
Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
PEER REVIEWED AND REFEREED INTERNATIONAL JOURNAL
( Fulfilled Suggests Parametres of UGC by IJMER)
Volume:14, Issue:9(3), September, 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in


# A Note on Ga overpartitions of *n*


Dr Gudimella V R K Sagar

Senior Lecturer in Mathematics, Government Polytechnic for Minorities, Guntur.



**Abstract:**

The Mathematicians Corteel and lovejoy [2] derived *overpartitions* of $n$. Hanuma Reddy [3] proposed a formula for the sum of $i^{th}$ greatest parts of *overpartitions* of positive integer $n$. Sagar G.V.R.K[5] derived *Ga partitions* of positive integer $n$. In this chapter we define *Ga overpartitions* of $n$ and $r-$*Ga overpartitions* of $n$ and derive the generating functions for the number of the smallest parts and the sum of smallest parts of *Ga overpartitions* of positive integer $n$ by utilizing $r-$*Ga overpartitions* of $n$.



**Keywords:** Partition, r-partition, overpartition, *Ga* overartition, Smallest part of the *Ga* overartition.

**Subject classification:** 11P81 Elementary theory of Partitions.


*****

## 1. Introduction:

We define *Ga overpartition* of $n$. It is a non-increasing sequence of natural numbers whose sum is $n$ and smallest parts are of the form $a^{k-1}, k \in N$ in which first (equivalently, the final) occurrence of a number may be one time over lined. We denote the *Ga overpartitions* of $\lambda$, denoted by $\overline{Ga\,\lambda}$ and the number of *Ga overpartitions* of $n$ by $\overline{Ga\ p(n)}$. Since the over lined parts form a *Ga partition* into distinct parts.

For example, the number of *overpartitions* of 4 is 14 and are as follows:

$$4,\ \bar{4},\ 3+1, \bar{3}+1,\ 3+\bar{1},\ \bar{3}+\bar{1},\ 2+2,\ \bar{2}+2, 2+1+1,$$
$$\bar{2}+1+1, 2+\bar{1}+1,\ \bar{2}+\bar{1}+1,\ 1+1+1+1,\ \bar{1}+1+1+1.$$

Let $\overline{Ga\,\xi(n)}$ denote the set of all *Ga overpartitions* of $n$ and $\overline{Ga\,p(n)}$ the cardinality of $\overline{Ga\,\xi(n)}$ for $n \in N$ and $\overline{Ga\,p(0)}=1$. If $1 \le r \le n$ write $\overline{Ga\,p_r(n)}$ for the number of *Ga overpartitions* of $n$ each consisting of exactly $r$ parts, i.e $r-$*Ga overpartitions* of $n$. If $r \le 0$ or $r \ge n$ we write $\overline{Ga\,p_r(n)}=0$. Let $\overline{Ga\ p(k,n)}$ represent the number of *overpartitions* of $n$ using natural numbers at least as large as $k$ only.

Let $\overline{Ga\,spt(n)}$ denote the number of smallest parts including repetitions in all *overpartitions* of $n$.

**1.1 We derive** $\overline{p_r(n)} = \dfrac{q^r(-1,q)_r}{(q)_r}$ **as follows:**

$$\overline{p_1(n)} = (1+1)p_1(n) = \frac{(1+1)q}{(1-q)} = \frac{q(-1,q)_1}{(q)_1}$$

$$\overline{p_2(n)} = (1+1)^2 p_2(n) - (1+1)p_1\left(\left[\frac{n}{2}\right]\right)$$

$$= \frac{(1+1)^2 q^2}{(1-q)(1-q^2)} - \frac{(1+1)1q^2}{(1-q^2)}$$

$$= \frac{(1+1)q^2}{(1-q^2)}\left\{\frac{(1+1)}{(1-q)} - 1\right\} = \frac{2q^2(1+q)}{(1-q)(1-q^2)} = \frac{q^2(-1,q)_2}{(q)_2}$$

By induction, we get

$$\overline{p_r(n)} = \frac{q^r(1+1)(1+q)(1+q^2)\dots(1+q^{r-1})}{(1-q)(1-q^2)(1-q^3)\dots(1-q^r)} = \frac{q^r(-1,q)_r}{(q)_r}$$

and $$\overline{p_r(n-a)} = \frac{q^{r+a}(-1,q)_r}{(q)_r} \qquad (1.1)$$

Notations that are employed in this chapter are given here under.

**1.2 Some more notations:**

i) $\overline{Ga\,f}(a^{k-1},n)$ : number of $Ga$ *overpartitions* of $n$ with least part $a^{k-1}$.

ii) $\overline{Ga\,p_r}(a^{k-1},n)$ : number of $r-Ga$ *overpartitions* of $n$ with least part greater than or equal to $a^{k-1}$.

iii) $\overline{Ga\,f_r}(a^{k-1},n)$ : number of $r-Ga$ *overpartitions* of $n$ with least part $a^{k-1}$.

iv) $Ga\,n_s(\overline{\lambda})$ : number of the smallest parts including repetitions in $\overline{Ga\,\lambda}$.

v) $\overline{\lambda}$ : *overpartition* of $\lambda$.

vi) $\overline{\xi}(n)$ : set of all *overpartitions* of $n$.

vii) $\overline{p}(n)$ : number of *overpartitions* of $n$.

viii) $\overline{\xi_r}(n)$ : set of $r-$*overpartitions* of $n$.

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
PEER REVIEWED AND REFEREED INTERNATIONAL JOURNAL
( Fulfilled Suggests Parametres of UGC by IJMER)
Volume:14, Issue:9(3), September, 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

ix) $\overline{p_r}\ (n)$ : number of $r-overpartitions$ of $n$ .

x) $\overline{p}\ (a^{k-1},n)$ : number of $overpartitions$ of $n$ with least part greater than or equal to $a^{k-1}$.

xi) $\overline{f}\ (a^{k-1},n)$: number of $overpartitions$ of $n$ with least part $a^{k-1}$.

xii) $\overline{p_r}(a^{k-1},n)$ : number of $r-overpartitions$ of $n$ with least part greater than or equal to $a^{k-1}$.

xiii) $\overline{f_r}(a^{k-1},n)$ : number of $r-overpartitions$ of $n$ with least part $a^{k-1}$.

xiv) $\overline{spt}(n)$ : number of the smallest parts including repetitions in all $overpartitions$ of $n$.

xv) $n_s\left(\overline{\lambda}\ \right)$: number of the smallest parts including repetitions in $\overline{\lambda}$.

The Mathematicians Corteel and lovejoy [2] derived $overpartitions$ of $n$. By utilizing $r-overpartitions$ of $n$, we propose a formula for finding the number of smallest parts of $n$.

## 2. Generating function for $\overline{Ga\,spt(n)}$.

### 2.1 Theorem:

$$\overline{Ga\,spt(n)} = \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\overline{p\left(a^{k-1},n-t.a^{k-1}\right)}+\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\overline{p\left(a^{k-1}+1,n-t.a^{k-1}\right)} + 2\left\{d(n)\,|\,d(n)\text{ is divisors of form }a^{k-1}\right\}$$

**Proof:** Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2},\ldots,\mu_{l-1}^{\alpha_{l-1}},\left(a^{k-1}\right)^{\alpha_l}\right)$ be any $r-partition$ of $n$ with $l$ distinct parts. For corresponding to it there are $2^l$ times $r-Ga\,overpartitions$ of $n$. (2.1)

**Case 1:** Let $r>\alpha_l=t$ which implies $\lambda_{r-t}>a^{k-1}$

Subtract all $a^{k-1}$'s, we get $n-t.a^{k-1}=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2},\ldots,\mu_{l-1}^{\alpha_{l-1}}\right)$

Hence $n-t.a^{k-1}=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2},\ldots,\mu_{l-1}^{\alpha_{l-1}}\right)$ is a $(r-t)-Ga\,partition$ of $n-ta^{k-1}$ with $l-1$ distinct parts and each part is greater than or equal to $a^{k-1}+1$. For corresponding to it they are $2^{l-1}$ times $(r-t)-Ga\,overpartitions$ of $n-ta^{k-1}$. From (2.1), we know that the total number of $r-Ga\,overpartitions$ are $2^l$.

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
PEER REVIEWED AND REFEREED INTERNATIONAL JOURNAL
( Fulfilled Suggests Parametres of UGC by IJMER)
Volume:14, Issue:9(3), September, 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

Now we get, two times the number $\overline{Ga\, p_{r-t}\left(a^{k-1}+1, n-ta^{k-1}\right)}$ of $r-Ga\ overpartitions$ from $r-Ga\ partitions$ of $n$ with exactly $t$ smallest elements as $a^{k-1}$.

**Case 2:** Let $r>\alpha_l>t$ which implies $\lambda_{r-t}=a^{k-1}$

Omit $a^{k-1}$'s from last $t$ places, we get $n-ta^{k-1}=\left(\mu_1^{\alpha_1}, \mu_2^{\alpha_2}, \ldots, \mu_{l-1}^{\alpha_{l-1}}, \left(a^{k-1}\right)^{\alpha_l-t}\right)$

Hence $n-ta^{k-1}=\left(\mu_1^{\alpha_1}, \mu_2^{\alpha_2}, \ldots, \mu_{l-1}^{\alpha_{l-1}}, \left(a^{k-1}\right)^{\alpha_l-t}\right)$ is a $(r-t)-partition$ of $n-ta^{k-1}$ with $l$ distinct parts and the least part is $a^{k-1}$. For corresponding to it there are $2^l$ times of $r-overpartitions$ of $n-ta^{k-1}$ with least part $a^{k-1}$

Now we get the number of $r-Ga\ overpartitions$ with smallest part $a^{k-1}$ that occurs more than $t$ times among all $r-Ga\ overpartitions$ of $n$ is $\overline{f_{r-t}\left(a^{k-1}, n-ta^{k-1}\right)}$.

**Case 3:** Let $r=\alpha_l=t$ which implies all parts in the $Ga\ partition$ are equal and each part is of the form $a^{k-1}$. For each $r-Ga\ partition$ with equal parts have two times of $r-Ga\ overpartitions$ of $n$.

The number of $Ga\ partitions$ of $n$ with equal parts and each part is of the form $a^{k-1}$ is equal to the number of divisors of $n$ which are in the form $a^{k-1}$. Since the number of such divisors of $n$ is $\left\{d(n) \mid d(n) \text{is divisors of form}\, a^{k-1}\right\}$ the number of $Ga\ overpartitions$ of $n$ with all parts are equal is $2\left\{d(n) \mid d(n) \text{is divisors of form}\, a^{k-1}\right\}$.

From cases (1), (2) and (3) we get $r-Ga\ overpartitions$ of $n$ with smallest part $a^{k-1}$ that occurs $t$ times is

$$\overline{f_{r-t}\left(a^{k-1}, n-ta^{k-1}\right)}+2\overline{p_{r-t}\left(a^{k-1}+1, n-ta^{k-1}\right)}+2\beta \text{ where } \beta=\begin{cases}1 & \text{if } \frac{n}{r}=a^{k-1} \\ 0 & \text{otherwise}\end{cases}$$

$$=\overline{f_{r-t}\left(a^{k-1}, n-ta^{k-1}\right)}+\overline{p_{r-t}\left(a^{k-1}+1, n-ta^{k-1}\right)}$$
$$+\overline{p_{r-t}\left(a^{k-1}+1, n-ta^{k-1}\right)}+2\beta$$

$$=\overline{p_{r-t}\left(a^{k-1}, n-ta^{k-1}\right)}+\overline{p_{r-t}\left(a^{k-1}+1, n-ta^{k-1}\right)}+2\beta \qquad (2.2)$$

From [3], the number of smallest parts in $Ga\ overpartitions$ of $n$ is

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
PEER REVIEWED AND REFEREED INTERNATIONAL JOURNAL
( Fulfilled Suggests Parametres of UGC by IJMER)
Volume:14, Issue:9(3), September, 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

$$\overline{Ga\,spt(n)} = \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\overline{p\left(a^{k-1}, n-ta^{k-1}\right)} + \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\overline{p\left(a^{k-1}+1, n-ta^{k-1}\right)}$$
$$+2\left\{d(n)\,|\,d(n)\,\text{is divisors of form}\,a^{k-1}\right\}.$$

**2.2. Theorem:** $\overline{p_r\left(a^{k-1}+1, n\right)} = \overline{p_r\left(n-a^{k-1}r\right)}$ (2.3)

**Proof:** Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r), \lambda_i > a^{k-1}\ \forall i$ be any $r-overpartition$ of $n$.

Subtracting $a^{k-1}$ from each part, we get

$$n-a^{k-1}r = \left(\lambda_1-a^{k-1}, \lambda_2-a^{k-1}, \ldots, \lambda_r-a^{k-1}\right)$$

Hence $n-a^{k-1}r = \left(\lambda_1-a^{k-1}, \lambda_2-a^{k-1}, \ldots, \lambda_r-a^{k-1}\right)$ is a $r-overpartition$ of $n-a^{k-1}r$.

Therefore the number of $r-overpartitions$ of $n$ with parts greater than or equal to $a^{k-1}+1$ is $\overline{p_r\left(n-a^{k-1}r\right)}$.

**2.3. Theorem:** $\sum_{n=0}^{\infty}\overline{Ga\,spt(n)}q^n = \frac{(-1,q)_\infty}{(q)_\infty}\sum_{n=1}^{\infty}\frac{2q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)}\frac{(q)_{a^{n-1}-1}}{(-1,q)_{a^{n-1}+1}}$

**Proof:** From theorem (2.1) we have

$$\overline{Ga\,spt(n)} = \sum_{r=1}^{\infty}\sum_{t=1}^{\infty}\overline{p\left(a^{k-1}, n-ta^{k-1}\right)} + \sum_{r=1}^{\infty}\sum_{t=1}^{\infty}\overline{p\left(a^{k-1}+1, n-ta^{k-1}\right)}$$
$$+2\left\{d(n)\,|\,d(n)\,\text{is divisors of form}\,a^{k-1}\right\}$$

Replace $a^{k-1}+1$ by $a^{k-1}, n$ by $n-ta^{k-1}$ for first part and $n$ by $n-ta^{k-1}$ for second part in (2.3)

$$\overline{Ga\,spt(n)} = \sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\sum_{n=1}^{\infty}\overline{p_r\left(n-ta^{k-1}-r\left(a^{k-1}-1\right)\right)} + \sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\sum_{n=1}^{\infty}\overline{p_r\left(n-ta^{k-1}k-ra^{k-1}\right)}$$
$$+2\left\{d(n)\,|\,d(n)\,\text{is divisors of form}\,a^{k-1}\right\}$$

where $d(n)$ is the number of positive divisors of $n$.

From(1.1)

$$\overline{Ga\,spt(n)} = \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{q^{r+ta^{k-1}+r\left(a^{k-1}-1\right)}(-1,q)_r}{(q)_r} + \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{q^{r+ta^{k-1}+ra^{k-1}}(-1,q)_r}{(q)_r} + \sum_{k=1}^{\infty}\frac{2q^{a^{k-1}}}{1-q^{a^{k-1}}}$$
$$= \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{q^{ta^{k-1}+ra^{k-1}}(-1,q)_r}{(q)_r} + \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{q^{r+ta^{k-1}+ra^{k-1}}(-1,q)_r}{(q)_r} + \sum_{k=1}^{\infty}\frac{2q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
PEER REVIEWED AND REFEREED INTERNATIONAL JOURNAL
( Fulfilled Suggests Parametres of UGC by IJMER)
Volume:14, Issue:9(3), September, 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}q^{ta^{k-1}}\left[\sum_{r=1}^{\infty}\frac{\left(q^{a^{k-1}}\right)^{r}(-1,q)_{r}}{(q)_{r}}\right]+\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}q^{ta^{k-1}}\left[\sum_{r=1}^{\infty}\frac{q^{r}\left(q^{a^{k-1}}\right)^{r}(-1,q)_{r}}{(q)_{r}}\right]+\sum_{k=1}^{\infty}\frac{2q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$=\sum_{k=1}^{\infty}\frac{q^{k}}{\left(1-q^{k}\right)}\left[\left(1+\sum_{r=1}^{\infty}\frac{\left(q^{k}\right)^{r}(-1,q)_{r}}{(q)_{r}}\right)-1\right]$$

$$+\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\left[\left(1+\sum_{r=1}^{\infty}\frac{\left(q^{a^{k-1}+1}\right)^{r}(-1,q)_{r}}{(q)_{r}}\right)-1\right]+\sum_{r=1}^{\infty}\frac{2q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$=\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\left(1+\sum_{r=1}^{\infty}\frac{\left(q^{a^{k-1}}\right)^{r}(-1,q)_{r}}{(q)_{r}}\right)+\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\left(1+\sum_{r=1}^{\infty}\frac{\left(q^{a^{k-1}+1}\right)^{r}(-1,q)_{r}}{(q)_{r}}\right)$$

$$=\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\prod_{r=0}^{\infty}\left(\frac{1+q^{r}q^{a^{k-1}}}{1-q^{r}q^{a^{k-1}}}\right)+\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\prod_{r=0}^{\infty}\left(\frac{1+q^{r}q^{a^{k-1}+1}}{1-q^{r}q^{a^{k-1}+1}}\right)\qquad\text{from [1]}$$

$$=\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\prod_{r=0}^{\infty}\left(\frac{1+q^{r+a^{k-1}}}{1-q^{r+a^{k-1}}}\right)+\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\prod_{r=0}^{\infty}\left(\frac{1+q^{r+a^{k-1}+1}}{1-q^{r+a^{k-1}+1}}\right)$$

$$=\frac{(-1,q)_{\infty}}{(q)_{\infty}}\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\frac{(q)_{a^{k-1}-1}}{(-1,q)_{a^{k-1}}}+\frac{(-1,q)_{\infty}}{(q)_{\infty}}\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\frac{(q)_{a^{k-1}}}{(-1,q)_{a^{k-1}+1}}$$

$$=\frac{(-1,q)_{\infty}}{(q)_{\infty}}\sum_{k=1}^{\infty}\frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\frac{(q)_{a^{k-1}-1}}{(-1,q)_{a^{k-1}}}\left[1+\frac{\left(1-q^{a^{k-1}}\right)}{\left(1+q^{a^{k-1}}\right)}\right]$$

$$=\frac{(-1,q)_{\infty}}{(q)_{\infty}}\sum_{k=1}^{\infty}\frac{2q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)}\frac{(q)_{a^{k-1}-1}}{(-1,q)_{a^{k-1}+1}}$$

$$=\frac{(-1,q)_{\infty}}{(q)_{\infty}}\sum_{n=1}^{\infty}\frac{2q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)}\frac{(q)_{a^{n-1}-1}}{(-1,q)_{a^{n-1}+1}}$$

**2.4 Corollary:** The generating function for $\overline{Ga\,A_c(n)}$, the number of smallest parts of the $Ga\,overpartitions$ of $n$ which are multiples of $c$ is

$$\sum_{n=0}^{\infty}\overline{Ga\,A_c(n)q^{n}}=\frac{(-1,q)_{\infty}}{(q)_{\infty}}\sum_{n=1}^{\infty}\frac{2q^{ca^{n-1}}}{\left(1-q^{ca^{n-1}}\right)}\frac{(q)_{ca^{n-1}-1}}{(-1,q)_{ca^{n-1}+1}}$$

**2.5 Theorem:** The generating function for the sum of smallest parts of $Ga\,overpartitions$ of $n$ is

$$\sum_{n=0}^{\infty} \overline{sum\,Ga\,spt(n)q^n} = \frac{(-1,q)_\infty}{(q)_\infty} \sum_{n=1}^{\infty} \frac{2\,a^{n-1} q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)} \frac{(q)_{a^{n-1}-1}}{(-1,q)_{a^{n-1}+1}}$$

**References:**

[1] G.E.Andrews, The theory of partitions.

[2] Sylvie Coortel and J Lovejoy (2003) *overpartitions*, Transactions of the *American Mathematical Society*, 356, 4, *pp* : 1623 – 1635.

[3] Hanumareddy.K.(2010), A Study of *r - partitions* submitted to Acharya Nagarjuna University for award of Ph.D degree.

[4] Hanumareddy.K and Gudimella V R K Sagar, A Note on *M1partitions* of *n*, International Research journal of Mathematics, Engineering and IT, Vol.III, Issue XII, p.p:48-53,December 2016.

[5] Hanumareddy.K and Gudimella V R K Sagar, A Note on *Ga partitions* of *n*, International Journal of Latest Technology in Engineering,Management and applied scince (IJLTEMAS), VolumeV,Issue XII,December 2016,p.p:94-98, ISSN 2278-2540.

[6] Hanumareddy.K and Gudimella V R K Sagar, A Note on *M2 partitions* of *n*, International Journal of Latest Technology in Engineering,Management and applied scince (IJLTEMAS), VolumeVI,Issue II,p.p:63-66,ISSN 2278-2540.

[7] Hanumareddy.K and A.Manjusri, The number of smallest parts of Partition of *n*. IJITE.,Vol.03,Issue-03,(March2015),ISSN:2321 – 1776.

[8] Hanumareddy.K ,Manjusri and Gudimella V R K Sagar, The number of Smallest parts of *overpartitions* of *n*, International Research journal of Mathematics, Engineering and IT, Val.2, Issue 3, March 2015.

[9] Hanumareddy.K.(2010), A Note on *partitions*, International jational of Mathematical Scinces, 9, 3-4,pp. 313-322.

[10] G.E.Andrews, The number of smallest parts in the partitions of *n*. J.Reine Angew.Math., 624 : 133 – 142, 2008.

[11] Hanumareddy.K ,Manjusri and Gudimella V R K Sagar, The number of Smallest parts of $j^{th}$*overpartitions* of *n*, International journal of Research in Engineering and Applied Sciences, Vol-5, Issue-3, *p.p* : 29 – 37, March 2015.ISSN:2249-3905.