolution of Physics*. 1938. Simon and Schuster. p. 31.
> co-written with Leopold Infeld

In the philosophy literature, the use of speculative deductive reasoning to support a theoretical position is called "inferences to the best explanation." (For an excellent demonstration of this mode of argumentation, see Yale Professor Shelley Kegan's lecture at http://www.academicearth.org/lectures/arguments-for-existence-of-soul)

6 Causal Reasoning

6.1 Causal Explanations

6.1.1 Causal Hypotheses

A causal generalization is called a causal hypothesis in the hypothesis-testing paradigm of research. As in the case of correlational hypotheses, causal hypotheses call for explanations. Typically, but not necessarily, these are causal explanations.

Here are a few puzzling causal generalizations that call for explanations.

> If you add a few drops of lemon juice to a pot of boiling milk, the milk splits. Why does the milk split? Why does adding a few drops of tamarind or vinegar also make milk split, but not adding water or sugar syrup? Why doesn't lemon juice make boiling water or buttermilk split in the same way? How exactly does the lemon juice cause the milk to split? What are the specifics of the causal mechanisms of milk splitting?
>
> Heating a metal rod makes it expand. Why does it expand? Why doesn't cooling make it expand? If a metal rod and a wooden rod are both heated, why does the metal rod expand much more? Why does water expand both when it is heated and when it is cooled to 4ºC? What is the mechanism of expansion?
>
> If you hit a glass panel with a hammer, it shatters. Why does it shatter? Why doesn't a metal or rubber panel shatter in the same way? What is the mechanism of glass shattering?
>
> Why do ajinomoto (monosodium glutamate), smoke, and red lights cause migraine headaches? How do migraine medications cure migraine? What are the causal mechanisms of triggering and curing migraines?
>
> If you light a match and throw it on a heap of straw, it catches fire. Why doesn't a heap of metal wires catch fire in the same way? Why doesn't straw catch fire if it is wet with water? How come it catches fire if it is wet with petrol (by itself or with water)? What is the mechanism of burning?
>
> What makes detergent remove dirt from clothes? Water removes certain kinds of dirt, but not grease. Why? Why does soap remove grease? How does soap remove grease? What is the mechanism of removing dirt from fabric?
>
> Mating between a male rabbit and a female rabbit, or a male cat and a female cat, can cause the female to become pregnant. Mating between a male rabbit and a female cat, or a male

cat and a female rabbit doesn't cause pregnancy in the female. Why? We can create a test tube rabbit embryo with the sperm from a male rabbit and the egg from a female rabbit, or with sperm from a male cat and an egg from a female cat, but not sperm from a male cat and egg from a female rabbit, or sperm from a male rabbit and egg from a female cat. Why? What is the mechanism of embryogenesis that explains this?

6.1.2 Causal Hypothesis vs. Causal Theory

To understand the distinction between a causal hypothesis (as an observational generalization) and a causal theory (as a network of relations that explains such causal generalizations), consider the recommendation to eat well and exercise in Olivia Judson's article "Brain Damage" in *The New York Times* (http://opinionator.blogs.nytimes.com/2010/04/20/brain-damage). The reasoning in her recommendation can be summarized as follows:

Increase of body fat causes increased risk of brain damage.
Eating well and exercising decreases body fat.
Therefore, eating well and exercising decreases risk of brain damage.

The premises in the above reasoning are causal generalizations:

Obesity is a cause of brain damage.
Eating well causes obesity to decrease.
Exercising causes obesity to decrease.

What is the explanation for these propositions? The outline of the causal "theory" of the relation between obesity and brain damage on which Judson bases her recommendation to eat well and exercise can be summarized as follows:

The gene called FTO has a causal effect on both body mass and brain function.
Increase in fat cells causes an increase in body mass.
Eating well causes a decrease in fat cells.
Fat cells secrete hormones like leptin.
Leptin causes changes in the brain, which in turn cause the development of Alzheimer's.
Fat cells also secrete other substances that cause chronic inflammation of the brain.
Body mass is also a causal factor in sleep apnea, diabetes, hypertension, and heart disease.

Sleep apnea, diabetes, hypertension, and heart disease cause brain damage. A natural question to ask is, given that FTO is a causal factor in both brain damage and increase in fat cells, how do we know that eating well and exercising counteract or lessen brain damage? To answer this question we need to hold in our minds all the cause-effect relations that mediate between FTO, eating well, and exercising on the one hand and brain damage on the other. Such thinking can be made much easier if we develop a notation for the representation of a complex network of causal relations, to which we turn in the next section.

6.2 A Diagrammatic Notation for Causal Networks

6.2.1 Representing Causation

Recall the arrow notation we used to represent conditionals: we use the notation in (36b) to represent the content of (36a):

(**36**) a. If proposition P is true, then proposition Q is true.

b. $P \rightarrow Q$

We will adapt this notation to create a diagrammatic representation of causal relations as a visual aid to think though complex instances of causal reasoning. Let us begin by using a curved arrow to represent causation, representing (37a) as (37b):

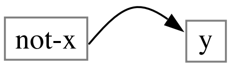
(**37**) a. x causes y

b. 

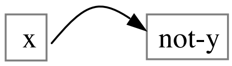
In this notation, the antecedent x is a cause and the consequent y is its effect. It is important to remember that the P and Q in propositional logic are propositions, but the x and y in causal logic are events, processes, states, and entities.

Let us borrow the negation “not” from propositional logic and use it in causal reasoning as follows:

(**38**) a. absence of x causes y (= not-x causes y)

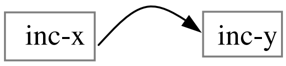
b. 

(**39**) a. x causes absence of y (= x causes not-y)

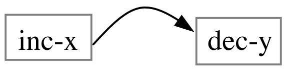
b. 

Instead of absence and presence, quantitative causal reasoning needs to consider increase and decrease. Hence, it would be better to replace “not” with inc(rease) and dec(rease), treating absence and presence as extreme cases of decrease and increase.

(**40**) a. increase of x causes increase of y

b. 

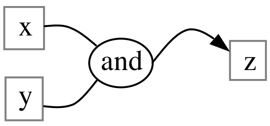
(**41**) a. increase of x causes decrease of y

b. 

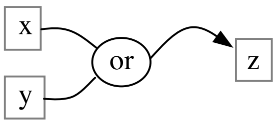
(41b) is the counterpart of “is directly proportional to” and “correlates positively with”, while (42b) is the counterpart of “is inversely proportional to” and “correlates negatively with.”

We now borrow the conjunction (“and”) and disjunction (“or”) of propositional calculus:

(**42**) a. the combination of x and y causes z

b. 

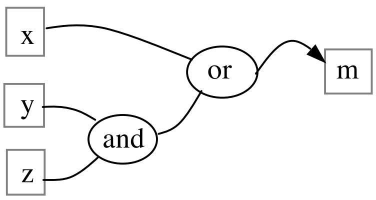
(**43**) a. either x or y causes z

b. 

In (42b), neither x nor y is a sufficient cause, but both are *necessary causes*. In (43b), neither x nor y is a necessary cause, but they are both *sufficient causes*.

A configuration of both conjunction and disjunction can result in causes that are neither necessary nor sufficient. Consider (44), for instance:

(**44**) a. Either x or the combination of y and z causes m

b. 

In (44), x is a sufficient cause. However, neither y nor z is either sufficient or necessary.

An awareness of the distinctions pointed to in (42)-(44) is crucial in research that seeks causal explanations. Take, for instance, the statement that ulcers are caused by a combination of bacteria and mental stress. Suicide is often caused by a combination of genetic, neurological, psychological, societal, and environmental factors. Human personalities and abilities are shaped by a combination of both nature and nurture. Picking out any one of these causes as ***the*** cause results in the kinds of futile controversies that are common in academia. And if we combine disjunctive and conjunctive causation, we have an instance of causation that is neither necessary nor sufficient, as indicated in (44), a structure that would have confounded Aristotle, but is quite common in real life.

6.2.2 Probabilistic Causation

To turn to a different issue, we note that in "commonsense" thinking, we tend to view causation as ***absolute causation***. That is to say, given the cause, the effect is inevitable. In real life, however, most causal relations are probabilistic. Instead of saying:

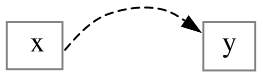
For any x, x's smoking will *necessarily cause* lung cancer in x.

we should be saying

For any x, x's smoking *tends to cause* lung cancer in x; or
For *most* x, x's smoking will cause lung cancer in x.

These are instances of ***probabilistic causation***. In our discussion of deductive propositional logic, we distinguished between the conditionals of classical logic, probabilistic logic and defeasible logic. We may extend these concepts to causal logic as well. We represent probabilistic causation as dotted curved arrows:

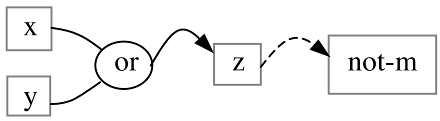
(**45**) a. x tends to cause y. b.



6.2.3 Other Concepts in Causal Networks

Just as we combined atomic conditionals into a network of conditionals when we represented (8) as (9) in propositional logic, we can combine atomic cause-effect relations to form complex causal networks. For instance, we may represent (46a) as (46b):

(**46**) a. either x or y necessarily causes x, and
z tends to cause absence of m



In (46), x and y are ***proximate*** (immediate, or *direct*) causes of z, and z is a proximate cause of not-m. In contrast, x and y are ***non-proximate*** (*indirect*) causes of m.

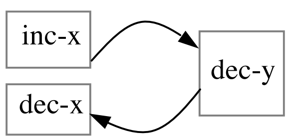
When we combine causal relations in this manner, we discover that cause-effect relations can be either ***synergistic*** (mutually reinforcing) or ***antagonistic*** (mutually negating). For instance, consider the following situation:

Intake of fatty foods increases the risk of cardiac problems.
Sedentary habits increase the risk of cardiac problems.
Regular exercise reduces the risk of cardiac problems.

The first two are synergistic, while the first (or second) and the third are antagonistic. To calculate the effects of synergistic and antagonistic cause-effect relations, we need to resort to ***defeasible causal reasoning***, analogous to defeasible deductive reasoning.

Now consider a feedback loop like that in (47):

(**47**) a. Increase of x causes decrease of y, and
decrease of y causes decrease of x.



Feedback loops of this kind result in dynamic equilibrium, as in the predator-prey relations. (Increase in the number of predators results in decrease in the number of prey, which results in the decrease in the number of predators that results in the increase in the number of prey which goes back to the increase in the number of predators.)

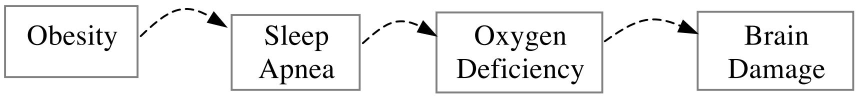
The classical notion of causation is that of a unidirectional (i.e., anti-symmetric) relation: if A influences B, then B cannot influence A. Given the existence of feedback loops in the physical, biological, mental, and social systems, we need to acknowledge ***bidirectional causation*** as well.

6.3 An Example of the Application of Causal Network Diagrams

As an illustration of the usefulness of our representational system for causation, let us take Olivia Judson's article on brain damage referred to earlier, which claims that obesity is a causal factor in brain impairment. She acknowledges that there are genetic causes for brain impairments as well, but goes on to say:

> "Obesity exacerbates problems like sleep apnea, which can result in the brain being starved of oxygen; this can lead to brain damage."

We can represent the causal structure of Judson's sentence as:

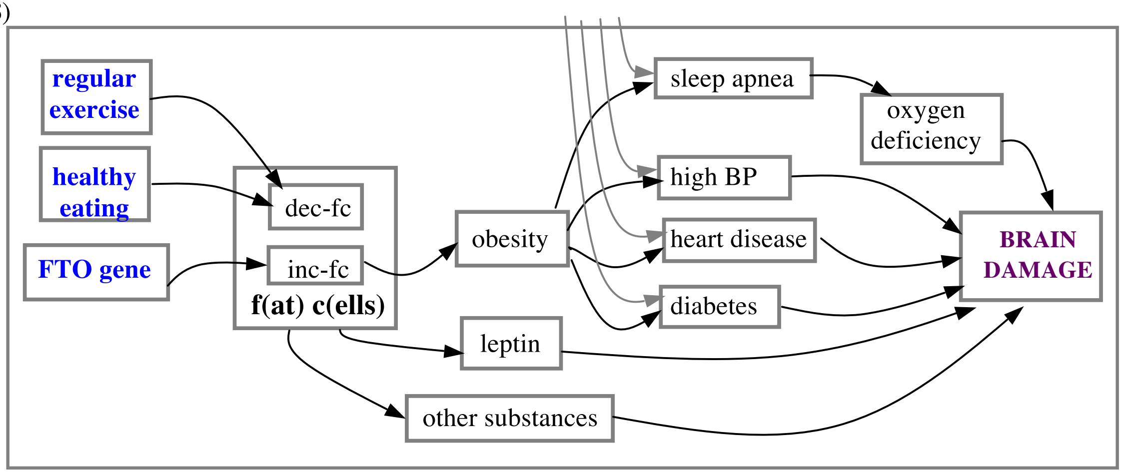


The paragraph in which this sentence appears is given below:

> "Obesity exacerbates problems like sleep apnea, which can result in the brain being starved of oxygen; this can lead to brain damage. Obesity often goes along with high blood pressure, heart disease and diabetes, all of which are bad for the brain in their own right. Indeed, one study has shown that if, in middle age, you are obese and have high blood pressure, the two problems gang up on you, increasing the chances of your getting dementia in old age more than either one would do on its own."

The causal network that forms the explanatory theory implicit in Judson's article can be diagrammed as follows:

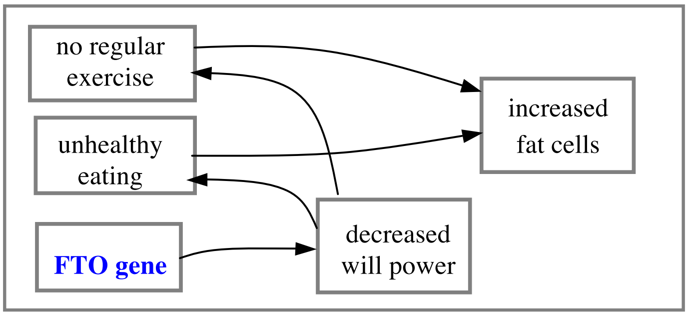
(**48**)



Experimental biology uses the terminology of proximate and ultimate causes for situations of this kind. (http://en.wikipedia.org/wiki/Proximate_and_ultimate_causation) In this terminology, the three causes in blue bold face (regular exercise, healthy eating, and FTO) are ***ultimate causes*** influence (increase or decrease chances of) brain damage. The statement, "Regular exercise and healthy eating are causal factors in reducing the probability of brain damage," is then an observational causal hypothesis. What a theory has to do is to provide the intermediate links of causation (the "mechanisms" that a how-explanation looks for) in terms of a chain of ***proximate*** causations.

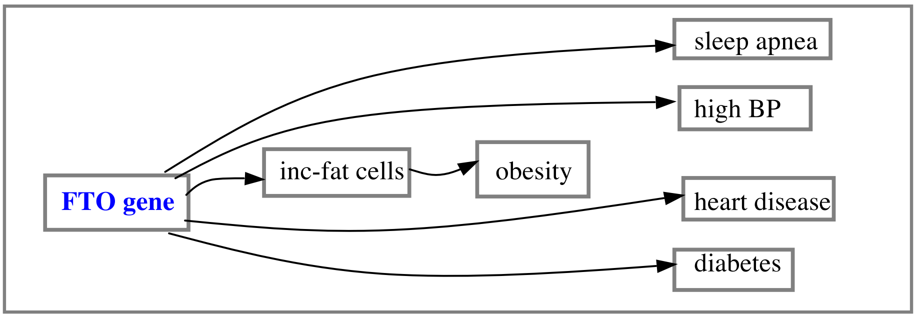
Once the theory is visually represented as a model along the lines illustrated in (48), there is an obvious question. What is the evidence to assume the directionality of causation along the lines asserted in (48)? Suppose we propose an alternative assumption, that the FTO gene allele apparently responsible for increase in fat cells actually causes decreased will power, resulting in unhealthy eating and lack of regular exercise, which in turn is responsible for increase in fat cells. The relevant aspects of this alternative are given in the competing model in (49):

(**49**)



Another obvious question is: assuming that the directionality of causation in the case of regular exercise, healthy eating and fat cells is as indicated in (48), what is the evidence to show that FTO does not directly cause sleep apnea, blood pressure, and heart disease? The relevant aspects of this alternative model are given in (50):

(**50**)



What is the evidence for accepting or rejecting the alternative models in (49) and (50)? Without answering that question, the justification for the model in (48) is incomplete.

Considering such alternatives are crucial in the critical evaluation of a model. Formulating the theory in terms of ordinary language often prevents us from noticing such plausible alternatives, or at least makes it very hard to do so. The advantage of the diagrammatic notation of the causal model sheds light on the crucial links that ought to be scrutinized and questioned.

7 Abductive Reasoning

Suppose we take an afternoon nap, and when we wake up, we find that the streets are wet. Now, we already know the observational causal generalization that:

Rain causes streets to be wet.

We also subscribe to the principle that:

Every event, entity, state of affairs, and property must have a cause.

Given the above, it is reasonable for us to conclude that:

It must have rained when we were taking a nap.

The mode of reasoning we used in arriving at this conclusion is called ***abductive reasoning***.

Compare the classical deductive reasoning in (51a) with the abductive reasoning in (51b):

(**51**) a. Whenever it rains, the streets get wet. It was raining last night. Therefore, the streets must have got wet.

b. Whenever it rains, the streets get wet. The streets are wet now. Therefore it must have rained.

(51a) is an example of the familiar Modus Ponens. Given that (i) rain causes wetness of the streets, and (ii) rain (the cause) is observed, we conclude that wetness of the streets (the effect) must have happened. In other words, *we infer the effect from the cause* in (51a).

In (51b), in contrast, we are *inferring the cause from the effect*, reversing the direction of inference. It was philosopher Charles Sanders Peirce who, noticing the use of this form of reasoning in everyday life and academic knowledge, called this mode of reasoning ***abduction*** (to distinguish it from ***deduction***).

Abductive reasoning is common in the justification of interpretations, in almost all academic fields outside of mathematics and logic. The form of reasoning that doctors use in diagnosing diseases from symptoms, for instance, is abductive. Take the following example:

(**52**) A person having a heart attack tends to have a feeling of choking, pain in the chest radiating to the left shoulder and arm, abnormal perspiration, breathlessness, and nausea.

Fanny Jenkins has just experienced a feeling of choking, pain in the chest radiating to the left shoulder and arm, abnormal perspiration, breathlessness, and nausea.

Therefore it is reasonable to conclude that Fanny Jenkins had a heart attack.

A great deal of the reasoning used in a law court is also abductive. For instance, consider this:

(**53**) You wake up in the morning to find the door wide open, your stereo system gone, and broken glass inside, near the window.

If someone shattered a windowpane from outside to break into the house, there would be broken glass inside, near the window.

It is therefore reasonable for you to conclude that someone entered the house by shattering the windowpane.

Now, it is important to distinguish valid abduction from the form of illegitimate reasoning illustrated by examples like the following:

(**54**) a. Whenever Leda teases Zeno, Zeno cries.
Zeno is crying now. Therefore, Leda must have teased him.

b. All horses are mammals. A rabbit is a mammal. Therefore a rabbit is a horse.

The Rule of Modus Ponens does not validate the reasoning either in (54a, b) or in (51b), (52), and (53). Despite this, (51b), (52), and (53) are examples of good reasoning, while (54a) and (54b) are not. To see why this is so, we need to spell out the rule of abduction:

Rule of Inference: ***Abduction***

Given that		*Theory T is justified;*
		P is the best available explanation for Q within T;
and		*Q is true;*
it is reasonable to conclude that:		*P is true*
in the absence of	(i) evidence to the contrary; and	
	(ii) a better/equally good alternative explanation for *Q* within *T*.	

The crucial ingredient here is: *in the absence of evidence to the contrary, and a better or equally good alternative explanation for Q within T*. This specification with minor differences appears in induction and speculative deduction, but is absent in classical deduction.

Why is the reasoning in (54a) and (54b) bad? In the case of streets being wet from rain ((51b)), the diagnosis of a heart attack ((52)), and the supposition of breaking and entering through the window ((53)), we cannot think of any obvious alternative explanations. Hence the reasoning in these cases is legitimate.

In (54a), however, we are aware of other potential causes for Zeno's crying: he could have fallen and hurt himself, or his sister may have scolded him, or his teacher may have given him a poor grade, and so on. The availability of obvious alternatives makes the reasoning invalid.

As for (54b), the relation between mammals and rabbits (if x is a rabbit, the x is a mammal) is neither causal nor an explanation, so abduction is not applicable in this case. (Also, we know there are many animals other than rabbits that are mammals.)

Let us go back to (51b) for a moment. From the wetness of the street, we concluded that it must have rained, because we couldn't think of another good cause. Suppose someone said:

> When the water truck goes down the streets watering the ground, the streets get wet.
> The streets are wet.
>
> Therefore, the water truck must have gone down the street.

We now have an equally good competing cause, and our previous conclusion is no longer valid. We must therefore look for further evidence to choose between the two. Suppose we examine the bushes near by, and find that they are not wet. The water truck hypothesis explains why only the streets are wet, while the rain hypothesis incorrectly predicts that the bushes would also be wet. Hence, we choose the water truck hypothesis. If, on the other hand, we find that the garden and the plants and the bushes are all wet, as expected if it rained, we choose the rain hypothesis. The truck hypothesis fails to explain why the garden and the plants and the bushes are all wet.

8 Analogical Reasoning

No survey of the modes of reasoning would be complete without at least a brief mention of analogical reasoning. Let us begin with an example. The English government has a law protecting animals in experiments, requiring that animals belonging to a 'privileged class' that feels pain should not be operated on without anesthesia. Now, pain is an internal experience that cannot be directly observed. How do we determine whether or not a given species of animal is capable of feeling pain?

How do each of us know that other human beings are capable of experiencing pain? The strategy is as follows. I know that under certain kinds of external stimuli (e.g., wounds, falls, burns) I feel pain, and when I feel pain, I tend to behave in certain ways (e.g., tears, certain facial expressions, body postures, noises). When other human beings behave in parallel ways in response to parallel stimuli, I infer that they have the parallel internal experience of pain. This inference is corroborated by what they say about their internal experience. It is therefore justifiable to conclude that human beings are capable of feeling pain.

We extend the same strategy to animals, except that they cannot report their pain to us. If we find that an animal responds to pain-inducing stimuli in ways parallel to the pain response of human beings, we conclude that the animal is experiencing pain. The type of reasoning that allows us to make such inferences is analogical reasoning.

Analogical reasoning involves making an inference about X based on (a) the analogy (equivalence) between X and Y, and (b) what we know about Y. When we observe a dog wounded in a traffic accident, for instance, we infer that the dog must be in pain, based on the analogy between humans and dogs and what we know about humans. The structure of this piece of reasoning can be unpacked as follows:

(**55**) Dogs are analogous to humans. When humans are wounded, they experience pain. Therefore, it is reasonable to conclude that when dogs are wounded, they experience pain.

Given below are additional examples of analogical reasoning:

(**56**) a. The human heart has a structure and a function. Its structure is broadly that of a complex piston. It pumps blood to other parts of the body. Now, we all agree that we cannot really understand the structure of the heart without understanding its function.

Likewise, we cannot understand the structure of language unless we understand what it is used for — its function.

b. We know that waves in water exhibit the property of interference.

If we assume that light travels in a vacuum analogous to waves in a medium, it would follow that light would also exhibit the property of interference.

This prediction is borne out. Hence it is reasonable to conclude that light travels as waves.

The reasoning in (56b) involves the analogy of light and waves, and hence is an example of speculative deductive analogical reasoning. The example in (56a) involves analogical reasoning but not speculative deduction. The central rule that sanctions analogical inferences is given below:

Rule of analogical reasoning

Given that:	*x is analogous to y*
and	*P is true of y*
it is reasonable conclude that:	*P is true of x*

Recall that the conclusions in many of the modes of reasoning discussed earlier need caveats like "in the absence of evidence to the contrary" and "in the absence of a better or equally good alternative". Such caveats apply to analogical reasoning as well, because they are characteristic of defeasible reasoning (in which the conclusion can be defeated by further evidence or by a better or equally good explanation), and analogical reasoning is also a form of defeasible reasoning.

9. Concluding Remarks

9.1 Ingredients of Scientific Inquiry

To review (and add to) the preceding discussion, some of the ingredients of scientific inquiry that a student of science needs to be aware of and understand can be summarized as follows:

- Mathematical inquiry arrives at, justifies, and critically evaluates conclusions based on ***axioms*** and ***definitions***, and previously established ***theorems***.

  Scientific inquiry arrives at, justifies, and critically evaluates conclusions based on ***data*** and previously established observational generalizations and theories.
- A body of data consists of ***data points***. A data point consists of one of more observations on a single entity or process in a ***sample***. A sample is a subset of a ***population*** under investigation.
- An individual observation is a (quantitative or qualitative) ***value*** of a given ***variable*** for an entity/process (a *measurement* being a quantitative observation).

  A variable is a ***parameter/coordinate/dimension*** along which a given entity/process may *vary*.

  If we view the logical possibilities of a population as a multidimensional space, we may say that a data point is the *location* of a given entity/process under investigation in terms of the coordinates/dimensions of that multidimensional space.
- An ***observational generalization*** is a *pattern* of data points.
- Observational generalizations are either ***correlations*** or ***causal generalizations***. In statistical research, correlations are often viewed as a *linear* pattern connecting the data

points, but a correlation can also be *non-linear*, and, in the case of *periodic* or *cyclic* patterns, a *wave*.

- A scientific theory is an ***explanation*** for the observational generalizations.

  A theory consists of a set of theoretical ***concepts*** and a set of theoretical propositions in terms of these concepts.

  The propositions may be a set of individual ***laws*** or a configuration of propositions that constitute a ***model***.
- The observational propositions (observational generalizations) that call for an explanation are the ***explanandum*** of the explanation, and the theoretical propositions that yield the explanations are the ***explanans***.

  A given set of explanans propositions provide an explanation if its ***predictions*** (the logical consequences) agree with the explanandum propositions.
- An ***interpretation*** of a given observation, data point, or observational generalization is a representation of the observation, data point, or observational generalization in terms of the concepts of the theory.
- ***Methodology*** in the context of the hypothesis-testing model of scientific research consists of the *use of tools, techniques, procedures, and protocols for gathering and processing the data points and arriving at observational generalizations from these data points*. [Training in this methodology does not include training in theoretical inquiry.]

9.2. The Place of Logics in Scientific (and Mathematical) Inquiry

To place the logics that we have discussed in the preceding sections in the context of the above summary:

- ***Descriptive statistics*** deals with the process of identifying a sample and establishing quantitative observational generalizations of the sample. It is supplemented by instrumentation and techniques for laboratory experiments and field experiments. This does not cover non-quantitative observations.
- The rules of inference of ***inductive logic*** provide the guidelines for generalizing observational generalizations from the sample to the population. Hence, inductive reasoning is sample-to-population reasoning. ***Inferential statistics*** provides the guidelines for sample-to-population reasoning when dealing with quantitative data.
- ***Deductive logic*** provides the guidelines for reasoning from the theory to its predictions, as well as from the observational generalizations on the population to the sample or to individual data points. For this, we may employ classical deduction, probabilitistic deduction, or defeasible deduction. Mathematical ***probability theory*** provides the guidelines for quantitative probabilistic reasoning.
- We justify theoretical propositions in scientific theory on the basis of observational generalizations, using speculative deductive reasoning. ***Speculative deductive logic*** provides the guidelines for this form of reasoning. In contrast, we use ***classical deductive reasoning*** to justify conjectures, and establish them as theorems in mathematical inquiry.
- We arrive at and justify interpretations on the basis of abductive reasoning, whose guidelines are provided by ***abductive logic***.
- ***Analogical reasoning*** allows us to discover patterns, justify them, and connect data points, patterns, interpretations, and theories across different fields/domains of inquiry.

In light of these remarks, we may view what we have sketched above as a (partial) trans-disciplinary foundation for reasoning and logic needed in scientific inquiry, which would be of value to science students who wish to acquire the capacity to engage in

(a) scientific inquiry (typically relevant at the high school or college levels), and
(b) independent research (typically relevant for Master's or PhD levels).

What we have summarized above may be thought of as the outline of a curriculum for a set of introductory and advanced courses on scientific inquiry as the foundations for scientific research. It is our hope that what we have presented here will also be of value to science teachers (in schools, colleges, and universities).

9.3. The Need for Theoretical Inquiry in Science Curricula

You must have noticed that even though we have covered observational inquiry/research, the typology of inquiry/research we have presented is of greater value in theoretical research. A good way of looking at this typology is perhaps through the perspective of thinking critically about scientific theories, which involves addressing the following questions:

A. What are the central propositions of the theory ("explanans")?
B. What observational generalizations ("explanandum") does the theory seek to explain?
C. What are the logical consequences ("predictions") of the theory?
D. How exactly are the propositions in C deduced from those in A?
E. How well do the predictions in D match the observational generalizations in B?
    1. Does the theory make incorrect predictions? (i.e., are there observational generalizations that are logically inconsistent with the predictions of the theory?)
    2. Does the theory cover all the relevant observational generalizations? (Is the theory "complete" with respect to the pool of available observational generalizations?)
    3. Does the theory make predictions that have not been tested yet? (Are there predictions for which there are no confirming or refuting observational generalizations?) Are these predictions testable?
F. On the basis of E, do we need to develop the theory further by:
    1. abandoning or revising some of the theoretical propositions of (A), and/or
    2. incorporating additional theoretical propositions into A?
G. Do alternatives to any of the propositions in A exist? If they do, which configuration of explanans propositions yields the best results for the explanandum in terms of the considerations in E?
H. In view of A-G, is the theory under consideration a good theory? If it is a good theory, is it the best theory?

It should be clear by now that the preceding discussion was structured such that its parts provide the foundations for addressing question H, which in turn calls for addressing questions A-G.

Exercises

1. Here are examples of reasoning with implicit premises. Make the reasoning explicit by supplying the missing premise(s) in each case. Check whether the premises you have unearthed are acceptable.

a. The murder weapon has Zeno's fingerprints on it.
Therefore, Zeno is the murderer.

b. Leda drinks whisky every day.
Therefore, Leda is immoral.

c. Dina is a prostitute.
Therefore Dina is immoral.

d. Blimpy flunked all her exams.
Therefore Blimpy is not intelligent.

e. On a scale of 1 to 10, Zeno has a score of 9.8 in his student feedback.
Therefore Zeno is an excellent teacher.

2. Convert the following statements into the "If… then…" format illustrated in section 2.1.1:

a. All slaidins are tall.
b. No mammal has wings.
c. Only birds have wings.

3. Represent the information in (2a) – (2c) in terms of Venn diagrams.

4. Consider the following promise from a teacher:

Anyone who wears shoes to this class will get an A for the assignment on reasoning.

You are told that Bill wears shoes but Susan doesn't. Mina has an A, but Jacob has a B. Your task is to determine if the teacher has kept his promise with respect to these students. You can interview them to elicit the relevant information, but you can ask each of them only one question. Also, you must pick as few students as possible. Which students would you interview to check the truth of the proposition? What question would you ask each one?

5. Consider the following scenarios:

You are asked to make an estimate of the density of trees (number of trees per square kilometer) in North America.

You are asked to estimate the proportion of red and white balls in a barrel containing ten thousand balls (without actually counting all the red and white balls in the barrel.)

What additional precautions do you need to take for the first task?

6. An argument is *valid* if its conclusion follows from its grounds. The argument below is valid in one mode of reasoning but invalid in another. What are the two modes of reasoning?

Wherever there is fire, there is smoke. There is smoke coming from that apartment. Therefore, in the absence of evidence to the contrary and an alternative explanation, it is reasonable to conclude that there is fire in that apartment.

7. Specify the modes of reasoning used to justify each of the following claims:

a. There is a strong correlation between IQ scores and language competence.
b. Based on the evidence of bruise marks around the neck, a forensic specialist concludes that the cause of death was strangling.
c. On the basis of evidence from passives, agreement, case marking, and topicalization, we conclude that the distinction between objects and adjuncts is legitimate.
d. Jane's 3-month old baby couldn't possibly have told her that he enjoys reading Emily Dickinson's poetry.