



Cover Page



A CRITICAL STUDY ON IMPACT OF ELASTICITY IN THE FIRM'S REVENUE

U. Naga Rekha Rani

Research Scholar, Department of Mathematics, Ekalavya University
Damoh, Madhya Pradesh

Faculty, Department of Mathematics, SR& BJNR Government Degree College
Khammam, Telangana State, India

Abstract

The elasticity is an important measure impacting on a firm's revenue. Hence, it is important for a firm to know how the proposed change in price of its product can affect its total revenue, when the product is to be sold in the new market condition at the new price. In this context, the measure of elasticity indirectly reflects how the buyers will react to the change in price and the new price to come. This implies that the elasticity of the product becomes a crucial measure to reflect what the percentage of income the firm can gain or lose, when the price change takes place for its respective product. This paper demonstrates in a new mathematically constructive approach as consistent with the existing accepted phenomena of elasticity that elastic product shows negative relationship between price change and change in total revenue; inelastic product can result in positive relationship between price change and change in total revenue; and unit elasticity product has no impact on change in total revenue as the response to a price change. Indicatively, this research paper explores three constructive, but similar and alternative, mathematical methods for the existing phenomena how the percentage of change in total revenue can be determined with respect to elasticity, and current and new prices and their respective quantities.

Keywords: Elasticity, Demand, Supply, Quantity, Market, Revenue.

INTRODUCTION

STATEMENT OF THE PROBLEM

Generally, the price, demand and supply analyses determine the equilibrium of a product in a market. The price as an independent variable determines both the demand and supply of a product and leads to market equilibrium. The elasticity is a measure of reflecting the changing rate of a quantity to a changing rate of price. Therefore, from a firm's perspective, the elasticity becomes a crucial measure in determining the price of the product, and any change in price affect the market and revenue of the firm. This implies that elasticity of the product has impacts on revenue of the firm, when there is a change in price of its product in the market. In this context this paper, provides the theoretical explanation and modelling how the elasticity of a product has impacts on the firm's revenue.

ELASTICITY AS THE DETERMINANT OF TOTAL REVENUE OF A PRODUCT

As there are different markets and firms in operations, it is important to know about how the elasticity becomes a crucial component in determining the revenue of the firm/market. To explore the relationship of elasticity to firm's revenue, consider the following revenue (TR) function of a firm/market.

$$TR = P \cdot Q \quad (1)$$

where TR = Total Revenue

P = Price of the product

Q = Quantity demanded for the product

Indicatively, variables P and Q are dependent on each other, since quantity becomes the function of price and price becomes the function of price as demand and supply become the two forces in determining the market equilibrium.

Therefore, differentiating TR for Marginal Revenue (MR) with respect to Q can result in

$$\frac{d(TR)}{dQ} = MR \cdot P + Q \left(\frac{dP}{dQ} \right) \quad (2)$$

$$\text{So that } MR \cdot P + Q \left(\frac{dP}{dQ} \right) \quad (3)$$



Cover Page



$$\text{But } E_d = \left(\frac{dQ}{dP} \right) \left(\frac{P}{Q} \right) \quad (4)$$

$$\rightarrow E_d = \left(\frac{Q}{P} \right) = \left(\frac{dQ}{dp} \right)$$

$$\rightarrow \left(\frac{E_d Q}{P} \right) = \left(\frac{dQ}{dp} \right)$$

$$\rightarrow \left(\frac{dP}{dQ} \right) = \left(\frac{P}{E_d \cdot Q} \right) \quad (5)$$

Substituting equation (5) in equation (3) can result in

$$MR = P + q \left(\frac{P}{E_d Q} \right) \rightarrow MR = P + \left(\frac{P}{E_d} \right)$$

$$MR = P \left(1 - \frac{1}{E_d} \right) \quad (6)$$

Notably, E_d is always negative in mathematical calculation, while interpreting it in positive term.

$$\text{Therefore, } MR = P \left(1 - \frac{1}{E_d} \right) \quad (7)$$

In equation 7, we can term that the MR is the result of multiplication of two components, namely: price (P) and elasticity

$$E_c = \left(1 - \frac{1}{E_d} \right).$$

$$\text{Since } E_c = \left(1 - \frac{1}{E_d} \right). \quad (8)$$

$$\text{Then } MR = P \cdot E_c \quad (9)$$

Now, it is possible to explore the relationship between the revenue and elasticity in the context of perfect (pure) competition market. In a perfect completion market, there might be various products, which can be primarily categorised into three types, based on the elasticity: (a) Inelastic, (b) Elastic, and (c) Unit-elastic products. As the MR in equation 7 consists of multiplying components of price and elasticity, the elasticity component $\left(1 - \frac{1}{E_d} \right)$ of equation 7 can be used to explore the impacting nature of elasticity on revenue.

Suppose consider an exhibit that the demand function of a product is given by

$$Q_d = -0.6P + 90 \text{ (in '000) and the initial quantity is 30 (000).}$$

Accordingly, it is known that the initial price of the product is:

$$30 = -0.6P + 90 \rightarrow P = (90 - 30)/0.6 = 100 \text{ and}$$

The respective elasticity is:

$$E_d = \left(\frac{dQ}{dP} \right) + \left(\frac{P}{Q} \right) = (0.6) \cdot \left(\frac{100}{30} \right) \rightarrow E_d = 2$$



Cover Page



Substituting $E_d = 2$ in equation (8) can result E_c in a positive term, i.e.,
 $E_c = 1 - (1/2) = (1/2)$.

As the elasticity of the product is $E_d = 2$ ($E_d > 1$ and elastic in the market), the change in price for an elastic product implies that its elastic nature will definitely have the negative impacts on the firm's revenue.

Suppose, the initial market price increases by 5% from $P = 100$ to 105, the quantity can fall to 27 ($= -0.6 \times 105 + 90$) from 30 as 10% decrease. This implies that

The revenue of the initial price ($P = 100$) and quantity ($Q = 30$) is $TR_0 = 3000$ and the revenue for the price changed ($P = 105$) and respective quantity changed ($Q = 27$) is $TR_1 = 2835$. Hence, the change in revenue as a percentage is:

$$\% \Delta TR = \left(\frac{TR_1 - TR_0}{TR_0} \right) = \frac{2835 - 3000}{3000} = \frac{-165}{3000} = 0.055$$

However, the reaction of suppliers to the price increase in the market has not been considered. It is obvious that the market equilibrium for a price increase reaches with the reaction of buyers and suppliers of the product to the price change. In this context, the approximate, but theoretically constructive, change in revenue of a firm can be determined with equation (9) above, i.e.

$$MR = P.E_c \quad (9)$$

$$\text{As } MR = \frac{\Delta TR}{\Delta Q} = P.E_c \rightarrow \frac{\Delta TR}{\Delta Q} = P.E_c \quad (10)$$

$$\text{So that } \% \Delta TR = \frac{\Delta TR}{TR} = \frac{P.E_c . \Delta Q}{TR} \quad (11)$$

According to the exhibit, $P = 100$, $E_c = (1/2) = 0.5$, and $\Delta Q = (27 - 30) = -3$.

Therefore, substituting the above values, since the price change from $P = 100$ leads to a change in income as:

$$\% \Delta TR, = 100.(0.5).(-3) = -150$$

Hence, the approximate percentage change in TR

$$\% \Delta TR = \frac{\Delta TR}{TR} = \frac{-150}{(100)(30)} = \frac{-150}{3000} = -0.05 = -5\%$$

To find the percentage change in $TR = \% \Delta TR$, the equation (11) can be constructively used as a mathematical instrument as shown below.

$$\% \Delta TR = \frac{\Delta TR}{TR} = \frac{P.E_c . \Delta Q}{TR}$$

Equation (11) can be further simplified with $TR = P.Q$ as shown below.

$$\% \Delta TR = \frac{\Delta TR}{TR} = \frac{P.E_c . \Delta Q}{TR} = \frac{P.E_c . \Delta Q}{P.Q}$$

$$\% \Delta TR = \frac{E_c . \Delta Q}{Q} = E_c (\% \Delta Q) \quad (12)$$

In the above exhibit, $E_c = 0.5$ and $\% \Delta Q = (\Delta Q/Q) = (-3/30) = -0.1$.

Therefore, alternatively using equation (12) can result in

$$\% \Delta TR = E_c (\% \Delta Q) = (0.5)(-0.1) = -0.05 = -5\%.$$



Cover Page



Further, it is possible to substitute $TR = P \cdot Q$ and $\left(1 - \frac{1}{E_d}\right)$

$$\text{So that, } \% \Delta TR = \frac{P \cdot \left(1 - \frac{1}{E_d}\right) \Delta Q}{P \cdot Q} = \frac{\Delta Q}{Q} \left(1 - \frac{1}{E_d}\right) = \left(\frac{\Delta Q}{Q} + \frac{\Delta Q}{Q}\right)$$

$$\text{Since mathematically } E_d = \left(\frac{dQ}{dP}\right) + \left(\frac{P}{Q}\right) = -\left(\frac{\Delta Q}{\Delta P}\right) \left(\frac{P}{Q}\right)$$

$$\% \Delta TR = \frac{\Delta Q}{Q} - \frac{\Delta Q}{Q} \left[\frac{1}{\left(-\frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}\right)} \right] = \frac{\Delta Q}{Q} + \frac{\Delta Q}{Q} \left(\frac{\Delta P \cdot Q}{\Delta Q \cdot P} \right)$$

$$\% \Delta TR = \frac{\Delta Q}{Q} + \frac{\Delta P}{P} \quad (13)$$

$$\% \Delta TR = \% \Delta Q + \% \Delta P \quad (14)$$

In the exhibit, when price change from $P = 100$ to $P = 105$, then the change in quantity is moving from $Q = 30$ to $Q = 27$.

$$\text{Hence, } \% \Delta Q = \frac{27 - 30}{30} = -0.1 = -10\% \quad \text{and} \quad \% \Delta P = \frac{105 - 100}{100} = 0.05$$

Using equation (14), now we can determine the percentage change in revenue ($\% \Delta TR$):

Notably, for a change in price of a product, how the revenue of a firm can get affected and determined in relation to respective change in quantity and relative elasticity is shown through two formulas above:

$$\% \Delta TR = \frac{P \cdot E_c - \Delta Q}{TR} \quad (11)$$

$$\% \Delta TR = \left(1 - \frac{1}{E_d}\right) (\% \Delta Q) = E_c \cdot (\% \Delta Q) \quad (12)$$

$$\% \Delta TR = \% \Delta Q + \% \Delta P \quad (14)$$

It is also possible to explore how a price decrease can be useful to determine the percentage change in revenue ($\% \Delta TR$).

Now consider the same exhibit of demand function above: $Q_d = -0.6P + 90$ (in '000) with the initial quantity 30 (000). If the price falls by 10% and moves from $P = 100$ to $P = 90$, the respective quantity can increase from $Q = 30$ to $Q = 36$.

For using equation (11), the values of $P = 100$, $Q = 30$, $TR = (100) \cdot (30) = 3000$



Cover Page



$$E_c = \left(1 - \frac{1}{E_d}\right) \left(1 - \frac{1}{2}\right) = 0.5 \text{ and } \Delta Q = (36-30) = 6.$$

$$\text{Accordingly, } \% \Delta TR = \frac{P.E_c - \Delta Q}{TR} = \frac{(100).(0.5).(6)}{3000} = 0.1 = 10\%$$

In the above exhibit, $E_c = 0.5$ and $\% \Delta Q = (\Delta Q/Q) = (6/30) = 0.2$.

Therefore, alternatively, using equation (12) can result in
 $\%TR = E_c (\% \Delta Q) = (0.5)(0.2) = 0.10 = 10\%$.

Similarly, for using equation (14), the values of $\% \Delta Q = (\Delta Q/Q) = (36 - 30)/30 = 0.2 = 20\%$ and $\% \Delta P = (\Delta P/P) = (90 - 100)/100 = -0.1 = -10\%$ needs to be substituted.

Accordingly, .

From the above, the following can be concluded:

For an Elastic Product ($E_d > 1$), there is negative relationship between the price change and total revenue change for a product of a firm, i.e.,

Increase in price for the product leads to decrease in revenue of a firm; and

Decrease in price for the product leads to increase in revenue of a firm.

The same approach can be shown for the products with $E_d < 1$ and $E_d = 1$.

Suppose consider the demand function of a product $Q_d = -0.6P + 90$ (in '000) and the initial quantity is 60 (000).

Accordingly, it is known that the initial price of the product is:

$$60 = -0.6P + 90 \rightarrow P = (90 - 60)/0.6 = 50 \text{ and}$$

The respective elasticity is:

$$E_d = \left(\frac{dQ}{dP}\right) \left(\frac{P}{Q}\right) = (0.6) \cdot \left(\frac{50}{60}\right) = (0.6) \cdot \left(\frac{50}{60}\right) \rightarrow E_d = 0.5 \text{ (the product is inelastic).}$$

Now assume that the price increases from $P = 50$ to $P = 55$, and this leads to decrease in quantity from 60 ('000) to 57 ('000).

For using equation (11), the values of $P = 50$, $Q = 60$, $TR = (50) \cdot (60) = 3000$

$$E_c = \left(1 - \frac{1}{E_d}\right) = \left(1 - \frac{1}{0.5}\right) = (1 - 2) = -1, \text{ and } \Delta Q = (57-60) = -3.$$

$$\text{Accordingly, } \% \Delta TR = \frac{P.E_c - \Delta Q}{TR} = \frac{(50).(-1).(-3)}{3000} = 0.05 = 5\%$$

In the above exhibit, $E_c = -1$ and $\% \Delta Q = (\Delta Q/Q) = (-3/60) = -0.05$.

Therefore, alternatively, using equation (12) can result in

$$\%TR = E_c (\% \Delta Q) = (-1)(-0.05) = 0.05 = 5\%.$$

Similarly, in using equation (14), the values of $\% \Delta Q = (\Delta Q/Q) = (57 - 60)/60 = -0.05 = -5\%$ and $\% \Delta P = (\Delta P/P) = (55 - 50)/50 = 0.1 = 10\%$ needs to be substituted.



Cover Page



Accordingly,

Now assume that the price decreases from $P = 50$ to $P = 45$, and this leads to increase in quantity from 60 ('000) to 63 ('000).

For using equation (11), the values of $P = 50$, $Q = 60$, $TR = (50) \cdot (60) = 3000$ $E_c = \left(1 - \frac{1}{E_d}\right) = \left(1 - \frac{1}{0.5}\right) = (1 - 2) = -1$, and

$$\Delta Q = -0.05 = -5\%$$

$$\text{Accordingly, } \% \Delta TR = \frac{P \cdot E_c - \Delta Q}{TR} = \frac{(50) \cdot (-1) \cdot (-3)}{3000} = 0.05 = 5\%$$

In the above exhibit, $E_c = -1$ and $\% \Delta Q = (\Delta Q/Q) = (3/60) = 0.05$.

Therefore, alternatively, using equation (12) can result in

$$\%TR = E_c (\% \Delta Q) = (-1)(0.05) = -0.05 = -5\%.$$

Similarly, in using equation (14), the values of $\% \Delta Q = (\Delta Q/Q) = (63 - 60)/60 = 0.05 = 5\%$ and $\% \Delta P = (\Delta P/P) = (45 - 50)/50 = -0.1 = -10\%$ needs to be substituted.

$$\text{Accordingly, } \% \Delta TR = \% \Delta Q + \% \Delta P = (5\%) + (-10\%) = 5\%$$

From the above, the following can be concluded:

For an Inelastic Product ($E_d < 1$), there is positive relationship between the price change and total revenue change for a product of a firm, i.e.,

Increase in price for the product leads to increase in revenue of a firm; and

Decrease in price for the product leads to decrease in revenue of a firm.

Now consider the same demand function of a product $Q_d = -0.6P + 90$ (in '000) and the initial quantity is 45 ('000).

Accordingly, it is known that the initial price of the product is:

$$45 = -0.6P + 90 \rightarrow P = (90 - 45)/0.6 = 75 \text{ and}$$

The respective elasticity is:

$$E_d = \left(\frac{dQ}{dP}\right) \left(\frac{P}{Q}\right) = (0.6) \cdot \left(\frac{75}{45}\right) \rightarrow E_d = 1 \text{ (unit elasticity product).}$$

Now assume that the price increases from $P = 75$ to $P = 80$, and this leads to decrease in quantity from 45 ('000) to 42 ('000).

For using equation (11), the values of $P = 75$, $Q = 45$, $TR = (75) \cdot (45) = 3375$,

$$E_c = \left(1 - \frac{1}{E_d}\right) = \left(1 - \frac{1}{1}\right) = (1 - 1) = 0 \text{ and } \Delta Q = (42 - 45) = -3,$$

$$\text{Accordingly } \% \Delta TR = \frac{P \cdot E_c - \Delta Q}{TR} = \frac{(75) \cdot (0) \cdot (-3)}{3375} = 0\% \text{ (no change in total revenue).}$$

In the above exhibit, $E_c = 0$ and $\% \Delta Q = (\Delta Q/Q) = (-3/45) = 0.067$.

Therefore, alternatively, using equation (12) can result in

$$\%TR = E_c (\% \Delta Q) = (0)(0.067) = 0 = 0\%.$$



Cover Page



Similarly, in using equation (14), the values of $\% \Delta Q = (\Delta Q/Q) = (42 - 45)/45 = -0.0667 = -06.67\%$ and $\% \Delta P = (\Delta P/P) = (80 - 75)/75 = 0.0667 = 6.67\%$ needs to be substituted in it.

Accordingly,

From the above, the following can be concluded:

For a Unit Elastic Product ($E_d = 1$), there is no change in total revenue for the change in price of the product of a firm, i.e.,

Increase in price for the product leads to no change in revenue of a firm; and

Decrease in price for the product leads to no change in revenue of a firm.

Indicatively, the products with different elasticity measures are available in perfect/ pure competition market, where huge numbers of buyers and sellers are available. Also, the market has no product differentiation, since all products have almost the same features and the buyers have no choice at all to buy a product selectively. In these markets, no buyer or seller can change the price of a product.

CONCLUDING REMARKS

As the elasticity becomes a one of the important factors in determining a firm's revenue with respect to a change in price of its product, it is important for the firm to know, before taking a decision on changing the price of its product, how the proposed change in price can affect its total revenue from the product to be sold in the new market at the new price. In this context, the measure of elasticity is very important that reflect how the buyers will react to the change in price and the new price to come. In this context, the elasticity of the product becomes a crucial measure to reflect what the percentage of income the firm can gain or lose, when the price change takes place for its respective product.

This paper demonstrates in a new mathematically constructive approach, but existing accepted fact, how the elasticity of a product can affect a firm's total revenue. This paper illustrates with an exhibit of considering three different types of products with elastic ($E_d > 1$), inelastic ($E_d < 1$), and unit elastic ($E_d = 1$) nature.

The above said mathematical constructive method is also consistent with the existing accepted phenomena of elasticity that elastic product shows negative relationship between price change and change in total revenue, inelastic product can result in positive relationship between price change and change in total revenue, and unit elasticity product has no impact on change in total revenue as the response to a price change. In this context, this paper explores three mathematically constructive, but similar and alternative, methods for the existing phenomena how the percentage of change in total revenue can be determined with respect to elasticity, and current and new prices and their respective quantities.

References

1. Ahuja, H. L. (2009). **Advanced Economic Theory**, 17th Ed., S. Chand & Company Ltd, Ram Nagar, New Delhi.
2. Anderson, Duncan, et al. "A Practitioner's Guide to Generalized Linear Models." CAS Discussion Paper Program (2004): 1-115.
3. Brockman, Michael J. "Statistical Motor Rating: Making Effective Use of Your Data." *Journal of the Institute of Actuaries* (April 1992): 457-543.
4. Gans, J., King, S., Stonecash, R., and Mankiw. N. G. (2005). *Principles of Economics*, 3rd ed. Tax Implementation and Market New Equilibrium – An Alternative Method Integral Review- A Journal of Management, Vol. 6 No. 1, June 2013 Thomson, Nelson, Australia Pty
5. Hastie, T., Tibshirani, and J.H. Friedman. *The Elements of Statistical Learning*. Springer, 2001.
6. Hirschey, M. and Pappas, J. L. (1993). **Managerial Economics**, 8th Ed., The Dryden Press, Fort Worth. Limited.
7. Holler, Keith D., David Sommer, and Geoff Trahair. "Something Old, Something New in Classification Ratemaking with a Novel Use of GLMs for Credit Insurance." *CAS Forum* (Winter 1999): 31-84.
8. Mankiw. N. G. (2022). *Principles of Economics*, 4th ed. Thomson, South-Western, USA. 3. Senthilnathan, S. (2009). The application of price on y-axis and volume on x-axis in demand and supply analyses – is it a muddle? Working paper series, Social Science Research Network, Viewed on 05-03-2013
9. McCullagh, P., and J.A. Nelder. *Generalized Linear Models*. 2nd ed. Chapman and Hall, 1989.
10. Mildenhall, Steve. "Systematic Relationship Between Minimum Bias and Generalized Linear Models." *CAS Proceeding LXXXVI* (1999): 393-487.



Cover Page



11. Murphy, Karl P., Michael J. Brockman, and Peter K. Lee. "Using Generalized Linear Models to Build Dynamic Pricing Systems." CAS Forum (Winter 2000): 107-139.
12. Welch, G. F. and Welch, P. J. (2012). Economics: Theory and Practice, 10th Ed., John Wiley & Sons, New York.

Related Websites

13. Learneconomicsonline.com>archieives
14. <https://opentoicscbc.ca> chapter1to5
15. <https://www.worldscientific.com>.