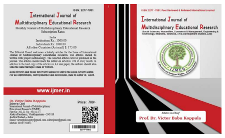

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

ANCIENT INDIA'S CONTRIBUTIONS TO THE ORIGIN OF MATHEMATICS : RELATED TO THE GEOMETRICAL WORK OF BAUDHAYANA AND DIHEDRAL SYMMETRY GROUP

Dr. Hiteshwar Singh
[Dept. M.L.S.M. College, Darbhanga]
Dhananjay Kumar Mishra [L.N.M.U. Darbhanga, Math Department]

Abstract

In this research paper we have tried to study the geometrical work of Baudhayana and correlate it to algebraic structure group theory. Many different applied geometrical figures have been studied by Baudhayana. We have presented these geometrical figures in the forms of group theory. As we know that many symmetrical geometrical figures can be shown as designs in group theory. In this research paper we have shown these Baudhayana geometrical figures into dihedral groups. This paper is mainly based on the Dihedral groups and Baudhayana geometrical figures. As we know that these groups show the symmetries of a regular polygon, which includes rotations and reflections. Dihedral groups are the simplest examples of finite groups. These dihedral groups play very important role in group theory, geometry and chemistry. These groups provide an idea to apply the group theory in geometry and in other branches of science. Dihedral groups bring the abstract algebra and geometry at one platform. There becomes 2n elements in dihedral group D_n. If r will be the rotation and s will be the reflection of a polygon then the number of elements in that dihedral group will be $\{e, r, r^2, r, r^{n-1}, s, rs, r^2s \ldots, r^{n-1}s\}$.

Keywords: Shulba Sutra, Shrauta Sutra, Rhombus, Sage Baudhayan, Yagya, pi, Hypotenuse, Dihedral group, symmetry group, finite group, reflection and rotation of regular polygon, geometry.

Introduction:

Geometry of Euclid is taught in secondary level all over the world, considering it to be authentic and genuine in the subject of geometry. But it should be remembered that even before the great Greek geologist Euclid, many geometry scientists and philosophers in India had discovered important laws of geometry, among those geometry philosopher, the name of Baudhayana is prominant. Geometry or Geometry in India at that time was written in a book name Shulva Shastra. The sutras of Baudhayana have been written in Vedic Sanskrit and are related to religion, daily rituals, mathematics as well daily life style etc. They related to the Taittiriya branch of the Krishna Yajurveda. These are probably the oldest texts in the Sutra texts. They were probably composed in the 8th-7th century BC. The most notably, Baudhayana's Shulbasutras contain many results and theorems of early mathematics and geometry, including an approximate value of the square root of 2, and a statement of the Pythagorean Theorem.

LIFE STORY OF BAUDHAYANA:

It is not possible to write a biography of Baudhayana as nothing is known about him except that he was the author of one of the earliest Sulbasutras. We don't know his dates exactly, even can estimate his life span, which is why we have given the same estimated birth year as death year. Born in 800 BC, Baudhayana composed more than two hundred religious texts. He gave many important principles of mathematics. It is told by a verse that in a rectangle the square of the hypotenuse is equal to the sum of the squares of the base and the perpendicular. This verse, described in his work Vriti Granth, is known as Baudhayana Theorem. The theorem was elaborated by the Greek scholar Pythagoras. Later on its basis Aryabhatta made discoveries in the field of space science.

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

The famous grammarian of Sanskrit, Panini, in his treatise Ashtadhyayi, has described Baudhayana as a guru. Baudhayana was a great scholar of philosophy, religion, mathematics and language. He wrote more than two hundred treatises. Among them, Vedavruti, Vedanta, Ratna Manjush, Dharmasutra and Grihasutra are important. He certainly must have been a man of much learning, but was probably not interested in mathematics for himself, only interested in using it for religious purposes. There is no doubt he wrote the Sulbasutra to provide rules for religious rites and it would seem almost certain that Baudhayana himself would have been a Vedic priest. The mathematics given in the Sulbasutras is meant to enable the precise construction of the altars required for the sacrifice. It is clear from the writings that Baudhayana must have been a priest as well as a skilled craftsman. He must have been proficient with life-useful use of mathematics, which he described as a craftsman who himself constructed sacrificial altars of the highest quality.

Contributions of Baudhayana:

1. **To find the square root of 2:**

Baudhayana (verse number i.61-2, explained in Apastamba i.6) describes the method of finding the length of a diagonal given the lengths of the sides of a square. In other words, it describes the method of finding the square root of 2. To get the value of the diagonal of a square, by adding one-third to the side, then adding one-fourth of it, then subtracting thirty-fourth of it, what is obtained is approximately the value of the diagonal.

2. **Constructing a circle of area equal to the area of the square:**

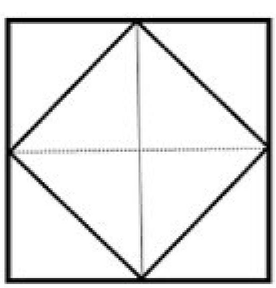
He said to draw half its diagonal about the centre towards the East-West line; then describe a circle together with a third part of that which lies outside the square. That is, if the side of the square is 2a, then the radius of the circle $r = [a+1/3(\sqrt{2}a - a)] = [1+1/3(\sqrt{2} - 1)]\ a$.

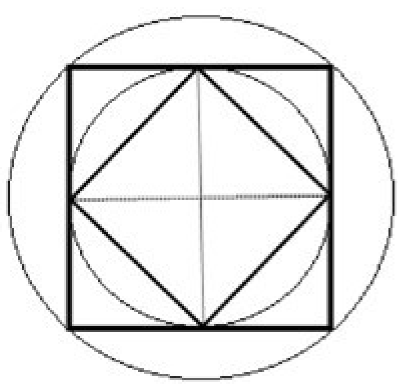
3. Constructing a Square of Area Equal to the Area of the Circle:

If you wish to turn a circle into a square, divide the diameter into eight parts and one of these parts into twenty-nine parts: of these twenty-nine parts remove twenty-eight and moreover the sixth part
(of the one part left) less the eighth part (of the sixth part).

4. How to circle a square?

Baudhayana was able to draw a circle roughly equal to the area of a square and vice versa. These processes are described in their sources (I-58 and I-59). Possibly for this purpose, in his quest to build circular altars, he constructed two circles enclosing the two squares shown below.





Cover Page
INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

Now, as in the area of squares, he realized that the inner circle should be exactly half of the larger circle in area. He knew that the area of a circle is proportional to the square of its radius and the above construction proves to be the same. By the same logic, as the circumference of two squares, the circumference of the outer circle must also be 22 times the circumference of the inner circle. This proves the known fact that the perimeter of a circle is proportional to its radius. This led to an important perusal made by Baudhayana, that the areas and perimeters of many regular polygons, including the squares above, can be related to each other as in the case of circles.

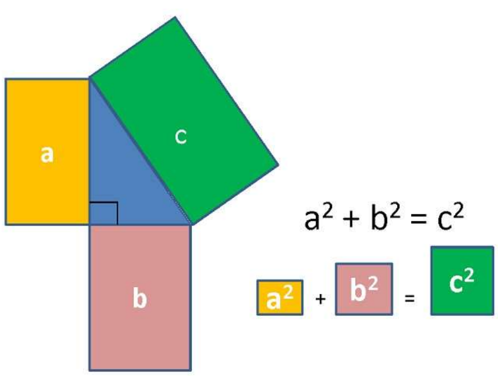
5. Baudhayana Right Angled Triangle Theorem:

Baudhayana listed the Pythagorean Theorem in his book Baudhayana Sulbasutra already much earlier than Greek philosopher Pythagoras.

For example Baudhayana used a rope in the above verse/verse, which can be translated as: The areas produced by the length and breadth of a rectangle separately are equal to the areas produced by the diagonal together. The described diagonals and sides are those of a rectangle, and the areas are those of the squares whose sides are those of these line segments. Since the diagonal of a rectangle is the hypotenuse of a right triangle formed by two adjacent sides, this statement appears to be equivalent to the Pythagorean Theorem. Various arguments and explanations have been given for this. While some have assumed that the sides relate to the sides of a rectangle, others say that the reference may be to a square. There is no proof to suggest that Baudhayana's formula is limited to right-angled isosceles triangles so that it can be linked to other geometric figures as well. So it is logical to assume that the sides he mentioned can be sides of a rectangle. Baudhayana seems to have simplified the learning process by encapsulating the mathematical result in a simple verse in a layman's language. As we see, it becomes clear that this is probably the most innovative way to understand and visualize the Pythagorean Theorem (and geometry in general).

6. Comparing his findings with Pythagoras' theorem:

In mathematics, the Pythagorean Theorem is a relation between the three sides of a right-angled triangle. According to this theorem area of square at hypotenuse is equal to sum of areas of square at other two sides. If c is the length hypotenuse of the right angled triangle with a and b being the other two then



The question may well be asked why the theorem is attributed to Pythagoras and not Baudhayana. Baudhayana used area calculations and not geometry to prove his calculations. He came up with geometric proof using isosceles triangles.

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

7. The value of π calculation as suggested by Baudhayana:

Baudhayana is considered among one of the first to discover the value of ‘pi’. There is a mention of this in his Sulbha sutras. According to his premise, the approximate value of pi is 3.3.Several values of π occur in Baudhayana's Sulbasutra, since, when giving different constructions, Baudhayana used different approximations for constructing circular shapes.

Some of the major theorems propounded by Baudhayan are as follow,

(*i*). Diagonals of a rectangle cut each other at their middle point.

(*ii*). Diagonals of a rhombus cut each other at ninety degree

(*iii*). The area of a square formed by joining the mid-points of the sides of a square is half the area of the original square.

(*iv*). A rhombus is formed by joining the mid-points of the sides of a rectangle whose area is half the area of the original rectangle. It is clear from the above description that Baudhayana had studied the properties and areas of rectangle, square, right angled triangle, rhombus. The Yaj was probably due to the importance of the 'Yagya Bhumika' being made for the Yagya at that time.

8. Groups of order 8, and Symmetrical Figures of square:

Let there be a group G, and G = D_4. Here D_4 is a dihedral group. It is also known as the group of symmetry of a square. Let us suppose that ABCD be a square with centre O. We present it in figure 4.

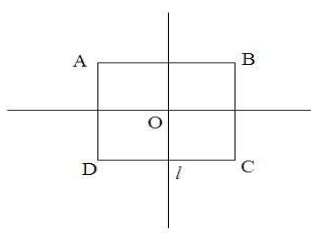


Fig. 1

Let l be a line which passes through the centre O and parallel to a side of the square. Now we suppose that a be a rotation through 90^0 in an anti-clockwise direction about an axes through O and perpendicular to the plane of the given square. Let b be a rotation about the line l through 180^0. We get all possible motion of the square in terms of a and b. If e be identity motion (element) then we have $a^4 = e$, $b^2 = e$ and $ba = a^3b$. Here ab means first carried out motion a then the motion b. Thus we find 8 distinct symmetries of the square as $e, a, a^2, a^3, b, ab, a^2b, a^3b$. These squares will be in figure 5 as follow,

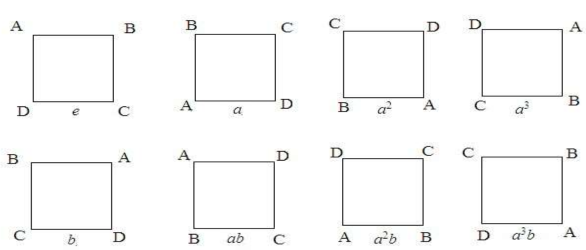
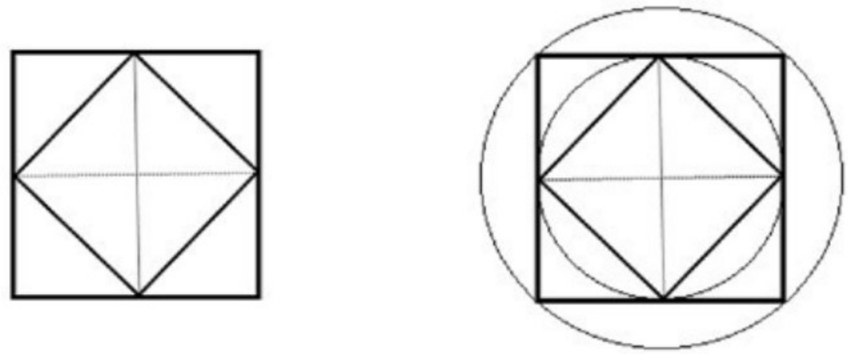


Fig.2 Therefore, $D_4 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$

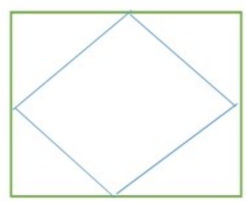
Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

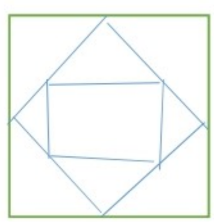
(*i*).



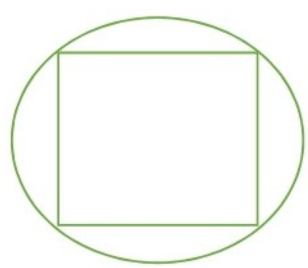
(*ii*).**This figure will be presented as 2D. D_4 symmetry group with two squares:**



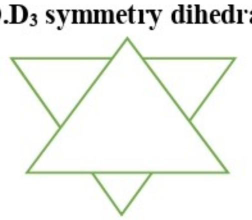
(*iii*). **This will be 2D .D_4 with three squares:**



(i*v*). **It is also a 2D. D_4 symmetry dihedral group with circle circumscribed a square:**



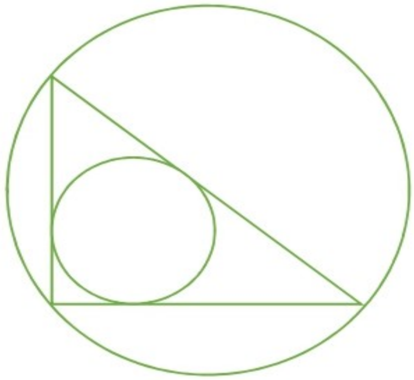
(*v*). **It is a 2D.D_3 symmetry dihedral group with two equilateral triangles:**



Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

(*vi*). **It is 2D.D_1 symmetry with two circles circumscribed and inscribed a right angled isosceles triangle as well as hypotenuse as diameter of outer circle while inner circle touching the hypotenuse at the centre of the outer circle:**



CONCLUSIONS:

Baudhayana's place in the field of mathematics will remain immortal for ages. Baudhayana's important contribution was to take science out of superstitions and give new thinking, new vision. He is the first mathematician in the world to discover famous theorem which gives the relation between sides of right angle triangle, but the credit is given to Pythagoras, which is wrong. With time people's thinking has changed and here along with the interest of the people of the world has also started to know and understand Indian culture. Perhaps it is the result of this that the attention of the people towards the great works of ancient sages and mathematicians. The biggest example of this is the theorem of Pythagoras, which is now known as the Baudhayana Pythagoras theorem. The aim of the Indians from the very beginning has been "Vasudhaiva-Kutumbakam". Everything discovered by them was dedicated to the welfare of the people. We forgot our great tradition and got lost in the depths of ignorance. Today there is a need to re-research on the beliefs established by our sages so that their discovery can be planned in the direction of public welfare. We have all heard our parents and grandparents talk of the Vedas. Still, there is no denying that modern science and technology owes its origins to our ancient Indian mathematicians, scholars etc. Many modern discoveries would not have been possible but for the legacy of our forefathers who made major contributions to the fields of science and technology. Be it fields of medicine, astronomy, engineering, mathematics, the list of Indian geniuses who laid the foundations of many an invention is endless. This research paper will enhance the connectivity of abstract algebra to geometry as well as to the symmetrical figures of Chemistry. A number of beautiful figures can be drawn with the help of symmetry of the polygon. Thus this paper will encourage the new researcher to apply the group theory in the world of symmetrical figures of the polygon by relating the different positions of polygon after rotation and reflection to the elements of dihedral group.

References:

1. Weisstein, Eric W. "Dihedral group ↗, MathWorld.
2. Dummit, David S.; Foote, Richard M. (2004). Abstract Algebra (3rd ed.) John Wiley & Sons. ISBN 0-471-43334-9.

3. "Dihedral groups: Notation" ↗, Math images Project. Archived from the original ↗ on 2016 – 03 – 20. Retrieved 2016 – 06 – 11.

Cover Page

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH
ISSN:2277-7881(Print); IMPACT FACTOR :9.014(2025); IC VALUE:5.16; ISI VALUE:2.286
Peer Reviewed and Refereed International Journal(Fulfilled Suggests Parametres by UGC)
Volume:14, Issue:3(1), March: 2025
Scopus Review ID: A2B96D3ACF3FEA2A
Article Received: Reviewed : Accepted
Publisher: Sucharitha Publication, India
Online Copy of Article Publication Available : www.ijmer.in

4. Cameron, Peter Jephson (1998). Introduction to Algebra, Oxford University Press, p. 95, ISBN 9780198501954.

5. Lovett, Stephen (2015), Abstract Algebra: Structures and Applications, CRC Press, p.71, ISBN 9781482248913.

6. Humphreys, John F. (1996). A Course in Group Theory. Oxford University Press. p. 195. ISBN 9780198534594.

7. Cayley, Arthur. "On the Theory of Groups', American Journal of Mathematics, Vol. 11, No. 2 (Jan1889), pp. 139-157. Available at JSTOR.

8. Weyl, Hermann (1982) [1952]. Symmetry, Princeton: Princeton University Press. ISBN 0-691-02374-3.

9. Singer, David A. (1998). Geometry: Plane and Fancy. Springer Science & Business Media.

10. David A. (1998). Geometry: Plane and Fancy. Springer Science & Business Media.

11. G G Joseph, The crest of the peacock (London, 1991).

12. R C Gupta, Baudhayana's value of $\sqrt{2}$, Math. Education 6 (1972), B77-B79.

13. S C Kak, Three old Indian values of π, Indian J. Hist. Sci. 32 (4) (1997), 307-314.

14. G Kumari, Some significant results of algebra of pre-Aryabhata era, Math. Ed. (Siwan) 14 (1) (1980), B5-B13.

15. S. Balchandra Rao Indian Mathematics and Astronomy: Some Landmarks. Jnana Deep Publications, Bangalore, 1998. ISBN 81-900962-0-6.

16. Vincent J. Katz. A History of Mathematics: An Introduction, 2nd Edition Addison-Wesley, 1998.ISBN 0-321-01618-1.

17.https://www.jagran.com/bihar/sitamarhi-today-the-birthplace-of-mathematician-bodhayan-will-be-developed-as-atourist-center-18810738.html

18. https://mathshistory.st-andrews.ac.uk/Biographies/Baudhayana.