Region of Inexactness and Related Concepts

1. Region of Inexactness

Suppose Plato says that he is six feet tall. On measuring him, we find that he is 5 feet 11.9 inches, not exactly 6 feet. Is Plato's statement false?

Though there is a difference of 0.1 inch between 6 ft and 5 ft 11.9 in, we are unlikely to say that Plato's statement is false. The difference is so small that it is negligible. For all practical purposes, 6' and 5'11.9" count as the same; we do not interpret "I am 6 feet tall," as "I am *exactly* six feet tall, not more, not less." Hence the difference of 0.1 inch is not *significant*.

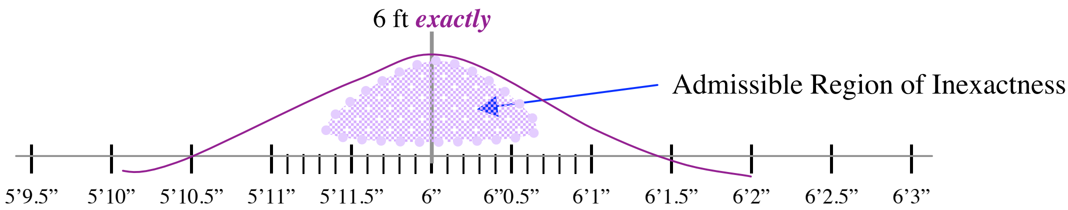
When we measure Plato, what if he turns out to be 5'6"? This time, we would say that Plato's statement about his height is false: a difference of 6" is significant. What if he is 5'7"? Now, what if he is 5'11.5"? You will agree that a difference of 0.5" is not as negligible as 0.1", but you might still be willing to judge Plato's statement as true. In other words, 6' and 5'11.5" are not distinct when we evaluate the truth of: "Plato is six feet tall." What this example shows is that there is a certain degree of slack, or fuzz — a *region of inexactness* — in our statements about the world, even when stated in terms of numbers. We are permitted a certain degree of deviation from the strict interpretation of what a sentence asserts.

In other words, there is bound to be a ***Region of Inexactness*** (RI) in our propositions about reality. In the above cases, Plato's actual height is slightly less than what he states. But RI also holds when the actual height is more than what he states. If Plato is 6'1', his statement about his height is still true. But it is false if he is 7', or 6'6" tall. Thus, RI extends in both directions.

How much of such inexactness are we prepared to tolerate? Suppose Plato is 5'3", and he says that he is 6' tall. You point to the 9" difference between his claim and the reality, and he says, "Come on, give me some slack. You know that there is some degree of inexactness in our statements." Would you grant him a slack of 9 inches? No.

We may represent the idea of RI in the form of a graph, with what the sentence asserts as the middle, and negligible deviations on either side:

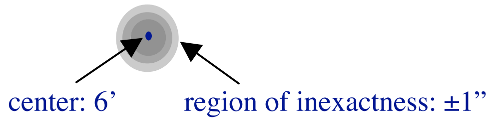
(1) Inexactness in: "Plato is 6 ft tall."



When someone says, "I am six feet tall," what he means is, his height is 6', plus or minus _" (or one inch)," or something to that effect. In order to be precise, we need to specify the degree of inexactness that we expect, by adding "plus or minus x" in our assertions.

In the figure in (1), we have a ***center*** (6 ft) and a ***region*** surrounding that point (between 5'11" and 6'1"). We may also visualize the center and region as in (2):

(2)



If the difference between the asserted and the actual goes beyond the admissible region of inexactness, we treat it as ***significant***. Thus, in the evaluation of the truth of Plato’s statement that he is six feet tall, a deviation of 6 inches is significant, but not that of 0.01 inch.

Those who are familiar with statistics would immediately note that the center in (1)-(2) corresponds to ***central tendency*** in statistics (mean, median, or mode). The region of inexactness corresponds to the range of ***dispersion***.[1]

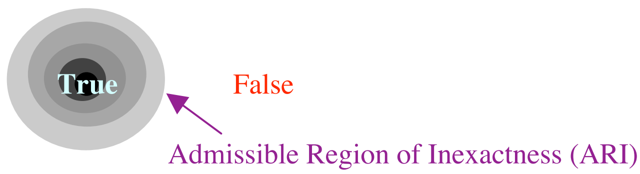
2. RI and Testability

One crucial demand on science is that the assertions it makes be ***testable***. Let us state this criterion explicitly as follows, in such a way that it applies to academic inquiry as a whole:

(3) *Assertions of truth in academic inquiry should indicate under what conditions we would judge them to be true, and under what conditions we would judge them to be false.*[2]

By stating: “Plato is six feet tall, plus or minus one inch,” we are specifying a criterion for testing the assertion. If Plato’s height is between 5’11” and 6’1”, the statement is true; but if it is less than 5’11”, or more than 6’1”, the statement is false. The practice of specifying the admissible region of inexactness as “plus or minus x” facilitates testability, and contributes to good intellectual hygiene.

(4)

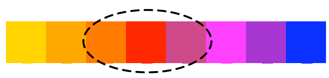


3. RI and Qualitative Claims in Academic Inquiry

Is RI relevant only in quantitative statements? Or is it important to specify ARI in qualitative statements as well? The answer is a resounding yes.

Suppose Plato says, “The colour of mother’s hair is red.” You wish to evaluate the truth of this statement, so you meet his mother and pay attention to her hair. Let us imagine that the colour of her hair is one of the patches given below:

Which of them would you treat as red? The answer, most likely, can be expressed in terms of the dotted ellipse below:



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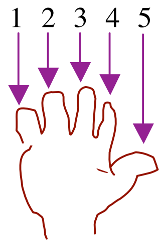
1 See http://en.wikipedia.org/wiki/Statistical_dispersion

2 The testability criterion in (3) is not identical to Popper’s falsifiability criterion, which calls only for formulations that tell us under what conditions we would judge a scientific theory to be false. Condition (3) views testability as a criterion relevant not only for refutability but also for justifiability, and not only for scientific theory, but also for any academic claim of truth.

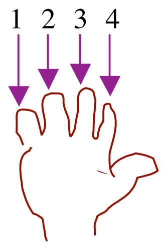
If you reflect on the colour region that you would treat as red, as opposed to white, black, orange, violet, brown, and so on, you will see in your mind's eye the same type of region with a center and a fuzzy boundary as we saw in the case of Plato's height.

Let us take an example of a different kind of inexactness. Suppose Plato says, "I have four fingers on my right hand." You examine his right hand, and it looks like this:

You count the number of fingers:



And you say, "Plato, you are wrong. You have five fingers on your right hand." He responds, saying, "No. I have exactly four fingers. You counted wrong." And he counts:

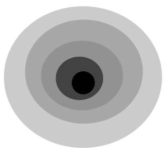


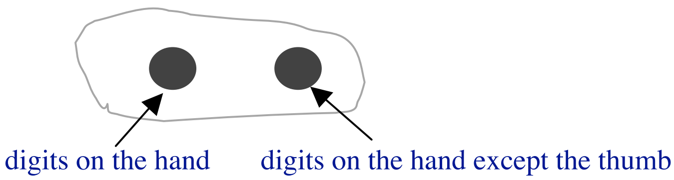
You point out that he didn't count his thumb. "Oh," he says, "but the thumb is not a finger."

In the English language, the word *finger* has two meanings. It can refer either to all the digits on the hand (meaning 1) or to all the digits except the thumb (meaning 2). If we assign the first meaning to the word 'finger' in "I have five fingers on my right hand," Plato's statement is false. But if we assign the second meaning, the statement is true.

This example illustrates inexactness that arises from ambiguity, in contrast to that of Plato's height, which illustrates inexactness arising from vagueness.

(5) Vagueness Ambiguity





The problem disappears once we clarify the meaning. In this case, we are not merely reducing the degree of inexactness, but eliminating the ambiguity.

Let us take another example. Plato claims that all scientific knowledge is ideology. We are surprised, and the following conversation ensues:

Us: You are wrong, Plato. Take the knowledge that the earth goes around the sun. Or the knowledge that petrol is inflammable, or that small pox is caused by a virus. Are you saying that these are all ideological?

Plato: They **are** all ideological. The belief that the earth goes round the sun and the belief that the sun goes round the earth are both ideologies.

Us: So any belief is an ideology?

Plato: Yes, of course.

Us: But scientific beliefs are rationally justified. In contrast, beliefs that many people regard as ideologies, say, the beliefs that blacks are inferior to whites, that women cannot be rational, that kings have a divine right to rule, and so on, have no rational support. Would you group both these types of beliefs as ideologies?

Plato: Yes, I would.

Given Plato's clarification that for him, all beliefs are ideologies, and given that what we call "knowledge" is a body of propositions that we *believe* to be true, it follows that for him, all scientific knowledge is ideological. He is simply saying that scientific knowledge is a body of propositions that we believe to be true; this is nothing new. In contrast, if we define ideology as beliefs that are not rationally justified, Plato's position would be wrong.

Let us take another example. Plato makes the following claim:

(6) Faculty appointments in American universities have a strong gender bias.

We ask him to support his statement with adequate evidence. And he comes up with the following justification:

(7) We surveyed a randomized sample of 100,000 university administrators in American universities. More than 95% of them said they have a preference for candidates who appear more rational and more confident, over those who appear more emotive and more diffident.

But how does this data show that there is gender bias? Gender bias, as we know it, is a prejudice for or against males or females. It is not a preference for rationality or for confidence. When we raise this issue with Plato, he says:

(8) There is an important difference between gender and biological sex. Biological sex involves the distinction between male and female, while gender involves the distinction between masculine and feminine. Masculine and feminine are non-biological properties that are associated with male and female. In terms of biology, for instance, a book is neither male nor female, but in Hindi the word *kitaab* 'book' is assigned feminine gender, while *aam* 'mango' is assigned masculine gender. The claim in (5) is about the properties of rationality and confidence prototypically associated with males.

The meanings of words in ordinary language have considerable fuzz around them. Most of them are vague and many of them are multiply ambiguous. Gender is one such word. Quite often, we use 'gender' as a synonym for biological sex, but then there are also contexts in which we distinguish between gender and sex. For most people, 'gender' in 'gender bias' refers to prejudice based on biological sex, but the confusion disappears if the concept is clarified as in (8). This means that claim (6) should be reformulated as in (9):

(9) Faculty appointments in American universities have a strong gender bias.
[*Note:* By 'gender' in 'gender bias', we mean non-biological properties that are prototypically associated with male and female. A property is masculine if it is prototypically associated with males, and feminine if it is prototypically associated with females. We assume that rationality and confidence are masculine, while emotionality and diffidence are feminine.]

Given this clarification, Plato's conclusion translates in effect as: "Faculty appointments in American universities have a strong bias against emotionality and lack of confidence." This follows uncontroversially from the grounds he cites. If we interpret 'gender bias' in its ordinary sense, the conclusion translates as: "Faculty appointments in American universities have a strong bias in terms of biological sex." This does not follow from the grounds he cites. Satisfying condition (3), therefore, crucially requires a note of clarification on the concept of 'gender bias', like the material in parentheses in (9). This material provides the ARI for the term 'gender bias' that allows us to critically evaluate the claim in (9), and on the basis of the additional clarificatory note, arrive at a judgment of its truthhood.

Let us take one more example. Suppose Plato says:

(10) a. The flagella of bacteria and the legs of cows are homologous organs.
 b. The wings of bats and those of birds are homologous organs.

Are these statements true? To answer the question, we need to know what "homologous organs" means. Consider the following candidates as definitions of the term:

(11) a. Two organs are homologous if and only if they have the same structural design.
 b. Two organs are homologous if and only if they serve the same function.
 c. Two organs are homologous if and only if they have the same structural design and serve the same function.
 d. Two organs are homologous if and only if the have the same structural design, serve the same function, and are derived from the same evolutionary ancestor.

Under (11a), statement (10a) is false, but under (11b) it is true. Statement (10b) is true under (11a) and (11b); however, it is false under (11d), because the wings of bats and birds are not derived from the same evolutionary ancestor.

As the examples discussed above show, definitions and conceptual clarifications in qualitative statements serve the same function as the specification of the admissible range of inexactness in quantitative statements. They provide boundaries — either crisp or fuzzy — allowing us to determine whether or not an assertion is true, and thereby conform to the guidelines in (3).

Having recognized this property of definitions and conceptual clarifications, we can now extend the idea of ARI to the qualitative domain, and say that definitions and conceptual clarifications also constitute ARIs.

Abstract terms in academic pursuits, including 'science', 'democracy', 'species', 'language', 'modernity', 'postmodern', 'ideology', 'feminist logic', and 'positivism', often involve multiple ambiguities entangled with vagueness. Without the intellectual hygiene of being accompanied by ARI specifications, they end up violating (3).

4. Factors that Influence ARI

Let us return to Plato's height. Given "Plato is six feet tall," on what basis do we judge "plus or minus one inch" as an admissible range of inexactness? To generalize, on what basis do we arrive at a judgment on the appropriateness of the RI of a claim or conclusion?

Suppose Plato has just measured the diameter of a copper wire, and tells us that it is 0.4 cm. When we measure the wire, we find it that it is 0.1 cm. So we tell Plato that his measurement is wrong. And he says, "Come on, when I said the diameter is 0.4 cm, I meant it is 0.4 cm plus or minus 0.3 cm." Would you accept Plato's response? One inch may be a reasonable slack for six feet, but is 0.3 cm permissible slack for 0.4 cm?

The answer clearly is no. And that answer suggests that one of the parameters that determines the appropriateness of RI is *proportionality*. One inch is less than 1.5% of 6 ft, but 0.3 cm is 75% of 0.4 cm, which is way beyond what is reasonable. If proportionality is a consideration, one would expect that two feet would be within ARI in the specification of the height of a ten story building. And this expectation is correct: if Plato says his apartment complex is 150 feet high, he wouldn't be uttering a falsehood if the actual height is 148 feet.

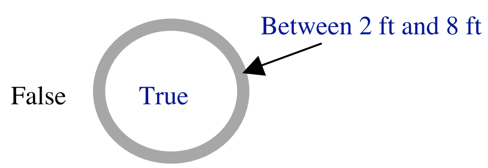
But proportionality can't be the only consideration. If Plato says that his apartment complex is 148 ft 8 in, would you judge his assertion as true if the actual height is 150 ft? Clearly not. This suggests that another factor is the degree of *precision* implied by the statement itself. We expect specifications of numbers like 100 and 200 to be rounded up (or down) from one or two units of the same category, but a specification such as 98 ft 5 in. carries a degree of precision that disallows any rounding up or down. Having specified the degree of precision in terms of inches, the only permissible RI is in terms of inches, not feet.

The *purpose* of the degree of precision is yet another factor. Take the specification of a door as being 6'x3'. If the purpose is to decide how much wood we would need to build the door, plus or minus 2 inches would be quite reasonable, but if the door is meant to fit an existing frame, the same slack would be unacceptable.

5. ARI, Confidence Interval, and Confidence Level

Suppose we are told that Xena is a thirty-year old Indonesian woman, and we are asked what her height is. Our initial reaction might be, "How do I know? I haven't seen her, and I have no information about her other than that she is a thirty-year old Indonesian woman."
But let us think about this carefully, using the thinking tool of ARI. Is there *nothing* we can say about Xena's height? Does the information that she is a human being provide no constraints on her height? Had she been a mouse, her height could be 3 inches, but being a human being, Xena can't be 3 inches tall. And unlike a giraffe, she can't be 18 ft tall either. So at least we can say with reasonable confidence that her height must be between, say, 2 feet and 8 feet:

(12) Xena's height

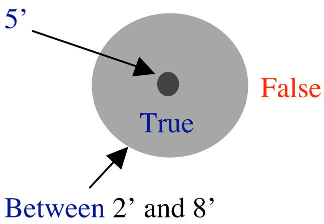


Recall that when specifying Plato's height in (2), we used a center (6 ft) and a region (±1 in). In (12), however, we only have a region with a boundary (between 2 ft and 8 ft), but not a center. Is it possible to specify the center as well for Xena's height?

The answer is yes. To find the center, we have to find the average height of adult Indonesian women. The Wikipedia entry for human height specifies the mean height of a female adult in Indonesia as 4ft 11in.[3] For convenience of exposition, let us pretend that it is 5' rather than 4'11". Using this information, we can specify Xena's height as 5', plus or minus 3':

[3] See http://en.wikipedia.org/wiki/Human_height

(13) Xena's height revised



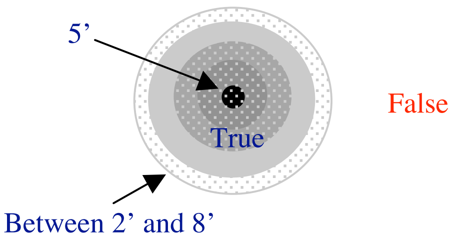
What would happen if we reduced the RI? Compare the following specifications:

(14) a. 5 ft, plus or minus 3 ft.
b. 5 ft, plus or minus 1 ft.
c. 5 ft, plus or minus 6 in.
d. 5 ft, plus or minus 2 in.

If we use (14a), it would take something extraordinary for our conclusion to be false. If we go by (14b), there is some probability that we are wrong: she might be 3'10" or 6'3". If we use (14c), there is a greater probability of our being wrong, and an extremely high probability that we are wrong if we use (14d). And if we say, "5 ft exactly, not a fraction of an inch less or more," we are almost certainly wrong.

Given these degrees of uncertainty, we may revise (13) as (15), distinguishing the different regions of inexactness:

(15) Xena's height revised further



As must be clear by now, this example illustrates the following generalization about ARI:

(16) The broader the ARI, the greater our confidence that our conclusion is correct; conversely, the narrower the ARI, the less confident we are that the conclusion is correct. (To put it differently, the greater the accuracy/precision in our specification, the less confident we are that the conclusion is correct.)

In the literature on statistical inquiry, ARI is called the ***confidence interval***, and the two end points of the interval are called ***confidence limits*** (e.g., in (14b), the limits are 4' and 6'). The degree of confidence that the conclusion is correct is called the ***confidence level***. The greater the required confidence level, the greater the confidence interval.[4]

In an earlier discussion, we listed three factors that influence our judgments on the admissible range of inexactness: *proportionality, precision,* and *purpose*. Given what we have discovered about the relation between confidence level and ARI, we need to add one more factor to the list, namely, the degree of *confidence* we would like to have in the correctness of our conclusion.

[4] See http://en.wikipedia.org/wiki/Confidence_interval

6. ARI and Significance

Suppose Xena says that she is taller than Plato, and he disagrees. They go to a height-measurer to settle their dispute. The results of the measurement are:

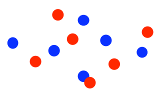
Xena: 5' 11.53"
Plato: 5' 11.52"

Clearly, the specification of these heights assumes a high degree of precision. Now, an ordinary ruler gives a 10-point calibration of an inch. So if we are measuring the length of a straight line on paper, we can reliably distinguish between 0.3" and 0.4". With some care, we might distinguish between 0.3 and 0.35, but no ordinary ruler can give us precision up to 0.01". Hence, if the measuring *instrument* is an ordinary ruler, the accuracy of the above measurements of Xena's and Plato's heights is highly suspect. For that kind of precision, we would need something like a vernier caliper, not an ordinary ruler.

Let us imagine that we have a highly precise measuring instrument that can distinguish between 5'11.53" and 5'11.52", and that this is the instrument that Plato and Xena's height-measurer used. Would you say, on the basis of these measurements, that Xena is taller than Plato?

The answer is no. This is because the elasticity of the human body would easily cause small differences in the measurement depending on how straight one stands, how the head is tilted, how the hair falls, and so on. It is perfectly possible that if we measured Xena and Plato a second time, the measurements would be 5'11.52" for Xena and 5'11.57" for Plato, and yet another time, it could be 5'11.61" for both Xena and Plato. Measurements of human height can vary randomly in terms of very small fractions of an inch. If we represent Xena's height as a blue dot and Plato's as a red dot, the dots would be as given in (17):

(17)



The difference of 0.01" found in the initial measurement is the result of picking one random red-blue pair from these dots. Had we picked some other pair of dots, the results might have been just the opposite. Hence, the difference of 0.01 is not significant.

When discussing Plato's claim that he is six feet tall when he is actually 5'11.9", we pointed out that for the purposes of this task, 6' and 5'11.9" count as non-distinct in terms of the permissible degree of deviation from the center, and that 0.1 inch is not "significant". What exactly do we mean by the term *significant*? Given the notion of admissible region of inexactness, we can now give the following answer:

(18) A result is significant if it is beyond the ARI.

In measurements of human height, the elasticity of the human body imposes a limit on the exactness of our knowledge, so that a difference of 0.01 cannot be significant. This shows that there are contexts in which exactness beyond a certain degree is meaningless.[5]

Like mean, dispersion, confidence level, and confidence interval, significance is a technical concept in statistics.[6] When we say that a result is not statistically *significant*, we mean that it could be due to random chance. For a result to be statistically significant, it should be beyond the range of chance or randomness fluctuations. In other words, it should be beyond ARI.

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[5] Watch the first 10 minutes of: http://ocw.mit.edu/courses/physics/8-01-physics-i-classical-mechanics-fall-1999/video-lectures/lecture-1/

[6] See http://en.wikipedia.org/wiki/Statistical_significance

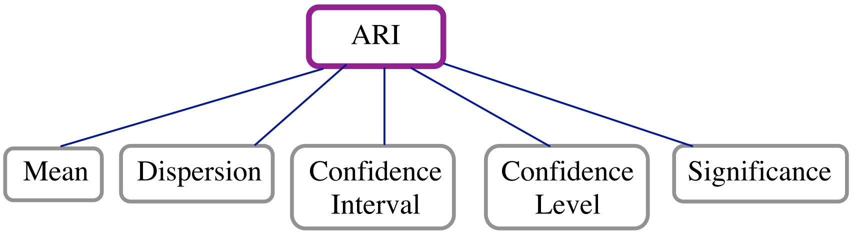
7. Summing up

At the beginning of this exploration, we introduced the concept of Admissible Region of Inexactness (ARI) in terms of the mismatch between a *conclusion* (e.g. Plato is 6' tall) and the *grounds* for the conclusion (e.g., measurement: 5'11.5"). In the formulation of the conclusion, what is the permissible degree of slack? For instance, the statement that Plato is 6' tall (the claim), when strictly interpreted (6', not a fraction of an inch less, or more) is inconsistent with the measurement 5'11.5" (the grounds). Do the grounds support or refute the claim?

For a claim to be meaningful, it should be accompanied by a statement that indicates the conditions under which it would be true, and the conditions under which it would be false ((3)). One important requirement for this criterion to be satisfied is specification of ARI.

In the course of pursuing ARI, we connected it to a cluster of related concepts:

(19)



ARI may have a center. In statistical research, this center translates as the central tendency (mean, mode, or median). The region of inexactness itself translates as dispersion .

The picture in (19) points to the presence of ARI in both grounds and conclusions. Hence, we may frame the central questions related to ARI as follows:

(20) a. What are the factors that affect the ARI of conclusions?
b. How does the ARI in grounds affect that of the conclusion and vice versa?
c. Is there an irreducible RI that functions as an unavoidable limit on human knowledge?

Our answer to (20a) recognizes *proportionality, precision, purpose*, and the *degree of confidence* expected. Of these, the last factor is of particular interest: all else being equal, the greater the degree of confidence, the greater the RI, and hence the less the degree of precision (16). In the language of statistics, this is expressed in terms of confidence level and confidence interval.

Given that two points within the same RI count as practically non-distinct for the evaluation of truth claims, it follows that a difference between two points is significant only if they are not within the same RI. In terms of statistics, RI is random variation, and hence a result is significant only if we can show that it couldn't be due to chance.