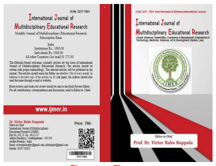
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PRE-ΔOPEN SET IN A TOPOLOGICAL SPACE

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Abstract:

In this paper I studied some topological properties of pre-Δopen sets using the concept of pre-open set and Δopen set in a topological space. The term pre-Δlimit point,pre-Δderived set,pre-Δclosure,pre-Δinterior point are discussed.

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Keywords: pre-Δlimit point, pre-Δderived set,pre-Δclosure,pre-Δinterior point.

1 Introduction

Mashhour et al. first gives an idea on pre-open sets[3]. Δopen sets are defined and studied by veera[6] and semi-Δopen set by T.M Noor and AHMAD Mustafa JABER[5].In this paper I Introduce the notion of pre-Δlimit point, pre-Δderived set,pre-Δclosure and pre-Δinterior of a set by using the concept of pre-open set and Δopen set and studied their topological properties.

2 Preliminaries

The pair(Z,τ) denote the topological space throughout this paper on which no separation axiom are assumed unless explicitly mentioned.A subset M of Z is said to be pre-open[3] if $M \subseteq int(cl(M))$.The complement of a pre-open set is a pre-closed set.The subset M is pre-open if and only if there exists an open set H in Z such that $M \subseteq H \subseteq cl(M)$[1]. A subset M of a space Z is called Δopen[6] if $M = (S - T) \cup (T - S)$ where S and T are open subsets of Z and semi-Δopen[5] if $M = (S - T) \cup (T - S)$ where S and T are semi-open[2] subsets of Z.The complement of a Δopen set is called Δclosed.The intersection of all Δclosed sets containing the set M is called the Δclosure of M.In this paper I take the symbols intΔ, clΔ, $\tau\Delta$ to denote the Δinterior, Δclosure and the family of all Δopen sets respectively w.r.to the topology τ.The set of all Δlimit points of M will be denoted by DΔ(M).

3 Main results

Definition 3.1 *A subset M of a space (Z,τ) will be called pre-Δopen if $M = (S - T) \cup (T - S)$, where S and T are pre-open sets in Z.*

The family of all pre-Δopen sets in Z will be denoted by $\tau\Delta$p.The complement of a pre-Δopen set will be called pre-Δclosed set and pre-Δclosure of M will be denoted by clΔp(M) which is the intersection of all pre-Δclosed sets containing M.

From the following example it is clear that every Δopen set and also every pre-open set is pre-Δopen but in general its converse applications are not true.

Example 3.2 *Let $Z = \{r,s,t\}$, then $\tau = \{Z,\phi,\{r\}\}$ be a topology on Z.*

Closed subsets of Z are $Z,\phi,\{s,t\}$.Then $cl\{r\} = Z$, $cl\{s\} = \{s,t\}$, $cl\{t\} = \{s,t\}$, $cl\{r,s\} = Z$, $cl\{r,t\} = Z$, $cl\{s,t\} = \{s,t\}$.

Therefore, $int(cl\{r\}) = int(cl\{r,s\}) = int(cl\{r,t\}) = Z$ and $int(cl\{s\}) = int(cl\{t\}) = int(cl\{s,t\}) = \phi$.

Hence the family of pre-open sets $\tau p = \{X,\phi,\{r\},\{r,s\},\{r,t\}\}$.Let $S = \{r,t\}$ and $T = \{r\}$ then $(S - T) \cup (T - S) = \{t\} \cap \phi = \{t\}$.

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Thus the set $\{t\}$ is pre-Δopen but it is neither Δopen nor pre-open.

Definition 3.3 *Let M be a subset of a topological space Z. A point $m \in Z$ will be called pre-Δlimit point of M if for all $R \in \tau\Delta p$, $m \in R$ implies that $R \cap (M\backslash\{m\}) \neq \phi$.*

The set of all pre-Δlimit points of M will be called pre-Δderived set of M and it is to be denoted by DΔp(M).

Example 3.4 *Let $Z = \{r, s, t\}$ with the topology $\tau = \{Z, \phi, \{s\}\}$, then $\tau p = \{Z, \phi, \{s\}, \{r, s\}, \{s, t\}\}$ and $\tau\Delta p = \{X, \phi, \{r\}, \{s\}, \{t\}, \{r, s\}, \{r, t\}, \{s, t\}\}$.Here $\tau\Delta = \{Z, \phi, \{s\}, \{r, t\}\}$.Take $M = \{s, t\}$.For $r \in Z$, Δopen sets containing the element r are Z and $\{r, t\}$ .Now $M\backslash\{r\} = \{s, t\}$ .Since $Z \cap \{s, t\} = \{s, t\} \neq \phi$ and $\{r, t\} \cap \{s, t\} = \{t\} \neq \phi$, $r \in D\Delta(M)$.Similarly we can easily check that $s, t \notin D\Delta(M)$ and hence $D\Delta(M) = \{r\}$.Considering $pre-\Delta open$ sets we get $D\Delta p(M) = \phi$.*

Definition 3.5 *Let M be a subset of a topological space Z. A point $m \in Z$ will be called pre-Δinterior point of M if there exists a pre-Δopen set R such that $m \in R \subseteq M$.*

The set of all pre-Δinterior points of M will be called the pre-Δinterior of M and it is to be denoted by intΔp(M).

Example 3.6 *Let $Z = \{r, s, t\}$ with the topology $\tau = \{Z, \phi, \{r, s\}\}$.If $M = \{s, t\}$ then $int\Delta(M) = \phi$, $int\Delta p(M) = \{s, t\}$.*

Proposition 3.7 *Let M and N be two arbitrary subsets of Z.Then the following statements are true:*

1. intΔp(M) is the union of all pre-Δopen subsets of M.

2. M is pre-Δopen if and only if $M = int\Delta p(M)$.

3. $M \subseteq N \Rightarrow int\Delta p(M) \subseteq int\Delta p(N)$.

4. $int\Delta p(M) \cup int\Delta p(N) \subseteq int\Delta p(M \cup N)$.

5. $int\Delta p(M \cap N) \subseteq int\Delta p(M) \cap int\Delta p(N)$.

Proof. 1. Suppose that the collection of all pre-Δopen subsets of M is $\{R_k | k \in \Lambda\}$.If $m \in int\Delta$p(M),then there exists $i \in \Lambda$ such that $m \in R_i \subseteq M$.Thus $m \in \bigcup_{k\in\Lambda} R_k$ and so $int\Delta p(M) \subseteq \bigcup_{k\in\Lambda} R_k$.Now, if $n \in \bigcup_{k\in\Lambda} R_k$, then $n \in R_i \subseteq M$ for some $i \in \Lambda$.Hence $n \in int\Delta p(M)$ and so $\bigcup_{k\in\Lambda} R_k \subseteq int\Delta p(M)$.Hence $int\Delta p(M) = \bigcup_{k\in\Lambda} R_k$

2. Straightforward.

3. Suppose that $m \in int\Delta p(M)$,then there exists a pre-Δopen set R such that $m \in R \subseteq M$.Now as $M \subseteq N, m \in R \subseteq M \subseteq N$ so that $m \in int\Delta p(N)$.

4. Since $M \subseteq M \cup N$ and $N \subseteq M \cup N$, $int\Delta p(M) \subseteq int\Delta p(M \cup N)$ and $int\Delta p(N) \subseteq int\Delta p(M \cup N)$ .Hence $int\Delta p(M) \cup int\Delta p(N) \subseteq int\Delta p(M \cup N)$.

5. Since $M \cap N \subseteq M$ and $M \cap N \subseteq N$, $int\Delta p(M \cap N) \subseteq int\Delta p(M)$ and $int\Delta p(M \cap N) \subseteq int\Delta p(N)$. Hence $int\Delta p(M \cap N) \subseteq int\Delta p(M) \cap int\Delta p(N)$.

Proposition 3.8 *Let M and N be two arbitrary subsets of Z. Then the following statements are true:*

1. $D\Delta p(M) \subseteq D\Delta(M)$.

2. If $M \subseteq N$ then $D\Delta p(M) \subseteq D\Delta p(N)$.

3. $D\Delta p(M) \cup D\Delta p(N) \subseteq D\Delta p(M \cup N)$.

4. $D\Delta p(M \cap N) \subseteq D\Delta p(M) \cap D\Delta p(N)$.

Proof. 1. Let $m \in D\Delta p(M) \Rightarrow \forall R \in \tau\Delta p, m \in R$ we have $\{R \cap M\}\backslash\{m\} \neq \phi \Rightarrow m \in D\Delta(M)$ since every Δopen set is pre-Δopen.

2. Suppose that $m \in D\Delta p(M)$ and let $R \in \tau\Delta p$ with $m \in R$.Then $R \cap (M)\backslash\{m\}) \neq \phi$.Now as $M \subseteq N, R \cap (N\backslash \{m\}) \neq \phi$ so that $m \in D\Delta p(N)$.

3. Since $M \subseteq M \cup N$ and $N \subseteq M \cup N, D\Delta p(M) \subseteq D\Delta p(M \cup N)$ and $D\Delta p(N) \subseteq D\Delta p(M \cup N)$ and hence $D\Delta p(M) \cup D\Delta p(N) \subseteq D\Delta p(M \cup N)$.

4. Since $M \cap N \subseteq M$ and $M \cap N \subseteq N, D\Delta p(M \cap N) \subseteq D\Delta p(M)$ and $D\Delta p(M \cap N) \subseteq D\Delta p(N)$ and hence $D\Delta p(M \cap N) \subseteq D\Delta p(M) \cap D\Delta p(N)$.

Theorem 3.9 *Let M be a subset of Z and* $m \in Z$*.Then the following two statements are equivalent:*

(i) $m \in R \Rightarrow M \cap R \neq \phi \ \forall R \in \tau\Delta p$.

(ii) $m \in cl\Delta p(M)$.

Proof. $(i) \Rightarrow (ii)$

If $m \notin cl\Delta p(M)$, then there exists a pre-Δclosed set F such that $M \subseteq F$ and $m \notin F$.Hence $Z\backslash F$ is pre-Δopen set containing m and $M \cap (Z\backslash F) \subseteq M \cap (Z\backslash M) = \phi$.This is a contradiction and hence (ii) is valid.

$(ii) \Rightarrow (i)$.

Let $m \in cl\Delta p(M)$.Then $m \in M \cup D\Delta p(M) \Rightarrow$ either $m \in M$ or $m \in D\Delta p(M)$.If $m \in M \Rightarrow M \cap R \neq \phi$ since $m \in R$.

If $m \in D\Delta p(M)$, then by definition $R \cap (M\backslash\{m\}) \neq \phi$ and hence $R \cap M \neq \phi$

Corollary 3.10 $D\Delta p(M) \subseteq cl\Delta p(M)$*, M is any subset of Z.*

Proof. Straightforward.

Theorem 3.11 *If M be an arbitrary subset of Z then* $cl\Delta p(M) = M \cup D\Delta p(M)$.

Proof. Let $m \in cl\Delta p(M)$.Let $m \notin M$ and $R \in \tau\Delta p$ with $m \in R$.Then $R \cap (M\backslash\{m\}) \neq \phi$.Hence $m \in D\Delta p(M)$, and therefore $cl\Delta p(M) \subseteq M \cup D\Delta p(M)$.

Now since $M \subseteq cl\Delta p(M)$ and by the corollary 3.10 we have $M \cup D\Delta p(M) \subseteq cl\Delta p(M)$.

Lemma 3.12 *A subset M of Z is a pre-Δopen if and only if there exists a Δopen set I in Z such that* $M \subseteq I \subseteq cl(M)$.

Proof. Since M is a pre-Δopen, there exits pre-open sets S and T in Z such that $M = (S - T) \cup (T - S)$.Also since S and T are pre-open, there exists open sets J and K in Z such that $S \subseteq J \subseteq cl(S)$ and $T \subseteq K \subseteq cl(T)$ and conversely.

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Therefore, $(S-T)\cup(T-S)\subseteq(J-K)\cup(K-J)\subseteq[cl(S)-cl(T)]\cup[cl(T)-cl(S)]\subseteq cl(S-T)\cup cl(T-S)\subseteq cl[(S-T)\cup(T-S)]$.

Hence the result.

Corollary 3.13 *The intersection of an open set and a Δopen set is a Δopen.*

Proof. Let K be an open set and *I* be a Δopen set in Z. Also let $I=(P-Q)\cup(Q-P)$ where P and Q are open sets in Z.

Then, $K\cap I=K\cap[(P-Q)\cup(Q-P)]=[K\cap(P-Q)]\cup[K\cap(Q-P)]=[(K\cap P)-(K\cap Q]\cup[(K\cap Q)-(K\cap P)]$.

Since $K\cap P$ and $K\cap Q$ are open sets,the result follows.

Theorem 3.14 *The intersection of an open set and a pre-Δopen set is a pre-Δopen.*

Proof. Let K be an open set and M be a pre-Δopen set in Z. Then there exists a Δopen set I in Z such that $M\subseteq I\subseteq cl(M)$. Thus we can write $K\cap M\subseteq K\cap I\subseteq K\cap cl(M)\subseteq cl(K\cap M)$.

Since $K\cap I$ is Δopen then by the lemma 3.12 $K\cap M$ is pre-Δopen.

Theorem 3.15 *If M be a subset of Z, a discrete topological space where every open set is a pre-Δopen set.Then* $D\Delta p(M)=\phi$

Proof. Let m be an element of M.By the the statement, since every subset of Z is Δopen and so pre-Δopen.The singleton set $S=\{m\}$ in particular, is a pre-Δopen.But $m\in S$ and $S\cap M=\{m\}\cap M\subseteq\{m\}$.Hence m is not a pre-Δlimit point of M and hence $D\Delta p(M)=\phi$.

Theorem 3.16 *For every subset M of Z,we have*

M is pre-Δopen if and only if $D\Delta p(M)\subseteq M$.

Proof. Assume that M is pre-Δclosed. let $m\notin M$ that is $m\in Z\backslash M$.Since $Z\backslash M$ is pre-Δopen,m is not a pre-Δlimit point of M,that is $m\notin D\Delta p(M)$, because $(Z\backslash M)\cap(M\backslash\{m\})=\phi$.Hence $D\Delta P(M)\subseteq M$.

Theorem 3.17 *Let M be a subset of Z.If W be a pre-Δclosed super set of M, then* $D\Delta p(M)\subseteq W$.

Proof. From the proposition 3.8 we have the result that if $M\subseteq N$, then $D\Delta p(M)\subseteq D\Delta p(N)$ and from the theorem 3.16 we have the result $D\Delta p(M)\subseteq M$.This two results together implies that $D\Delta p(M)\subseteq D\Delta p(W)\subseteq W$.

Theorem 3.18 *Let M be a subset of Z.If a point* $m\in Z$ *is a pre-Δlimit point of M,then m is also a pre-Δlimit of* $M\backslash\{m\}$.

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Proof. Since $m \in Z$ is a pre-Δlimit point M, then for all $R \in \tau\Delta p, m \in R$ implies that $R \cap (M \setminus \{m\}) \neq \phi$ and hence it is straightforward that m is also a pre-Δlimit point of $M \setminus \{m\}$.

Theorem 3.19 *Every open set is always a pre-Δopen set.*

Proof. Since every open set is a pre-open set,it is straightforward that every open set is always a pre-Δopen.

Theorem 3.20 *Every Δopen set is always a pre-Δopen set.*

Proof. Since every Δopen set is a pre-open set,it is straightforward that every Δopen set is always a pre-Δopen.

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