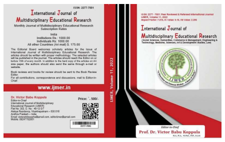
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ENHANCING MATHEMATICAL REASONING AND AESTHETIC SENSITIVITY OF STUDENTS USING FRACTAL GEOMETRY CONCEPTS

[1]Dr. Tara S. Nair and [2]Harsha M
[1&2]Assistant Professor
[1]Postgraduate Department of Education
[1]N.S.S. Training College and [2]Jameela Beevi Memorial Centre for Teacher Education
[1]Pandalam, Pathanamthitta and [2]Kayamkulam
Kerala, India

Abstract

Mathematics offers a way of doing things through problem solving in a systematic manner. Understanding when and how a mathematical technique is to be used always have importance as a basic skill. In modern times the teaching of Mathematics has shifted from knowing and doing approach rather than traditional algorithmic approach. Fractal geometry is a subfield of mathematics that has readily established powerful illustration in inquiry of irregular geometric objects with infinite nesting structures. This study explored the ability of students to assimilate knowledge, thoughts and emotions for identifying fractal patterns to comprehend mathematical concepts that enable them to solve problems. An Instructional Package considering individualized instruction, interaction, motivation, self pacing and interesting incorporating varied cognitive and aesthetic experiences with constructive feedback was developed on certain concepts of Fractal Geometry. The results reveal that learning fractal geometry concepts enhanced the mathematical reasoning and aesthetic sensitivity of students.

Keywords: Mathematics, Fractals, Fractal Geometry, Instructional Package, Sierpinski Triangle, Koch curve, Mathematical Reasoning, Aesthetic Sensitivity.

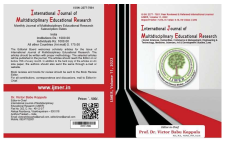
1. Introduction

An experimental science as old as civilization itself, Mathematics is one of the most useful and fascinating branches of human knowledge, a discipline that is still evolving and making new discoveries. It is the scientific study of quantities and its measurements, relationships and operations expressed as symbols and numbers. In the modern world it implies observations, data and inferences of the world we live quantifying structure, space and change. Mathematics education implies the practice of teaching and learning mathematics that enable solving problems through formulae and algorithms to perform computations and research. NCERT (2006) aims the teaching of Mathematics education in schools for the mathematization of the learner's thinking to evolve clarity of thought and pursue assumptions to logical conclusions. Being a compulsory subject of study in school education right from the foundational years, access to quality Mathematics education is every child's right which should be coherent, ambitious and important as stated by NCERT. The basic content areas of school mathematics like arithmetic, algebra, trigonometry, and geometry offer a methodology for abstraction, structuration and generalization inculcating habits of thought and communication.

2. Theoretical Overview

2.1 Fractal Geometry

Geometry is the branch of Mathematics concerned with the questions of shape, size, relative positions of figures and properties of space which aims to give a clear idea of the physical world we live in. It offers enormous applications in domestic activities, art, architecture, space study and astronomy. Of the different types of geometry that are studied namely algebraic geometry, complex geometry and Riemannian geometry, it is Fractal geometry that establishes connections with many branches of Mathematics including number theory, probability theory and dynamic systems. Mandelbrot (1957) defines 'Fractal Geometry' as the study of non-regular geometric shape that has the same degree of non-regularity at all scales. It deals with irregular geometrical objects with an infinite nesting of structure at all scales combining mathematics with visual images that are fascinating in the intricacy at the same time strikingly beautiful. Naturally occurring fractals features geographical objects, turbulent flow, human systems, and galaxies. Nevertheless, artificial fractals are seen in objects such as electrode position, electrolyte deposition and viscous fingering. The simplest fractal shape is the Sierpinski Triangle generated by connecting midpoints of sides of equilateral triangle, described by Polish mathematician Waclaw Sierpinski in 1915. Another example is the Koch snowflake curve following a simple process of iteration introduced by the Swedish mathematician Helge von Koch in 1904. Researchers have studied Fractal geometry and its dimensions in its varied levels (Simmt & Davis, 1999; Lornell & Aesterberg, 1999; Gleik, 1987; Bringer & Ury, 2002) and identified its applications in predicting natural disasters, malfunctioning of systems in the field of medicine and in modern digital devices and Wi-Fi.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2.2 Mathematical Reasoning

Reasoning plays a significant role in adjustment to one's environment. It depicts higher type thinking that is purposeful, systematic and organized. Central to mathematics learning, Mathematical Reasoning refers to the organization and employment of those habits of thought necessary to make logical deductions and to comprehend functional relationships (Good, 1959). It is a critical skill that enables students to make use of all other mathematical skills. Reasoning sequences in problem solving occurs as an implementation strategy (edges) connected by instances (vertices) which indicate a momentary state of knowledge with implicit subtasks that the reasoned formulates during the solving process (Lithner, 2008). He identified two types of reasoning: imitative and creative. Creative mathematically founded reasoning is featured by novelty, plausibility and mathematical foundation. As a critical skill, mathematical reasoning enables a student to utilize all other mathematical skills and that students' mathematical reasoning is still dominated by intuitive reasoning. The study by Venugopal (2012) found that there is a significant correlation between reasoning ability and achievement in mathematics of students. For the development of mathematical reasoning, it is inevitable to know the actual developmental level of students to accommodate them in their right schemas.

2.3 Aesthetic Sensitivity

Aesthetics is contrived as the critical reflection of art, culture and nature by scholars. In modern times it reflects the principles underlying the works of a particular art or theory. An experience beyond description of qualities of subtle beauty is understood as aesthetic sense of an individual. John Dewey in 'Logic of Inquiry' proposed new theory of art and aesthetic experience in daily events and scenes of life. Aesthetic Sensitivity refers to the ability to recognize something of aesthetic value and to discern the qualities that make it aesthetically sensitive. This enables the capability to perceive and appreciate beauty in all its forms. It takes time, practice and effort to develop aesthetic qualities as aspects of perception, intellectual thought and activity. As Hardy (1940) posits "a mathematician like a painter or poet is a maker of patterns. The mathematician's patterns like the painters' or poets' must be beautiful; the ideas like the colours and words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics." Kantian characterization of mathematical beauty visualizes creative intellectual processes as an emotional response that would lead to mathematical knowledge amidst experiencing strong sense of beauty. Adding importance to aesthetic images in Mathematics Education satisfies the questions of the very nature of Mathematics as a subject discipline, role of knowing through emotional responses and success for all individuals.

3. Rationale for the study

Fractals are representative patterns that display characteristics of scaling and self similarity. They are seen all over nature spanning a huge range of scales from the tiny branching of our blood vessels and neurons to the branching of trees, lightning bolts and river networks. Geometric fractals like Sierpinski triangle and Koch curve are made by repeating a simple process called iteration. Amazing fractals are extremely simple to make. Studying fractals is a topic of widespread interest in recent years as a powerful determinant of mathematical inquiry often unaddressed in school curriculum. It allows students to explore mathematical concepts by drawing pictures of constructive iteration of classical fractals in nature. Forecast of natural phenomena are now done based on the application of fractals for planning and responding to natural disasters. These are also used in medicine to examine the functioning of physiological systems. The study by Karakus (2012) identified fallacies in students' understanding of fractals and found that they encountered difficulties in identifying pattern rules. Experiencing something that personally affects our mind is valuable to improve efficiency in social life.

4. Objectives of the study

1. To prepare and validate an Instructional Package on Fractal Geometry based on the topics: Self- similarity, Koch curve, Sierpinski Triangle, Introduction to Complex numbers and Fractals in Nature for Standard IX students.
2. To test the effectiveness of the Instructional Package on Fractal Geometry for enhancing Mathematical Reasoning and Aesthetic Sensitivity of Secondary School Students by comparing the pre-test, post-test scores of total sample and sub sample based on gender.

5. Hypotheses of the study

1. When the treatment group is exposed to experimental teaching there will be significant difference in pre-test and post-test scores for Mathematical Reasoning of total sample and subsample based on gender.
2. When the treatment group is exposed to experimental teaching there will be significant difference in pre-test and post-test scores for Aesthetic Sensitivity of total sample and sub-sample based on gender.

6. Methodology

Experimental design was adopted in the study. 40 Ninth Standard students of a school from Alappuzha district in Kerala was selected for conducting the experiment aimed for testing the effectiveness of the Instructional Package based on Fractal Geometry for enhancing Mathematical Reasoning of students. Instructional Package on Fractal Geometry was the independent variable and Mathematical Reasoning as the dependent variable. The tools and learning materials used for the present study were: 1) Instructional Package on Fractal Geometry including topics in Fractal Geometry namely Self- similarity, Koch Snow Flake, Sierpinski Triangle, Introduction to Complex Numbers, and Fractals in the Nature. These lessons were organized into three phases: Introduction meant to establish a positive learning environment and connect to prior experiences, Presentation meant to set context for learning and to practice and apply learning; and Evaluation to provide opportunities for reflection and consolidation. Images, videos, Power Point presentations and activity cards were used to develop the concepts that ensure active participation and interest of students. 2) Test of Mathematical Reasoning including test items of the components namely mathematical logical thinking, mathematical connection, basic operation skills and problem solving skill. Content and face validity of the test was assured by expert validation. Reliability of the test was found to be 0.68. 3) Test of Aesthetic Sensitivity included items pertaining to the components namely, aesthetic awareness, aesthetic creativity, aesthetic imagination and sensitivity to aesthetic principles in mathematics. Both the tests contained 25 multiple choice questions.

7. Results and Discussion

7.1: Effectiveness of Instructional package on Fractal Geometry for enhancing Mathematical Reasoning of Secondary School students

Table 1: Test of Significance of difference between means of Pre-test and Post-test scores of Mathematical Reasoning

Treatment group	Mean	N	S.D.	r	CR	Level of Significance
TOTAL SAMPLE						
Pre-test	10.088	45	4.347	0.785	8.907	Significant at 0.05 level
Post-test	14.622	45	5.540			
BOYS						
Pre-test	9.656	32	4.942	0.862	6.639	Significant at 0.05 level
Post-test	12.823	32	5.322			
GIRLS						
Pre-test	11.154	13	2.115	0.543	10.475	Significant at 0.05 level
Post-test	19	13	3.118			

The above table shows that the critical ratio obtained for the total sample and subsample gender is greater than the table value 1.96 at 0.05 level of significance. The post-test mean scores are greater than the pre-test scores for all the samples. Hence it can be concluded that after teaching with the instructional package on Fractal Geometry, Mathematical Reasoning of Secondary School students has increased.

7.2: Effectiveness of Instructional Package on Fractal Geometry for enhancing Aesthetic Sensitivity

Table 2: Test of Significance of difference between means of Pre-test and Post-test scores of Aesthetic Sensitivity

Treatment group	Mean	N	S.D.	r	CR	Level of Significance
TOTAL SAMPLE						
Pre-test	11.888	45	2.234	0.672	11	Significant at 0.05 level
Post-test	16.288	45	3.382			
BOYS						
Pre-test	11.816	32	3.728	0.711	7.773	Significant at 0.05 level
Post-test	15.656	32	3.651			
GIRLS						
Pre-test	12.076	13	1.552	0.543	11.65	Significant at 0.05 level
Post-test	17.846	13	1.951			

Table 2 shows that the critical ratio obtained for the total sample and subsample gender is greater than the table value 1.96 at 0.05 level of significance. The post-test mean scores are greater than the pre-pest scores. Hence it can be concluded that the teaching of fractal geometry has enhanced the Aesthetic Sensitivity of students.

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7.3: Analysis of ratings of experts regarding the purposiveness of instructional package on Fractal Geometry

The number and percentage of ratings of experts in ten items of the rating scale are given below.

Table 3: Percentage of responses of experts for the items in the rating scale

Sl.No.	Items	% of responses		
		Great Extent	Some Extent	Low Extent
1	Fulfills curricular objectives	70	30	0
2	Suits normal classroom	80	20	0
3	Enhances mathematical reasoning	90	10	0
4	Simple and easy to learn	90	10	0
5	Enhances aesthetic sensitivity	90	10	0
6	Follows maxims of teaching	100	0	0
7	Minimizes stress in learning	80	20	0
8	Economical in implementation and assessment	70	30	0
9	Content is relevant	90	10	0
10	Concepts learnt could be applied in real and novel contexts.	90	10	0

From the above table it can be inferred that all the expert teachers are of the opinion that organization of the content follows maxims of teaching. 90% noted that the Instructional package is suitable for enhancing mathematical reasoning, simple and easy to learn, appropriate for developing aesthetic sensitivity, content is relevant and concepts learned could be applied in real and novel contexts. 80% have the opinion that the package suits normal classrooms and minimizes stress in learning. 70% said that the package fulfils curricular objectives and is economical in implementing and assessment.

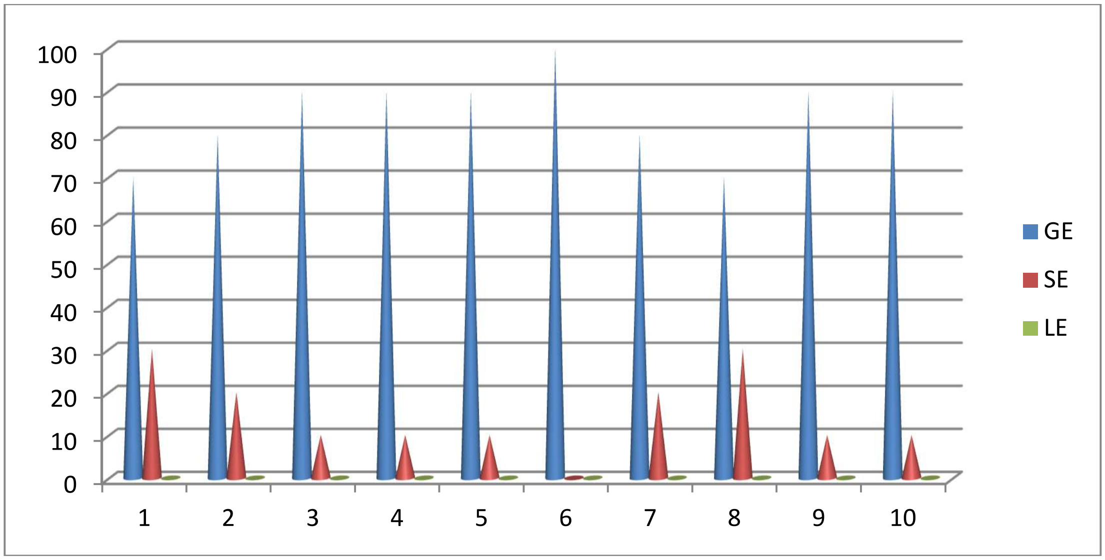
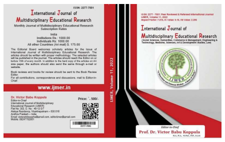


Figure 1: Graphical representation of expert opinion regarding purposiveness of Instructional package on Fractal Geometry

8. Conclusion

Teaching through the instructional package on Fractal Geometry is effective for enhancing mathematical reasoning and aesthetic sensitivity of secondary school students. The findings agree with the study of Karakus (2013) in that students can identify and determine fractal patterns and mathematical operations with them. This also substantiates the study by Muller (2014) in that students can learn with practice and careful analysis of task design to improve mathematical reasoning to higher level. The study reveals that learning fractal geometry offers applicability in mathematics education to exercise the development of faculties namely intuition, ingenuity and sensibility. It would enable students to appreciate the beauty in mathematical concepts and encourage them to improve their sensibility to visualize complex principles and theorems in an easy interesting way. Also, would teachers instruct in meaningful ways that are both appealing and everlasting.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